低维量子气体的单一性极限和稳定激发态

Unitary limit and stable excited state of low-dimensional quantum gas

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大连，*2010*，*8*，*3*

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提　　纲

- Brief introduction to relevant experiments
- Super Tonks-Girardeau gas （stable excited gas-like state）: bosonic STG gas, fermionic STG gas
- Realization of STG gas and repulsively bound state in optical lattice

References:

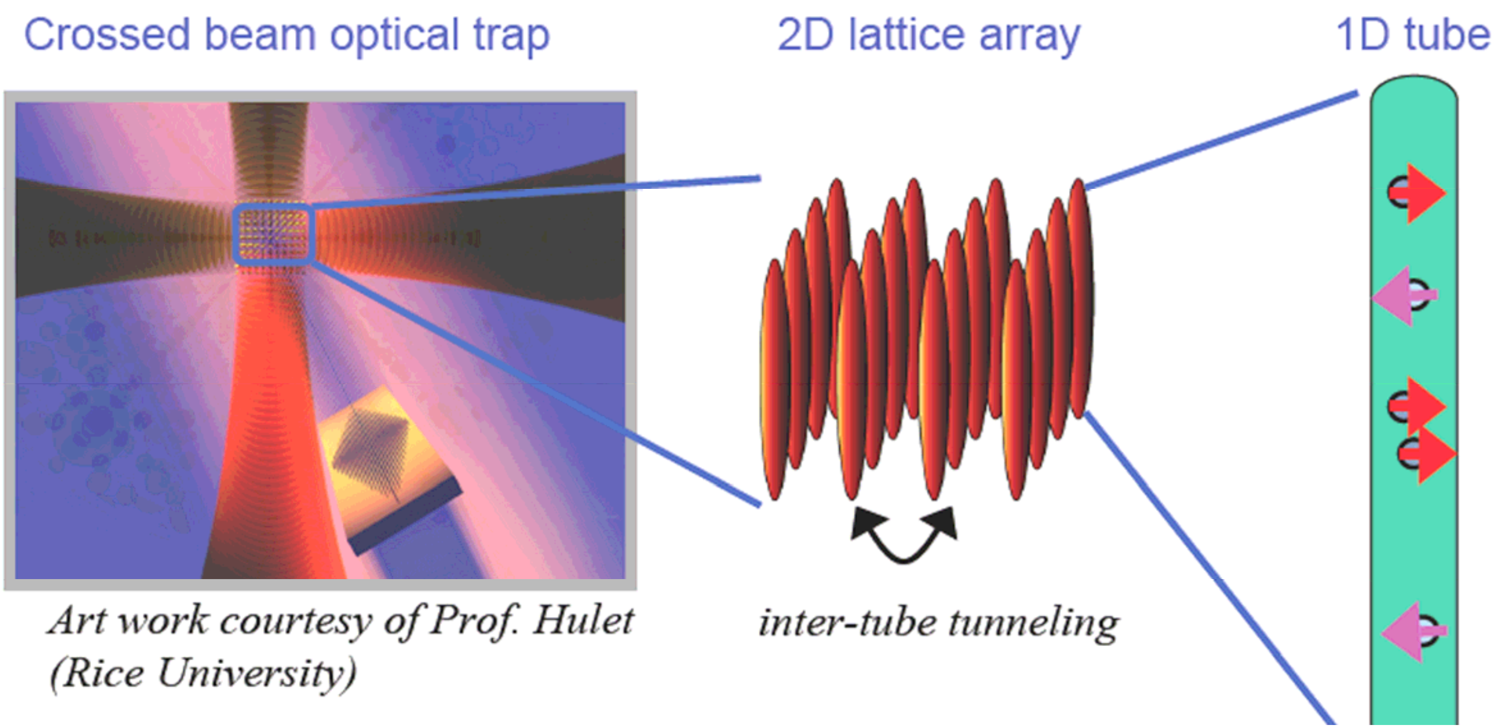
S Chen, L Guan, X Yin, Y Hao, and X-W Guan, Phys. Rev. A **81**, 031609(R) (2010)

S Chen, X-W Guan, X Yin, L Guan, M Batchelor, Phys. Rev. A **81**, 031608(R) (2010)

L Wang, Y Hao, and **S Chen**, Phys. Rev. A **81**, 063637 (2010)

L Guan, and **S Chen**, arXiv:1005.0461

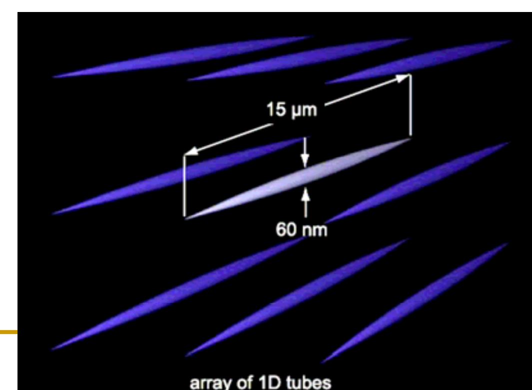
Experimental realization of 1D quantum gas



Art work courtesy of Prof. Hulet (Rice University)

Perfect model systems for a fundamental understanding of **1d quantum systems**.

