第四届全国冷原子物理和量子信息青年学者学术讨论会

原子分子转化系统的BERRY相位和分数磁单极

傅立斌

北京应用物理与计算数学研究所

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OUTLINE

+ Introduction

  -Review of Berry phase and conception of monopole

+ Berry phase of atom-molecule conversion system

  -Berry phase under mean-field treatment

  -Fractional monopole

  -Many-body effect of Berry phase

  -Hannay angle

+ Summary and discussion

WHAT'S THE BERRY IDEA

- Suppose that the Hamiltonian H(r) depends on a set of parameters

$$\mathrm{R} = (r_1, r_2, \ldots, r_d)$$

- For each value of R, the following eigenvalue equation is valid:

$$H(\mathrm{R})\left|n(R)\right\rangle = E(R)\left|n(R)\right\rangle$$

If the parameters r change slowly in time, the adiabatic theorem assures that the system remains in the eigenstate of the Hamiltonian, i.e.

$$\left|\psi(0)\right\rangle \propto \left|n(R(0))\right\rangle \quad \Rightarrow \quad \left|\psi(t)\right\rangle \propto \left|n(R(t))\right\rangle$$

- What if for same value of t=T: R(T)=R(0)?

Certainly:

$$\boxed{\left|\psi(T)\right\rangle \propto \left|\psi(0)\right\rangle}$$

BERRY PHASE

M. BERRY, PROC. ROY. SOC. A 392, 45 (1984)

Then the state returns to its initial form but since eigenstates are defined up to a phase factor, the state could acquire a phase due to the **adiabatic** and **cyclic** evolution that took place.

$$\left|\Psi(t)\right\rangle = e^{i\beta_n(t)} e^{i\gamma_n(t)} \left|\Psi(0)\right\rangle$$

$$\beta_n(T) = -\int_0^T E_n(t)dt$$

dynamical phase

$$\gamma_n(T) = i\oint \left\langle n(R(t)) \left| \frac{d}{dt} \right| n(R(t)) \right\rangle dt$$

$$= \oint A \cdot dR$$

Geometric phase

$$A(R) = \left\langle n(R) \left| \frac{d}{dR} \right| n(R) \right\rangle$$

Berry connection

Developments of Berry phase

Frank Wilczek and A. Zee* **Phys. Rev. Lett. 52, 2111–2114 (1984)**
"Appearance of Gauge Structure in Simple Dynamical Systems"

The conception was generalized to degenerate system

Y. Aharonov and J. Anandan **Phys. Rev. Lett. 58, 1593–1596 (1987)**
Phase change during a cyclic quantum evolution

The conception was generalized to non-adiabatic evolution, so-called AA phase

Berry phase and AA phase are all geometric phases, but when one mentions the geometric phase, it often means AA phase.

What is meant by "geometrical" in the geometric phase?

A physically observable feature of a quantum system that depends only on the path described during its evolution.

The connections

For a local basis of a manifold, $\mathrm{e}_{\mu}(x)$

The connections build up the relations between local basis of one point and its neighbor

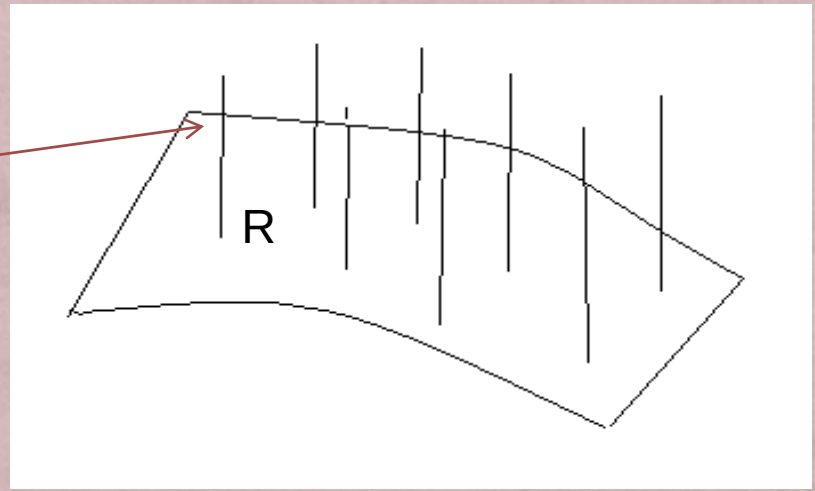
$$\Gamma_{\mu\nu} = \left[\left(\mathrm{e}_{\mu}(x+\mathrm{d}x), \mathrm{e}_{\nu}(x)\right) - \delta_{\mu\nu}\right] / dx$$

So given a set of connections just define a rule to transport basis on the manifold

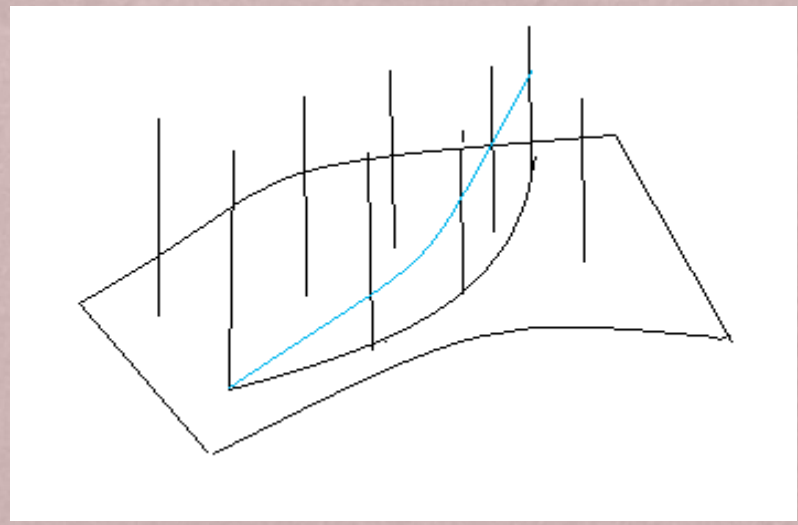
- Indeed the eigen equation build up a line bundle in parameter space

$$H(\mathrm{R})\left|n(R)\right\rangle = E(R)\left|n(R)\right\rangle$$

$$\mathrm{e}^{i\lambda}\left|n(R)\right\rangle$$



The adiabatic evolution of quantum system just yields a way of transport eigen vector along the given path in parameter space，so it defines a connection.