Experimental realization of 1D Bose gas



Experiment progress on TG gas

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

We report the observation of a one-dimensional (1D) Tonks-Girardeau (TG) gas of bosons moving freely in 1D. Although TG gas bosons are strongly interacting, they behave very much like noninteracting fermions. We enter the TG regime with cold rubidium-87 atoms by trapping them with a combination of two light traps. By changing the trap intensities, and hence the atomic interaction strength, the atoms can be made to act either like a Bose-Einstein condensate or like a TG gas. We measure the total 1D energy and the length of the gas. With no free parameters and over a wide range of coupling strengths, our data fit the exact solution for the ground state of a 1D Bose gas.

letters to nature

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

[1]*Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
[2]*Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany*
[3]*Institut für Physik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*
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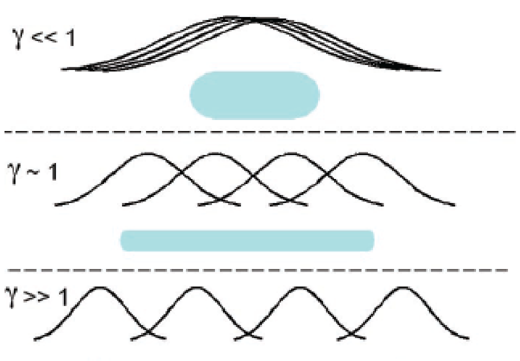
NATURE | VOL 429 | 20 MAY 2004 |

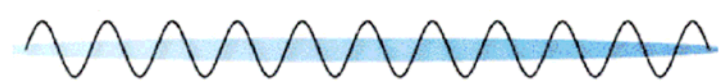
$$\gamma = \frac{mg}{\hbar^2 \rho}$$



$$\gamma = \frac{I}{K}$$

Interaction energy

Kinetic energy



$$\gamma = \frac{U}{J}$$

SCIENCE VOL 305 20 AUGUST 2004

T. Kinoshita, et. al, **Science**, 305, 1125 (2004)
B. Paredes, et. al, **Nature** 429, 277 (2004).

Theoretical description of 1D quantum gas

Confinement of atoms by strongly anisotropic harmonic trap

$$V = \frac{1}{2} m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

M. Olshanii

Quasi-1d: cigar-shape trap

$$\left(\omega_y = \omega_z = \omega_\perp\right) \quad \hbar\omega_\perp >> \hbar\omega_x, \qquad \hbar\omega_\perp >> k_B T$$

Transverse motion frozen, atoms move in the x direction

Effective 1d Hamiltonian

$$H = \int dx \hat{\psi}^+(x)\left[-\frac{\hbar^2}{2m}\partial^2_x + V(x,t) + \frac{g}{2}\hat{\psi}^+(x)\hat{\psi}(x)\right]\hat{\psi}(x)$$

$$g = -\frac{\hbar^2}{2 m a_{1D}} \qquad a_{1D} = -\frac{d_\perp^2}{2a_s}\left[1 - 1.46\frac{a_s}{d_\perp}\right] \qquad d_\perp = \sqrt{2\hbar / m\omega_\perp}$$

Olshanii PRL 81 (1998) 938

1D Bose gas in tightly confined trap

Transverse motion frozen → effective 1D interacting Bose gas

trap potential

相互作用强度可由 Feshbach 共振调节

$$\left[-\sum_{i=1}^{N}\left[\frac{\partial^2}{\partial x_i^2}+V(x_i)\right]+2c\sum_{1\le i<j\le N}\delta\left(x_i-x_j\right)\right]\Psi=E\Psi$$

M D Girardeau

$c=\infty$, Tonks-Girardeau gas

Fermi-Bose mapping

Girardeau 1960

Free Fermi wavefunction

$$\psi_B\left(x_1,\cdots,x_N\right)=A\left(x_1,\cdots,x_N\right)\psi_F\left(x_1,\cdots,x_N\right)$$

Bose wavefunction

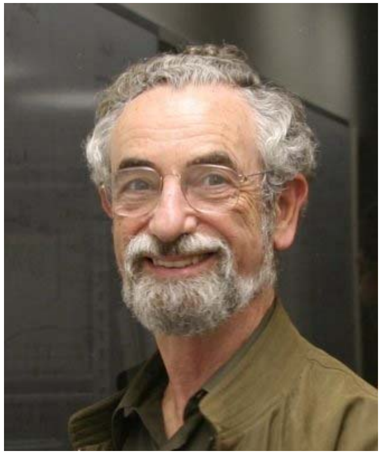
antisymmetry function $A(x_1,\cdots,x_N)=\prod_{i<j} sign(x_i-x_j)$

Evolution from BEC to fermionized Tonks-Girardeau gas

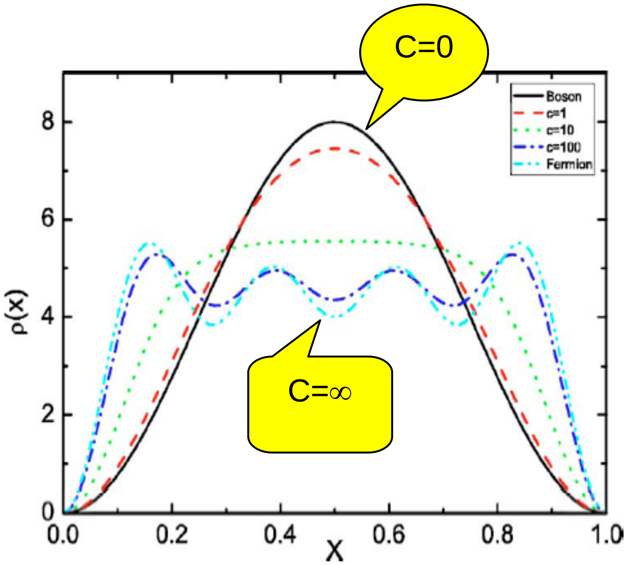
$$\left[-\sum_{i=1}^{N}\left[\frac{\partial^2}{\partial x_i^2}+V(x_i)\right]+2c\sum_{1\le i<j\le N}\delta\left(x_i-x_j\right)\right]\Psi=E\Psi$$

V=0, Lieb-Liniger model, exactly solvable for all c

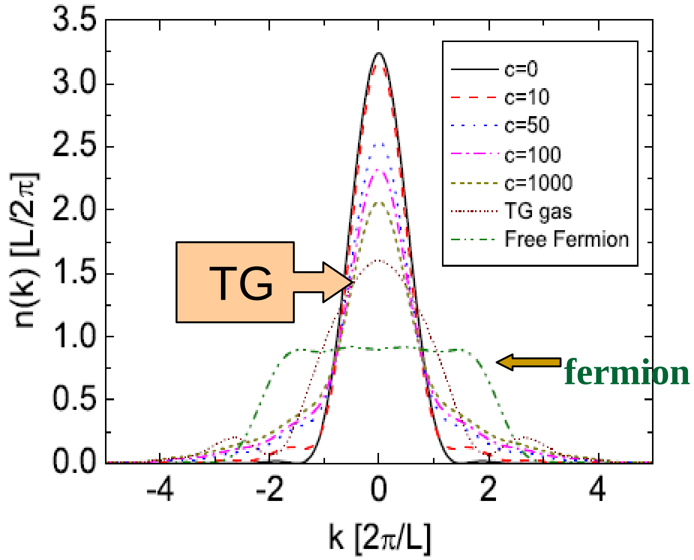
Exact solution for 1d hard-wall trap



Elliott H. Lieb



N=4



Y. Hao, Y. Zhang, J.Q. Liang and **S. Chen**, Phys. Rev. A **73**, 063617 (2006)

Y. Hao, Y. Zhang, and **S. Chen**, Phys. Rev. A 76, 063601 (2007)

PRL 95, 190406 (2005) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2005

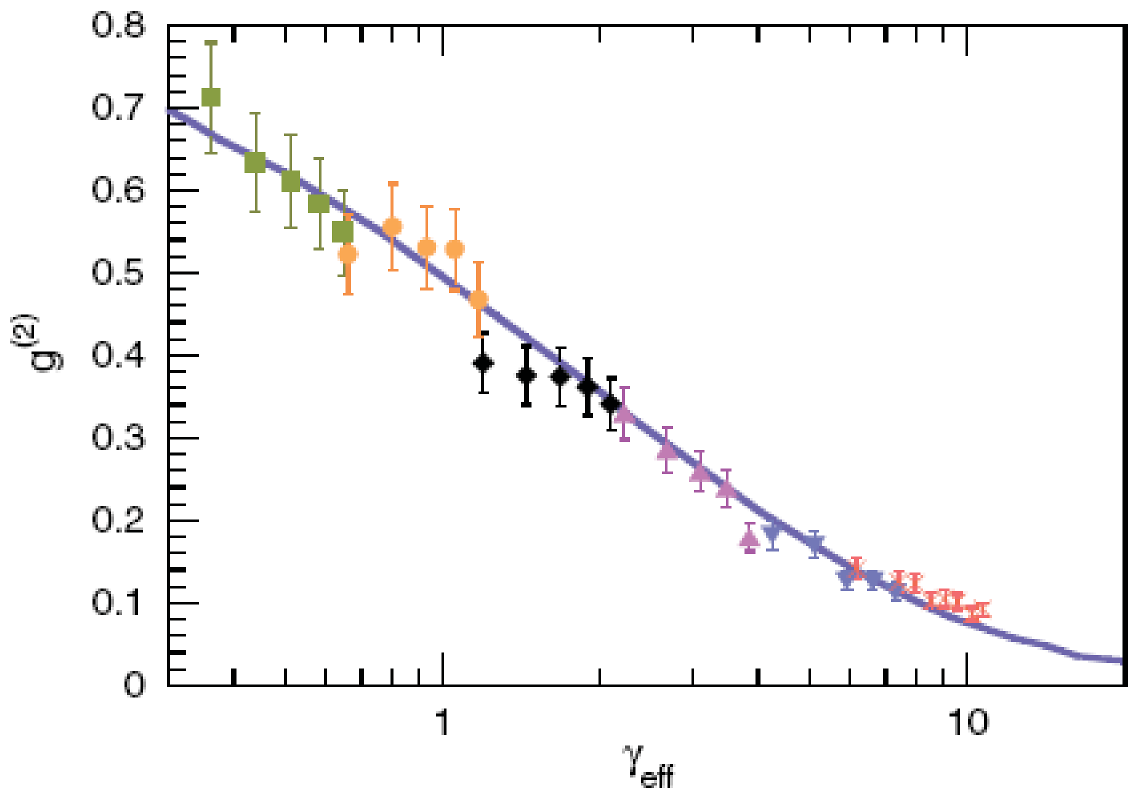
Local Pair Correlations in One-Dimensional Bose Gases

Toshiya Kinoshita, Trevor Wenger, and David S. Weiss

Department of Physics, The Pennsylvania State University, 104 Davey Lab, University Park, Pennsylvania, 16802 USA

(Received 14 July 2005; published 3 November 2005)

We measure photoassociation rates in one-dimensional Bose gases, and so determine the local pair correlation function over a wide range of coupling strengths. As bosons become more strongly coupled, we observe a tenfold decrease in their wave function overlap, thus directly observing the fermionization of bosons.



$$g_2 = \langle \psi^+ \psi^+ \psi \psi \rangle$$

can be calculated exactly from the results of Lieb-Liniger model!