The connections

For a local basis of a manifold, $\mathrm{e}_{\mu}(x)$

The natural connections is just

$$\Gamma_{\mu\nu} = \left(\mathrm{e}_{\mu}(x), \mathrm{d}\mathrm{e}_{\nu}(x)\right)$$

Berry connections are just the natural connection of eigenvectors (B. Simon PRL 51, 2167 （1983）)

$$A_{\mathrm{nm}}(R) = \left\langle \mathrm{n}(R) \middle| d \middle| m(R) \right\rangle$$

Gauge potential and Berry curvature

The connection of bundle is gauge potential

For a U(1) transformation $\mathrm{e}^{i\lambda(R)}\left|n(R)\right\rangle$

$$\mathrm{A}_{\mathrm{n}}(R) \to A_n(R) - id\lambda(R)$$

The differential two form give rise to Berry curvature

$$\mathrm{F}^{\mathrm{n}}{}_{\mu\nu}(R) = \partial_\mu A_\nu{}^n(R) - \partial_\nu A_\mu{}^n(R)$$

With the Stocks formula, we have

$$\gamma^n = \oint_C A \cdot dR = \int_S \mathrm{F} \cdot d\sigma$$

where $\mathrm{C} = \partial S$

Gauss-Bonnet-Chern Theorem

For a smooth, oriented compact manifold, the integral of curvature over the whole manifold will give rise to Chern number and its equals to Euler characteristic of the Manifold.

$$\frac{1}{2\pi}\int_S \mathrm{F}\cdot d\sigma = \mathrm{g}\chi$$

Here g is the charge and χ is the Euler characteristic, which can be obtained with relation

$$\chi = \mathrm{V} - E + F$$

where *V*, *E*, and *F* are respectively the numbers of vertices (corners), edges and faces in the polyhedron, which is topology homeomorphism with the manifold. Therefore ,for sphere it is 2.

Name	Image	Vertices *V*	Edges *E*	Faces *F*	Euler characteristic: *V* − *E* + *F*
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

MONOPOLE AND DIRAC STRING

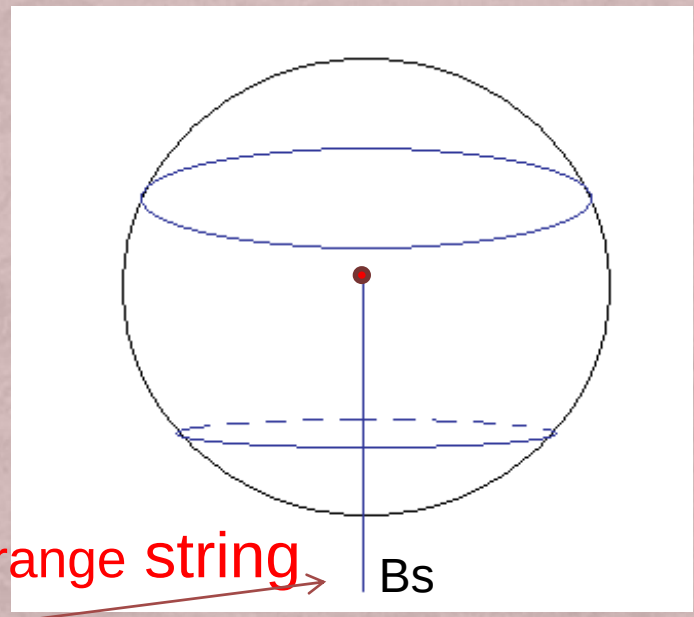
Monopole is an isolated charge with magnetic charge g. If it exist, its vector potential should be

$$\mathrm{A} = \frac{\mathrm{g}(1-\cos\theta)}{R\sin\theta}\hat{e}_{\varphi}$$

Then its field should be

$$\mathrm{B} = g\frac{\vec{R}}{R^3} + \mathrm{B}_s$$

$$\mathrm{B}_s = -4\pi g\delta(X)\delta(Y)\vec{e}_z \quad \text{for Z<0}$$



Indeed, Strange string guarantees the loop integral of gauge potential is unique, since each closed path relates with two surfaces, upper half sphere or under one. Dirac argue that it's the removable singularity of vector potential, so it should give nothing to electron motion, so

$$4\pi\mathrm{eg} = 2\pi \quad \Rightarrow \quad \mathrm{eg} = \frac{1}{2}$$

Berry phase of two level system and virtual monopole

The generic two level system

$$\mathrm{H} = -\vec{R}\cdot\vec{\sigma}, \quad \vec{R} = R(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

It has two eigenstates $|\mathrm{n}_-\rangle = \begin{pmatrix} \sin\frac{\theta}{2}e^{-i\phi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$, $|\mathrm{n}_+\rangle = \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix}$