Experiment progress on STG gas

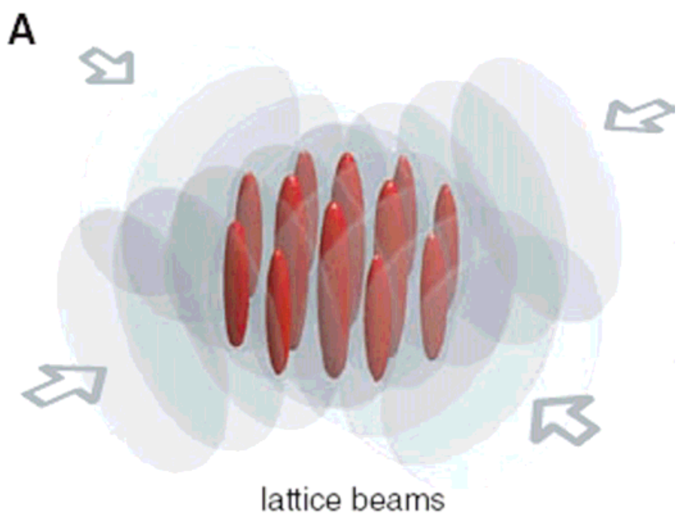
REPORTS

Realization of an Excited, Strongly Correlated Quantum Gas Phase

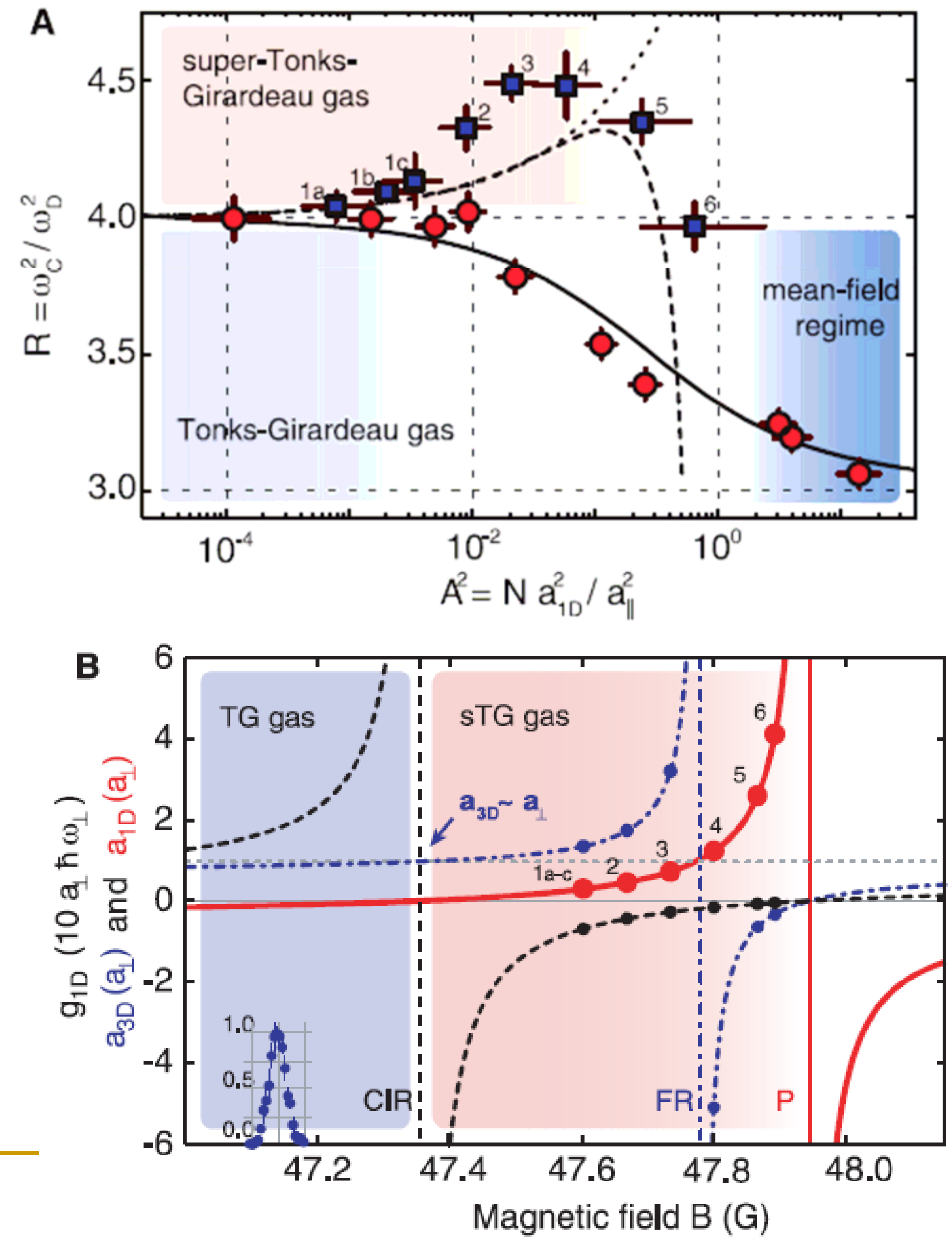
Elmar Haller,[1] Mattias Gustavsson,[1] Manfred J. Mark,[1] Johann G. Danzl,[1] Russell Hart,[1] Guido Pupillo,[2,3] Hanns-Christoph Nägerl[1]*

Ultracold atomic physics offers myriad possibilities to study strongly correlated many-body systems in lower dimensions. Typically, only ground-state phases are accessible. Using a tunable quantum gas of bosonic cesium atoms, we realized and controlled in one-dimensional geometry a highly excited quantum phase that is stabilized in the presence of attractive interactions by maintaining and strengthening quantum correlations across a confinement-induced resonance. We diagnosed the crossover from repulsive to attractive interactions in terms of the stiffness and energy of the system. Our results open up the experimental study of metastable, excited, many-body phases with strong correlations and their dynamical properties.

4 SEPTEMBER 2009 VOL 325 **SCIENCE**



$$g_{1D} = -\frac{2\hbar^2}{m a_{1D}} = \frac{2\hbar^2 a_{3D}}{m a_\perp^2} \frac{1}{1 - C\ a_{3D}/a_\perp}$$



PRL 95, 190407 (2005) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2005

Beyond the Tonks-Girardeau Gas: Strongly Correlated Regime in Quasi-One-Dimensional Bose Gases