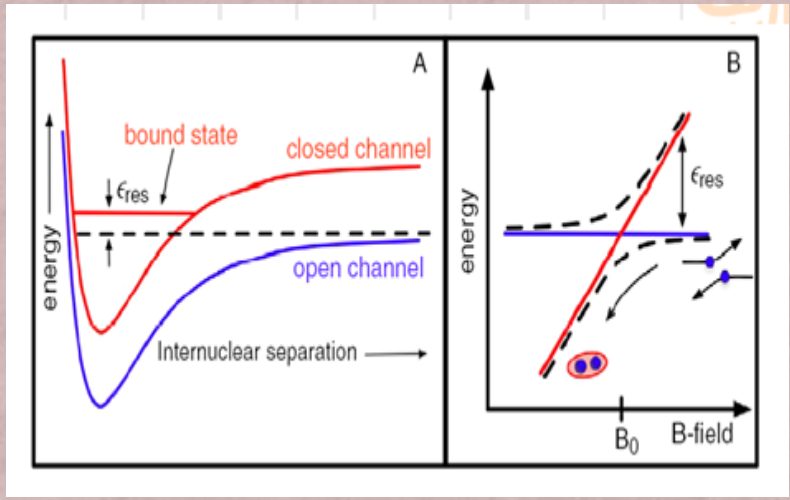
The Berry connection and curvature corresponding to ground state are

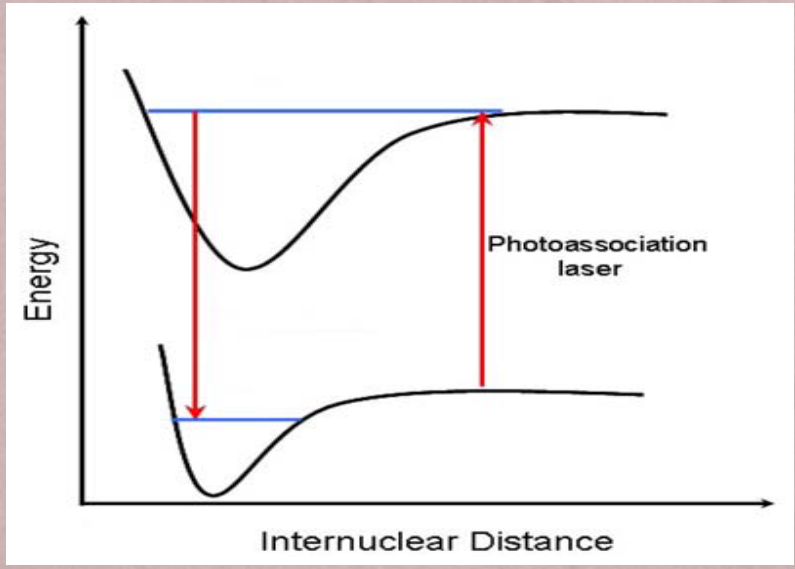
$$\vec{A} = \langle n_- | \nabla | n_- \rangle = \frac{1-\cos\theta}{2\mathrm{R}\sin\theta}\vec{e}_\phi, \quad \vec{\mathrm{F}} = \frac{\vec{\mathrm{R}}}{2\mathrm{R}^3}$$

Obviously, it's a field of monopole with charge g=1/2. Here we have ignored the strange string, which is dependent on specific gauge.

ATOM MOLECULE CONVERSION

Association of ultracold atoms into molecules is currently a topic of much experimental and theoretical interest with important applications ranging from the search for the permanent electric dipole moment to BCS-BEC crossover physics . Through Feshbach resonance or photoassociation , a pair of atoms can convert into a bounded molecule.





The simplest mode: single mode model

$$\mathcal{H} = \frac{R\cos\theta}{2}\left(\hat{\psi}_1^\dagger\hat{\psi}_1 - \hat{\psi}_2^\dagger\hat{\psi}_2\right) + \sqrt{\frac{3}{8}}\frac{R\sin\theta}{2}\left(e^{-i\phi}\hat{\psi}_1^\dagger\hat{\psi}_1^\dagger\hat{\psi}_2 + h.c.\right)$$

(ψ_1, ψ_2) and (ψ_1^+, ψ_2^+) are the annihilation and creation operators for atom and molecule respectively.

It is invariant under co-diagonal *U(1) transformation*

$$U(\eta) = e^{i\Theta(\eta)} \qquad \Theta(\eta) = \begin{pmatrix} \eta & 0 \\ 0 & 2\eta \end{pmatrix}$$

Mean-field approximation

Under mean-field treatment, we obtain the following equations

$$i\frac{d}{dt}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = H(\psi,\psi^*;\mathbf{R})\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$H(\psi,\psi^*;\mathbf{R}) = \begin{pmatrix} \frac{R\cos\theta}{2} & \sqrt{\frac{3}{8}}e^{-i\varphi}R\sin\theta\psi_1^* \\ \sqrt{\frac{3}{8}}e^{i\varphi}R\sin\theta\psi_1/2 & -\frac{R\cos\theta}{2}\end{pmatrix}$$