G. E. Astrakharchik,[1,2] J. Boronat,[3] J. Casulleras,[3] and S. Giorgini[1]

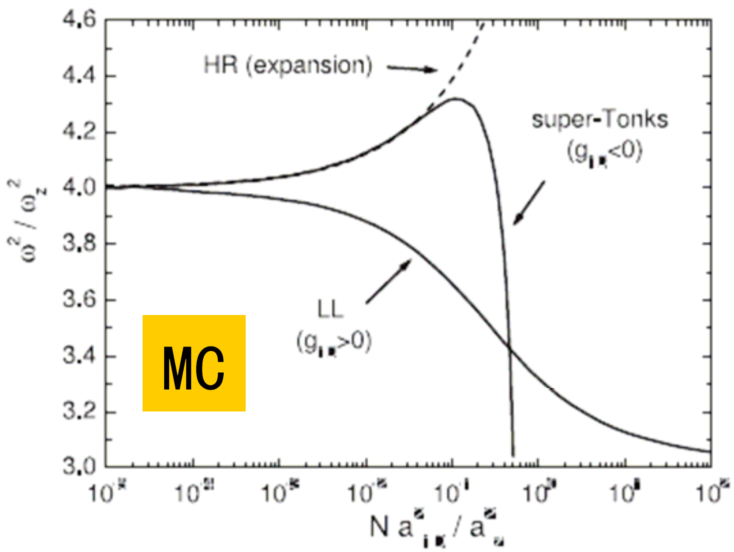
[1]*Dipartimento di Fisica, Università di Trento and BEC-INFM, I-38050 Povo, Italy*

[2]*Institute of Spectroscopy, 142190 Troitsk, Moscow Region, Russia*

[3]*Departament de Física i Enginyeria Nuclear, Campus Nord B4-B5, Universitat Politècnica de Catalunya, E-08034 Barcelona, Spain*

(Received 11 May 2004; revised manuscript received 17 May 2005; published 4 November 2005)

We consider a homogeneous 1D Bose gas with contact interactions and a large attractive coupling constant. This system can be realized in tight waveguides by exploiting a confinement induced resonance of the effective 1D scattering amplitude. By using the diffusion Monte Carlo method we show that, for small densities, the gaslike state is well described by a gas of hard rods. The critical density for cluster formation is estimated using the variational Monte Carlo method. The behavior of the correlation functions and of the frequency of the lowest breathing mode for harmonically trapped systems shows that the gas is more strongly correlated than in the Tonks-Girardeau regime.



MC

This is an excited state, instead of ground state!

这是一个新奇的量子气体态！

通常一个稳定的激发态很难在传统的凝聚态系统中实现。由于存在耗散，激发态会很快衰减到系统的基态。

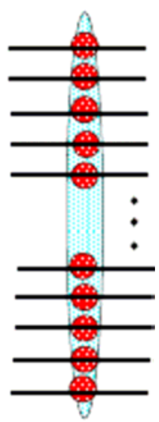
1D Bose gas with attractive interaction

C<0

$$\left[-\sum_{i=1}^{N}\left[\frac{\partial^2}{\partial x_i^2}+V(x_i)\right]+2c\sum_{1\le i<j\le N}\delta\left(x_i-x_j\right)\right]\Psi=E\Psi$$

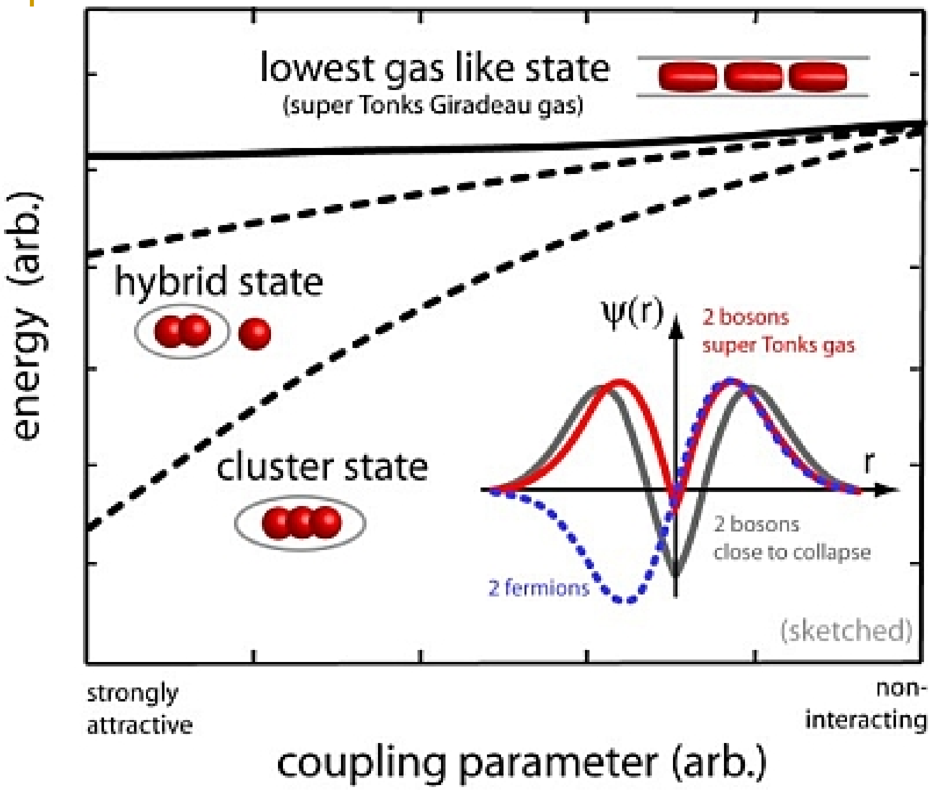
C->-∞, all bosons collapse into a bound state!