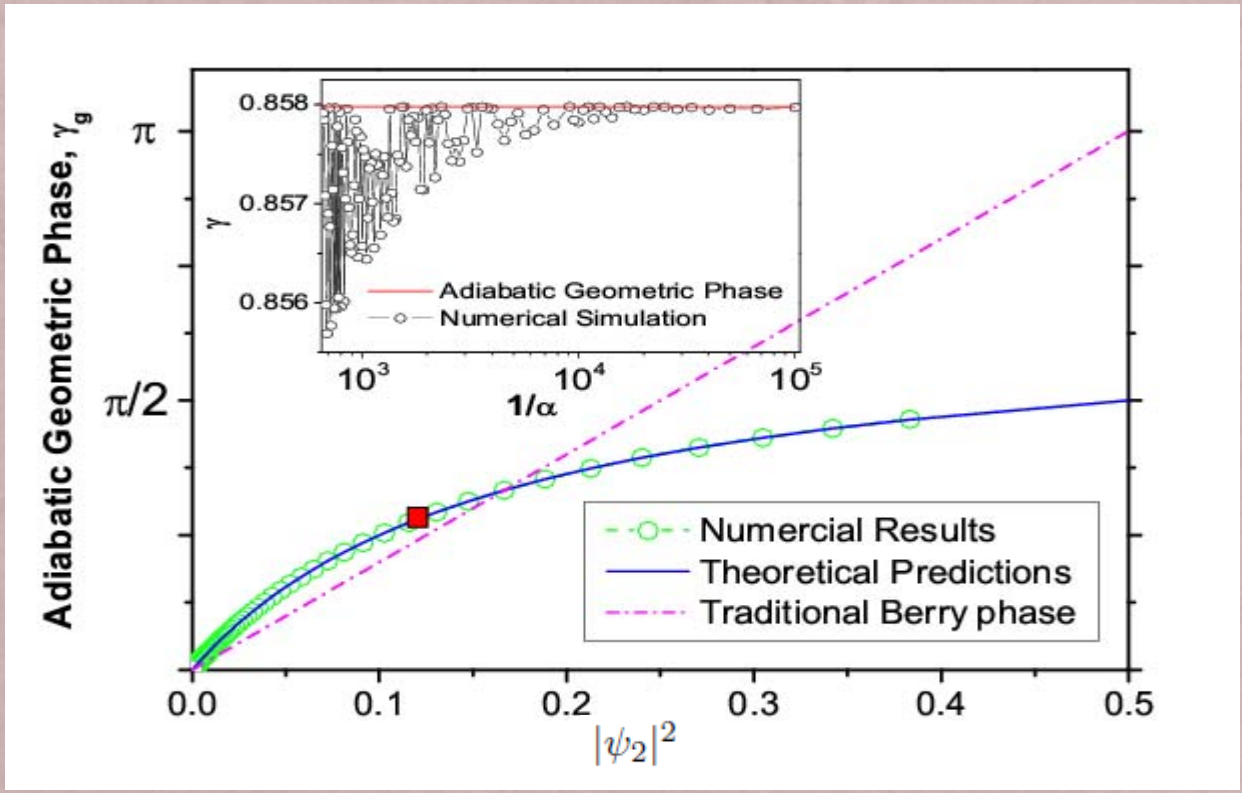
Its eigen equations are

$$H(\overline{\phi},\overline{\phi}^*;\mathbf{R})\begin{pmatrix}\overline{\phi}_1\\ \overline{\phi}_2\end{pmatrix} = \begin{pmatrix}\mu & 0\\ 0 & 2\mu\end{pmatrix}\begin{pmatrix}\overline{\phi}_1\\ \overline{\phi}_2\end{pmatrix}$$

$$\overline{\phi}_2^{\pm} = \frac{(-\cos\theta\pm 1)}{\sqrt{6}\sin\theta}, \overline{\phi}_1^{\pm} = e^{i\varphi/2}\sqrt{1-2|\overline{\phi}_2^{\pm}|^2},$$

Berry Formula fails to give the geometric phase in adiabatic limit for this system

Consider φ varies slowly with time, we calculate the geometric phase and comparing it with Berry formula we find it does not fulfill all the phase shift.



Calculating the geometric phase

For simplicity and without losing generality we denote $\lambda = \arg\psi_1$ and $q = -\arg\psi_2 + 2\arg\psi_1$ $p = |\psi_2|^2$ then from the Shrodinger Eq.

$$i\frac{d}{dt}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = H(\psi,\psi^*;\mathbf{R})\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$H(\psi,\psi^*;\mathrm{R}) = \begin{pmatrix}\frac{R\cos\theta}{2} & \sqrt{\frac{3}{8}}e^{-i\varphi}R\sin\theta\psi_1^*\\ \sqrt{\frac{3}{8}}e^{i\varphi}R\sin\theta\psi_1/2 & -\frac{R\cos\theta}{2}\end{pmatrix}$$

We get

$$\begin{aligned}\frac{d\lambda}{dt} &= p\frac{dq}{dt} - \mathcal{H}(p,q) - \Lambda(p,q),\\ \dot{p} &= -\frac{\partial\mathcal{H}}{\partial q}, \dot{q} = \frac{\partial\mathcal{H}}{\partial p},\end{aligned}$$

$$\mathcal{H}(p,q) = \tfrac{Z}{2}(1-3p) + \rho(1-2p)\sqrt{p}\cos(q-\varphi)$$

$$\Lambda(p,q) = \tfrac{\rho}{2}(1-2p)\sqrt{p}\cos(q-\varphi).$$

We assuming

$$p = \overline{p}(\mathbf{R}) + \delta p \text{ and } q = \overline{q}(\mathbf{R}) + \delta q$$

Here $\overline{p}(\mathbf{R})$ and $\overline{q}(\mathbf{R})$ are corresponding to eigen vector. Then we get

$$\frac{d\lambda}{dt} = -\mu(\mathbf{R}) + \overline{p}\frac{d\overline{q}}{d\mathbf{R}} \cdot \frac{d\mathbf{R}}{dt} - \frac{\rho}{2}\frac{(1-6\overline{p})}{2\sqrt{\overline{p}}}\delta p + o(\alpha^2)$$

$$\delta p = -\frac{1}{\rho}\frac{2\overline{p}\sqrt{\overline{p}}}{(1+6\overline{p})}\dot{\mathbf{R}}$$

$$\lambda = -(\gamma_d + \gamma_g)$$

$$\gamma_d = \int \mu dt$$

$$\gamma_g = \oint \overline{p} d\varphi + \oint \frac{(1-6\overline{p})\overline{p}}{1+6\overline{p}} d\varphi$$

Rewrite in terms of gauge potential

$$\begin{aligned}\gamma_g &= \oint \overline{p} d\varphi + \oint \frac{(1-6\overline{p})\overline{p}}{1+6\overline{p}} d\varphi \\ &= \frac{1}{6} \oint \left(1 - \frac{Z}{\sqrt{\frac{8}{3}\rho^2 + Z^2}}\right) d\varphi,\end{aligned}$$

$$\gamma_{\mathrm{g}} = \oint_C A \cdot dR$$

$$\mathbf{A} = \frac{1}{6\rho}\left(1 - \frac{Z}{\sqrt{\frac{8}{3}\rho^2 + Z^2}}\right)\widehat{e}_{\varphi}$$

For a co-diagonal U(1) transformation $\mathrm{e}^{i\eta[\lambda(R)]}\left|n(R)\right\rangle$

$$\mathrm{A}(R) \rightarrow A(R) - id\lambda(R)$$

Let us re-scale the parameter space as

$$R' \quad = \quad \left(X' = \sqrt{\tfrac{8}{3}}X, Y' = \sqrt{\tfrac{8}{3}}Y, Z' = Z\right)$$

$$\mathbf{A}' \quad = \quad \frac{1}{6}\left(1 - \frac{Z'}{\sqrt{X'^2+Y'^2+Z'^2}}\right)\frac{X'\hat{e}_{Y'} - Y'\hat{e}_{X'}}{X'^2+Y'^2}$$

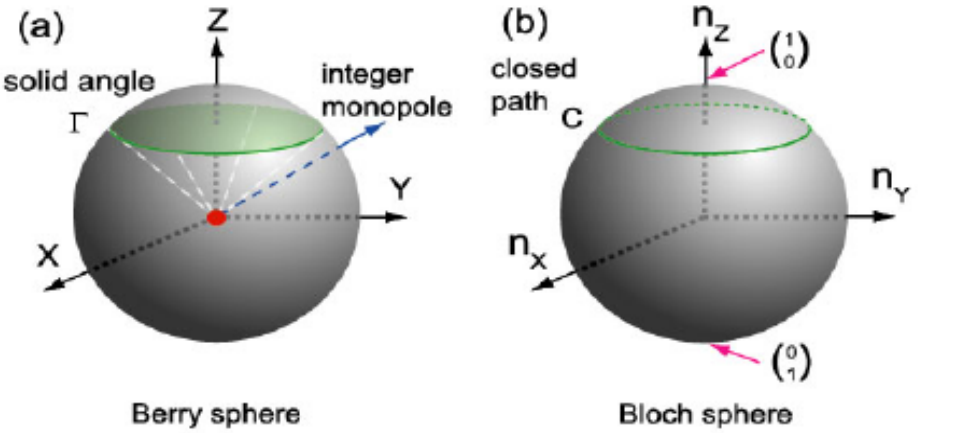
$$\mathbf{B} = \bigtriangledown \times \mathbf{A}' = g\frac{\mathbf{R}'}{R'^3}, \quad g = \frac{1}{6}$$

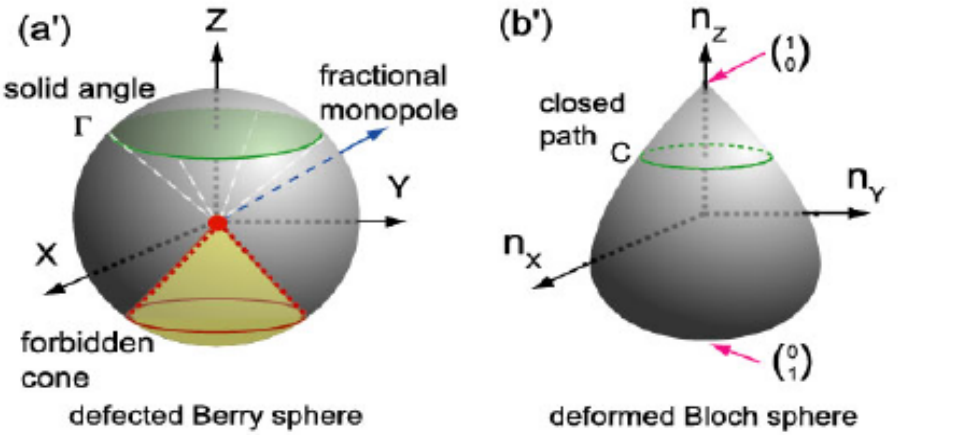
Obviously it is the field of monopole with charge

$$g = \frac{1}{3}g_0$$

$g_0 = 1/2$ is the elementary charge of monopole.

Comparing with conventional two level model





Two level system

Bloch vector is

$$\mathrm{n} = (2\mathrm{Re}[\psi_1^{*}\psi_2], 2\mathrm{Im}[\psi_1^{*}\psi_2], |\psi_2|^2 - |\psi_1|^2)$$

Atom-molecule system

Bloch vector is

$$\mathbf{n}_a = (2\sqrt{2}\, Re[(\psi_1^*)^2\psi_2],$$

$$2\sqrt{2} Im((\psi_1^*)^2\psi_2), |\psi_1|^2 - 2|\psi_2|^2)$$

The Berry sphere of this system is a broken sphere with a forbidden cone (yellow region) bounded by $\theta = \frac{2\pi}{3}$

For the two level system, the parameter space is a compact 3-D manifold, so

$$\frac{1}{2\pi}\int_S \mathrm{B}\cdot d\sigma / g_0 = \chi = 2$$

However, for the atom-molecule system, the parameter space is a 3-D manifold with boundary, its topology is equivalent with a 2-D surface. And we find

$$\frac{1}{2\pi}\int_S \mathrm{B}\cdot d\sigma / g = \chi + \frac{1}{2} = 1 + \frac{1}{2}$$

Here $\chi = 1$ is the topology index of 2-D surface.