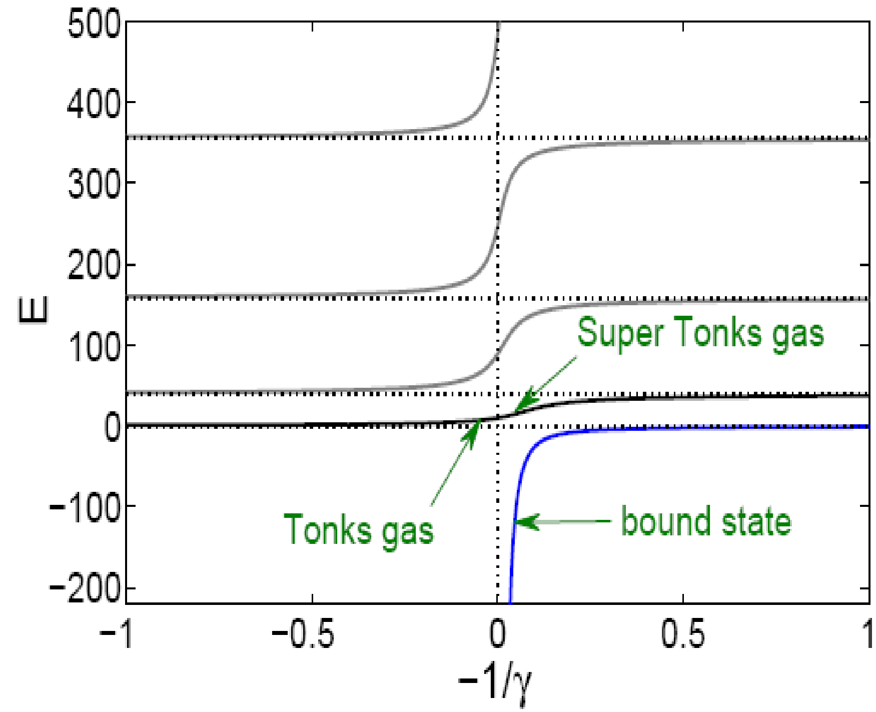
GS: Cluster state



$$E_0=-\tfrac{1}{12}c^2N(N^2-1)$$

J. B. McGuire, J. Math. Phys. (N.Y.) **5**, 622 (1964).



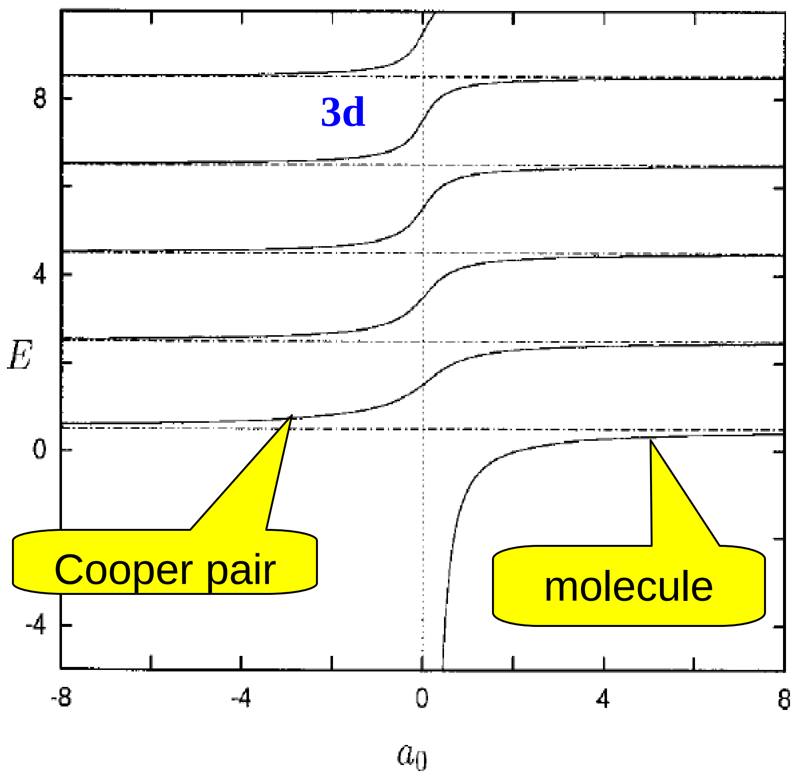


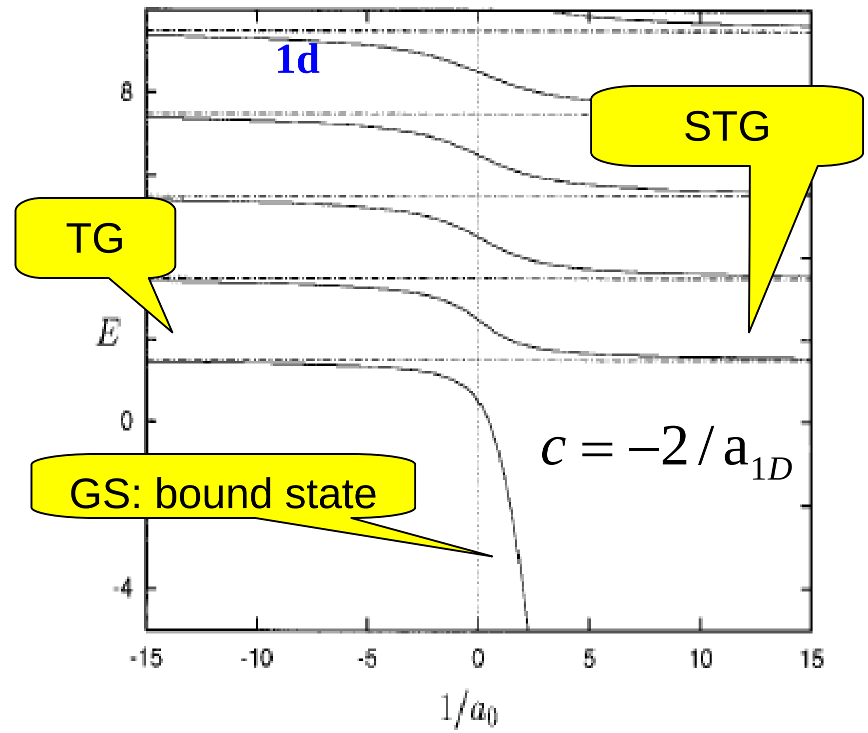
spectrum of 2-particle system

a sketch of possible many-body states of three attractively interacting atoms in 1D

http://www.uibk.ac.at/exphys/ultracold/projects/cs3/SuperTonksGirardeau/

Different physics across the unitary limit in 3d and 1d: spectrum of two-atom system in a trap





$$a_{1D} = -\frac{d_\perp^2}{2a_s}\left[1 - 1.46\,\frac{a_s}{d_\perp}\right]$$

Bush et al *Foundations of Physics, Vol. 28, No. 4, 1998*

many-body system ?