The term ½ may contain the geometric property of the boundary.

Many-body effect of Berry phase

For the quantum system

$$\hat{H} = \frac{R\cos\theta}{2}\left(\hat{\psi}_1^\dagger\hat{\psi}_1 - \hat{\psi}_2^\dagger\hat{\psi}_2\right) + \sqrt{\frac{3}{8}}\frac{R\sin\theta}{2}\left(e^{-i\phi}\hat{\psi}_1^\dagger\hat{\psi}_1^\dagger\hat{\psi}_2 + h.c.\right)$$

By choosing the Fock state

$$|N-2m, m\rangle = \frac{(\hat{\psi}_1^\dagger)^{N-2m}}{\sqrt{(N-2m)!}}\frac{(\hat{\psi}_2^\dagger)^m}{\sqrt{m!}}|0\rangle$$

We can get the matrix elements

$$H_{ij} = \langle N-2i, i|\,\hat{H}\,|N-2j, j\rangle$$

The ground state for $\varphi = 0$ is

$$|g(\varphi = 0)\rangle = \sum_{m=0}^{N/2} C_m(\theta)|N - 2m, m\rangle$$

with $C_m(\theta)$ are real numbers.

Then for other φ

$$|g(\varphi)\rangle = \sum_{m=0}^{N/2} C_m e^{i\varphi(N-2m)/2}|N - 2m, m\rangle$$

Using Berry's formula, we get

$$A_\theta^N = 0, A_\varphi^N = \frac{1}{2}\sum_{m=0}^{N/2}(N - 2m){C_m}^2$$

The total phase acquired during adiabatic evolution is

$$\lambda^N = \int E^N(t)dt + \int A^N_\varphi d\varphi$$

In mean-field treatment $|g\rangle = (|\phi\rangle)^{\otimes N}$

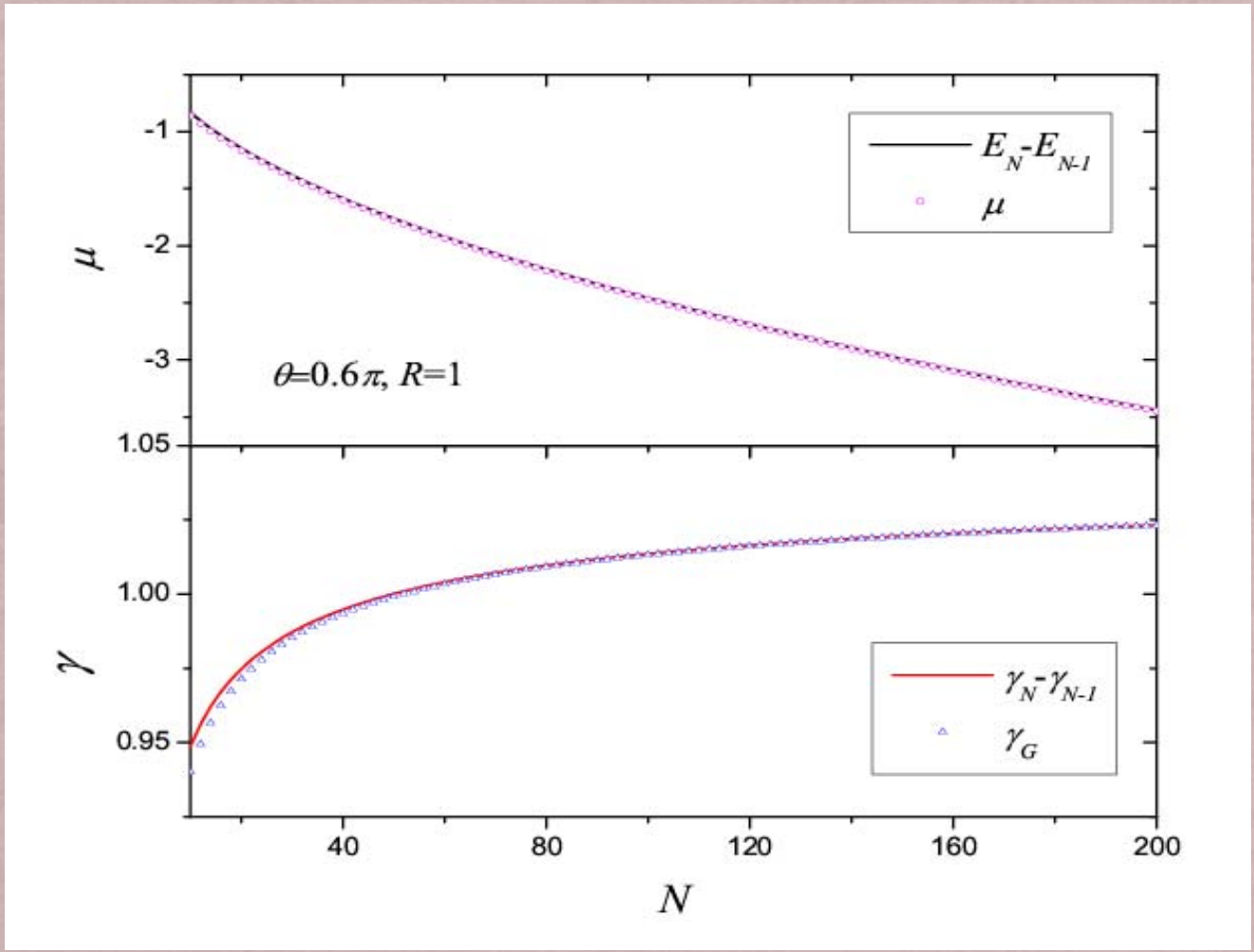
with single particle wave function $|\phi\rangle$