3d: BEC – BCS crossover

Question: how to understand the stable STG gas in the strong attractive regime? Why not decay into the cluster GS?

实验上实现**STG**气体本质上是一个动力学问题。

Step1: realization TG gas with $c \to \infty$

Step2: sudden swift of interaction from $c \to \infty$ to $c \to -\infty$

S Chen, L Guan, X Yin, Y Hao, and X-W Guan, Phys. Rev. A **81**, 031609(R) (2010)

Bethe ansatz solution of LL model

Hamiltonian for 1D interacting bosons

相互作用强度可由
Feshbach 共振调节

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g_{1D}\sum_{i<j}\delta\left(x_i - x_j\right)$$

exactly solved by BA method

$$\Psi(x_1, \ldots, x_N) = \sum_P A_P e^{i\sum_j k_{p_j} x_j}$$

$$c = \frac{m g_{1D}}{\hbar^2}$$

$$e^{ik_j L} = -\prod_{l=1}^{N}\left(\frac{k_j - k_l + ic}{k_j - k_l - ic}\right), \quad j = 1, 2, \ldots, N$$

$$E = \sum_{i=1}^{N} k_i^{\,2}$$

Real solutions -> scattering states; complex solutions -> bound states!

E. H. Lieb and w. Liniger, Phys Rev, **130**, 1605(1963)

TG gas and Super TG gas

N interacting bosons $$e^{ik_jL} = -\prod_{l=1}^{N}\left(\frac{k_j - k_l + ic}{k_j - k_l - ic}\right), \quad j = 1, 2, \ldots, N$$

$$\begin{cases} c > 0 \\ c < 0 \end{cases}$$

K only have real solutions

K may have complex solutions

GS for c>0: Real solutions; GS for c<0: complex solutions

- Repulsive bosons

$$\boxed{k_jL = 2\pi I_j - \sum_{i=`}^{N} 2\arctan\left(\frac{k_j - k_i}{|c|}\right)}$$

Ground state: $I_j = (N+1)/2 - j$

$c \to +\infty$ TG gas

$c \to -\infty$ STG gas

- Excited attractive bosons

$$\boxed{k_jL = 2\pi I_j + \sum_{i=`}^{N} 2\arctan\left(\frac{k_j - k_i}{|c|}\right)}$$

Super TG gas : $I_j = (N+1)/2 - j$

Real solutions for c<0

Not be considered untill very recently!

Bethe ansatz and Super TG gas

$$\gamma = \frac{cL}{N}$$

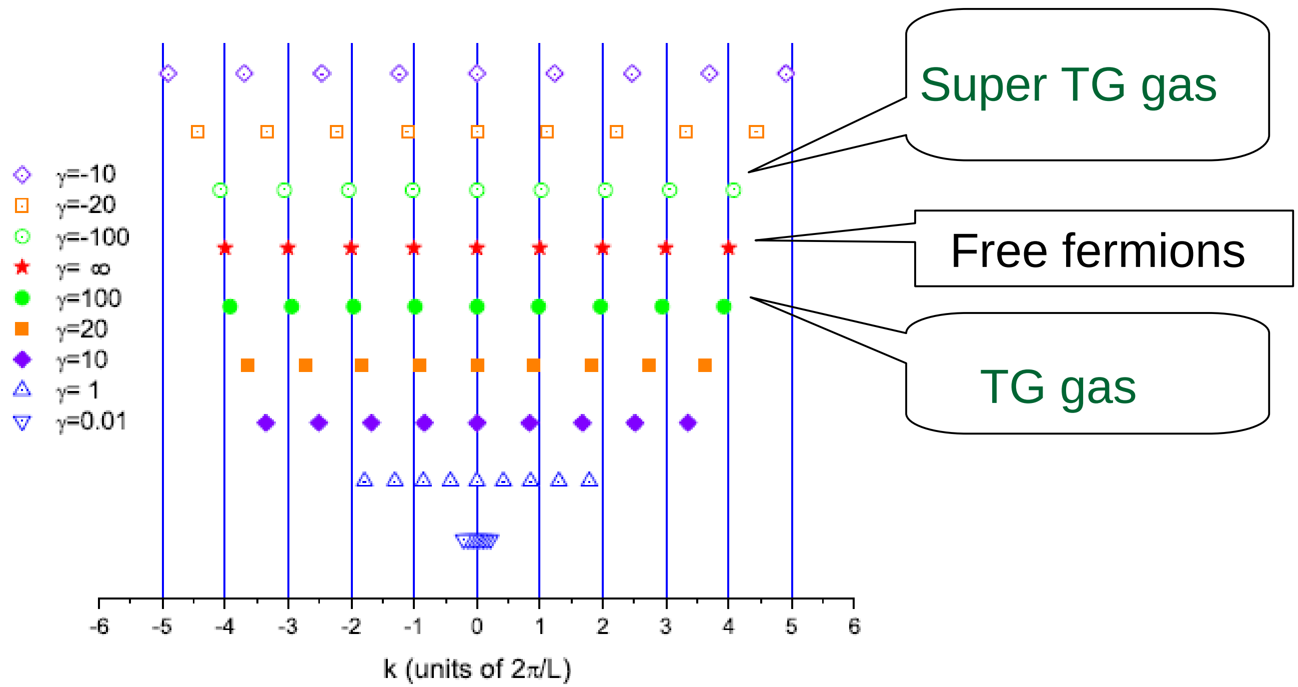


FIG. 1: (color online) Quasi-momentum distributions for the ground state of the repulsive and the STG gas phase of the attractive Bose gas with different values of γ.