Thus we expect

$$\mu = \frac{\partial E^N}{\partial N}, \quad \mathbf{A} = \frac{\partial \mathbf{A}^N}{\partial N}$$

Therefore $\gamma_g = \frac{\partial \gamma_g^N}{\partial N}$

The above results are verified by numerical simulation



HANNAY ANGLE

For a classical integrable system $\mathrm{H}(p,q;R)$, $\mathrm{p}=\{\mathrm{p}_i\}$ $q=\{q_i\}$

One can make a canonical transformation to action-agnle variables with generating function $\mathrm{S}(q,I;R)$

$$\{p,q\}\leftarrow S(q,I;R)\rightarrow\{\theta,\ \ I\},$$

$$p=\frac{\partial S}{q},\quad \theta=\frac{\partial S}{\partial I}.$$

And new Hamiltonian takes the form

$$\overline{H}(\theta,I;R)=H(\theta,I;\ \ \mathrm{R})+\frac{\partial S}{\partial R}\frac{\mathrm{d}R}{dt}$$

$$\frac{dI}{dt}=0,\quad \frac{d\theta}{dt}=\frac{\partial\overline{H}}{\partial I}$$

For system is time independent, one has

$$\frac{\mathrm{d}\theta}{dt} = \frac{\partial H}{\partial I} = \omega(I;R)$$

Consider R=R(t) varies with time slowly and R(T)=R(0). In the classical adiabatic evolution, the actions are invariant, but the angles should be

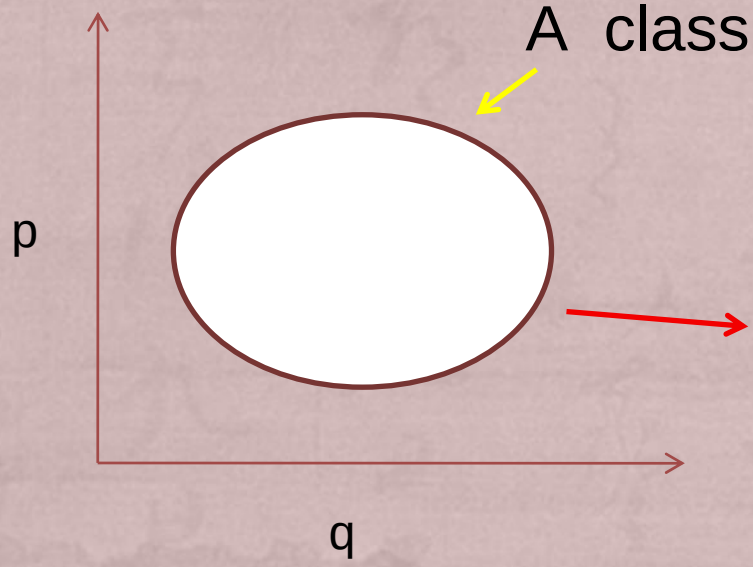
$$\theta_{\mathrm{i}} = \theta_i(0) + \int_0^T \omega_i(\mathrm{I}, R(t))dt + \Delta\theta_i(I;C)$$

where $\Delta\theta_i(I;C)$ is the so-call Hannay angles, which is only dependent on the path C.

$$\overline{H}(\theta, I; R) = H(\theta, I;\ \mathrm{R}) + \frac{\partial S}{\partial R}\frac{\mathrm{d}R}{dt}$$

$$\frac{dI}{dt} = 0, \quad \frac{d\theta}{dt} = \frac{\partial \overline{H}}{\partial I} \quad \Rightarrow \quad \Delta\theta_i(I;C) = \oint_C \frac{\partial}{\partial I_i}\left(\frac{\partial S}{\partial R}\right)dR$$

For a given R



$$\frac{1}{2\pi}\oint \mathrm{p}dq = (n+1/2)\hbar$$

Its counterpart in quantum description is an eigenstate

One can have hannay angle in classical description
And have Berry phase in quantum description

Berry pointed out in semi-classical limit, the Hannay angle and Berry phase have derivative relation

$$\Delta\theta\ (\mathrm{I}) = \frac{\partial\gamma_n}{\partial n}$$

MV Berry J PHYS A 18, 15(1985); JH Hannay J PHYS. A 18, 221(1985)

For a finite levels quantum system, its dynamic can be exactly casted Into a classical integrable Hamiltonian system. For example, a two-level system,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} z & \rho e^{-i\varphi} \\ \rho e^{i\varphi} & -z \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$q_1 = \text{-arg}(a) \quad \text{p}_1 = |a|^2 + |b|^2 \qquad q_2 = \arg(b) - \arg(a) \quad p_2 = |b|^2$$

$$\text{H}(p, q; R) = z(p_1 - 2p_2) + 2\rho\sqrt{(p_1 - p_2)p_2}\cos(q - \varphi)$$

$$\frac{\text{d}q_1}{dt} = p_2\frac{dq_2}{dt} - H(p, q; R)$$

One can obtain $$q_1 = \int_0^T E(t)dt + \gamma_B$$

It so happens $$\gamma_B = -\Delta\theta_2(I_2 = 0, C) = \oint_C pdq$$

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Exact integrability of the two-level system: Berry's phase and nonadiabatic corrections

Nilanjana Datta and Gautam Ghosh
Saha Institute of Nuclear Physics, 92, Acharya Prafulla Chandra Road, Calcutta 700 009, India

M. H. Engineer
Physics Department, Bose Institute, 93/1 Acharya Prafulla Chandra Road, Calcutta 700 009, India
(Received 25 August 1988)

The general time-dependent quantum two-level system is shown to admit an exact invariant. This leads to a classical formulation of the problem, which is then solved by standard techniques of Hamiltonian mechanics. Berry's phase and nonadiabatic corrections to it emerge as an asymptotic limit of the exact dynamics. A novel connection between Berry's phase and Hannay's angle is shown to exist in this case.

$$H=\varepsilon|1\rangle\langle 1|-\varepsilon|2\rangle\langle 2|+\gamma|1\rangle\langle 2|+\gamma^*|2\rangle\langle 1|$$

$$|\psi\rangle\ (=C_1|1\rangle+C_2|2\rangle)$$

$$L=(1/2\gamma^*)(\dot{C}_2-i\varepsilon C_2/\hbar)^2-\gamma C_2^2/(2\hbar^2)\ ,$$

$$p\equiv\frac{\partial L}{\partial\dot{C}_2}=(1/\gamma^*)(\dot{C}_2-i\varepsilon_2 C_2/\hbar)\ ,$$

$$H=(1/2)(\gamma^* p^2+2i\varepsilon p C_2/\hbar+\gamma C_2^2/\hbar^2)\ .$$

where an overdot indicates time differentiation. The corresponding equation for C_1 is obtained by the replacement $\gamma^*\rightarrow\gamma$, $\varepsilon\rightarrow-\varepsilon$.

HANNAY ANGLE FOR OUR MEDEL

$$\mathcal{H} = A\left(|\psi_1|^2 - |\psi_2|^2\right) + B|\psi_1|^2|\psi_2|\cos\left(\zeta_2 - 2\xi_1 - \varphi\right),$$

$\zeta_i = \arg(\psi_i)$

$q_1 = \zeta_1, \qquad p_1 = |\psi_1|^2 + 2|\psi_2|^2$

$p_2 = |\psi_2|^2$, and $q_2 = \xi_2 - 2\xi_1$

$$\mathcal{H}(p_1; p_2, q_2) = A\left(p_1 - 3p_2\right) + B(p_1 - 2p_2)\sqrt{p_2}\cos\left(q_2 - \varphi\right)$$

We then expand the Hamiltonian around $(\overline{p}_2, \overline{q}_2)$

$$\mathcal{H}^d(p_1; p_2, q_2) = \mathcal{H}_0(p_1) + T(p_1)\delta p + G(p_1)\delta p^2 + F(p_1)\delta q^2.$$

$$T(p_1) = \left.\frac{\partial \mathcal{H}(p_1; p_2, q_2)}{\partial p_2}\right|_{\overline{p}_2, \overline{q}_2} = B(p_1 - 1)/2\sqrt{\overline{p}_2},$$

$$G(p_1) = \left.\frac{\partial^2 \mathcal{H}(p_1; p_2, q_2)}{\partial p_2^2}\right|_{\overline{p}_2, \overline{q}_2} = -B(p_1 + 6\overline{p}_2)/4(\overline{p}_2)^{3/2}$$

$$F(p_1) = \left.\frac{\partial^2 \mathcal{H}(p_1; p_2, q_2)}{\partial q_2^2}\right|_{\overline{p}_2, \overline{q}_2} = -B(p_1 - 2\overline{p}_2)\sqrt{\overline{p}_2}.$$

$$S_1 = lq_1,$$

$$S_2 = S_g + \left(\overline{p}_2 - \frac{T}{2G}\right)(q_2 - \varphi)$$

$$S_g = \int_{\varphi}^{q_2} \left[\frac{\overline{E}}{G} - \frac{F}{G}(q_2 - \varphi)^2\right]^{1/2} dq_2 = 2J_2 \int_0^{\Gamma(q_2)} \left[1 - x^2\right]^{1/2} dx,$$

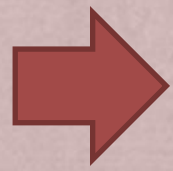
$$\Gamma(q_2) = \frac{\sqrt{F}}{\sqrt{E - \mathcal{H}_0 + \frac{T^2}{4G}}}(q_2 - \varphi).$$

$$\overline{\mathcal{H}}\left(J_1, \theta_1; J_2, \theta_2\right) = \mathcal{H}\left(J_1, J_2\right) + \frac{\partial S(J_1, J_2, q_1, q_2; \varphi)}{\partial t},$$

$$\mathcal{H}\left(J_1, J_2\right) = 2\left[F\left(J_1\right) G(J_1)\right]^{1/2} J_2 + \mathcal{H}_0(J_1) - \frac{T^2(J_1)}{4G(J_1)}$$

The Hannay angle is then obtained as

$$\theta_\alpha^h = \oint \frac{\partial}{\partial J_\alpha}\left(\frac{\partial S}{\partial \varphi}\right) d\varphi.$$



$$\theta_1^h = -\oint \frac{2\overline{p}_2}{1 + 6\overline{p}_2} d\varphi.$$

$$\gamma_g = -\theta^h$$

Summary

+ We investigate the Berry phase and Hannay angle of diabatic quantum evolution in an atom-molecule conversion system that is governed by a nonlinear Schrodinger equation.

+ We find that the Berry phase in such nonlinear system consists of two parts: the usual Berry connection term and a novel term from the nonlinearity brought forth by the atom-molecule coupling.

+ The total geometric phase can be viewed as the flux of the magnetic field of a monopole through the surface enclosed by a closed path in parameter space. The charge of the monopole, however, is found to be one third of the elementary charge of the usual quantized monopole.

+ We also derive the classical Hannay angle of a geometric nature associated with the adiabatic evolution. It exactly equals to minus Berry phase.


Thanks !