Transition probabilities from TG to Super TG gas

Quench dynamics

$$\left|\Psi(X,t)\right\rangle = \sum_{n=0}^{\infty} e^{-iE_n t} c_n \left|\psi_n(X,c')\right\rangle$$

$$c_n = \left\langle \psi_n(X,c') \,|\, \psi_0(X,c) \right\rangle$$

N=4

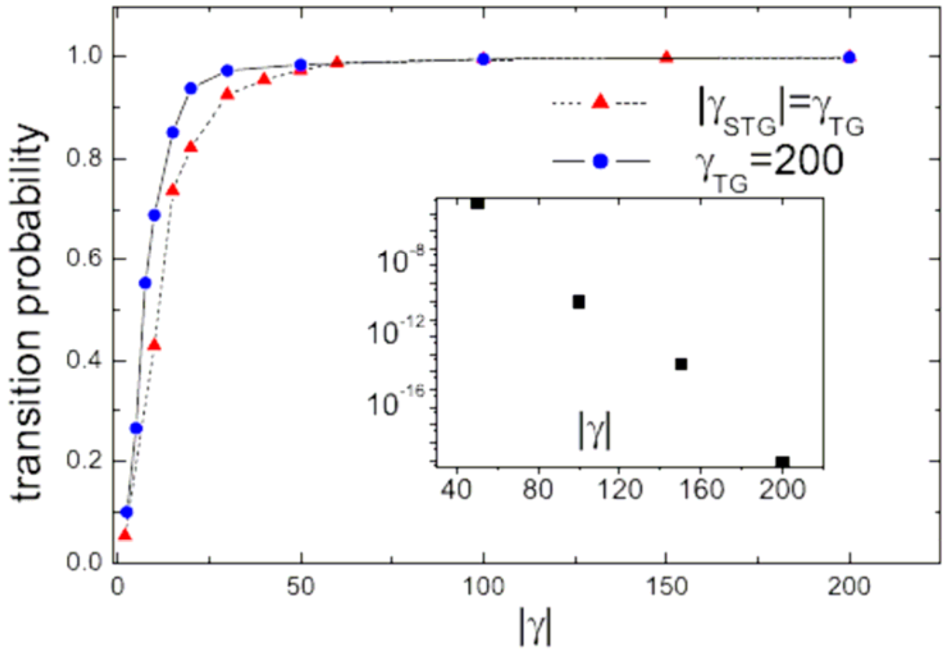


FIG. 2: (color online) Transition probabilities from the TG gas to STG phase. Inset: the transition probabilities from TG gas to the cluster state.

S Chen, L Guan, X Yin, Y Hao, and X-W Guan, Phys. Rev. A **81**, 031609(R) (2010)

Bethe ansatz and Super TG gas

In the thermodynamics limit with $N, L \to \infty,$

The energy per atom:

$$\varepsilon(\rho) = \frac{\hbar^2}{2m}\rho^2 e(\gamma)$$

$$e(\gamma) = \left|\frac{\gamma}{\lambda}\right|^3 \int_{-1}^{1} g(x) x^2 dx$$

$$g(x) = \frac{1}{2\pi} - \frac{1}{2\pi}\int_{-1}^{1} \frac{2\lambda g(x')}{\lambda^2 + (x-x')^2} dx', \qquad \lambda = |\gamma| \int_{-1}^{1} g(x) dx$$

Fit function:

$$\boxed{e(\gamma) = \frac{4\pi^2}{3} \frac{1 + p_1|\gamma| + p_2\gamma^2 + p|\gamma|^3/4}{1 + q_1|\gamma| + q_2\gamma^2 + p|\gamma|^3}}$$

$$p_1 = 0.075,\ p_2 = 0.013, q_1 = 0.227, q_2 = 0.034,\ p = 0.004$$

Calculation of the breathing mode of STG gas

Energy and correlation function

$$g_2 = \langle \psi^+ \psi^+ \psi \psi \rangle$$

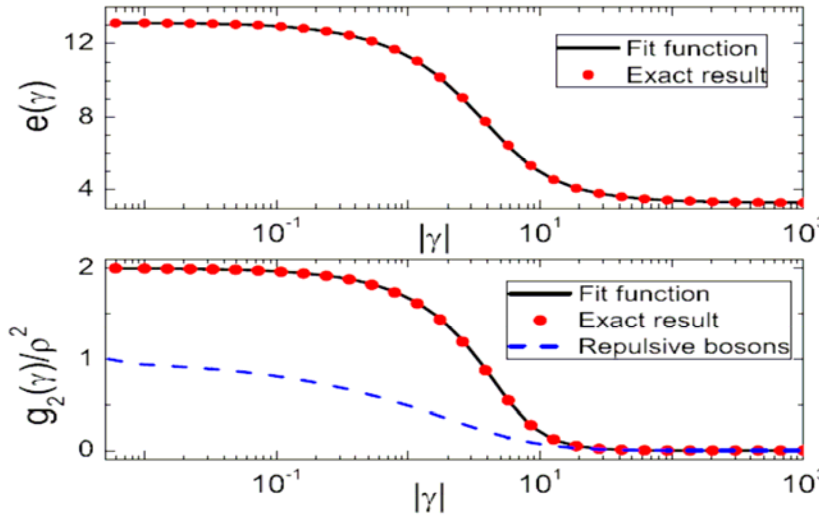


FIG. 3: $e(\gamma)$ and $g_2(\gamma)$ versus $|\gamma|$.

The Super TG gas in a harmonic trap

LDA approximation: $$\mu(\rho(x)) = \mu_0 - V_{ext}(x) \qquad V_{ext} = \frac{1}{2} m \omega_x^2 x^2$$

$$\mu(\rho) = \partial_\rho [\rho \epsilon(\rho)]$$

Breathing mode

$$\omega^2 = -2 \frac{\langle x^2 \rangle}{d \langle x^2 \rangle / d\omega_x^2} \qquad \langle x^2 \rangle = \frac{\int \rho(x) x^2 dx}{\int \rho(x) dx}$$

Calculation of the breathing mode of STG gas

The Super TG gas in a harmonic trap

$$V_{ext} = \frac{1}{2} m \omega_x^2 x^2$$

LDA approximation:

$$\mu_0 = \mu\left(\rho\left(x\right)\right) + V_{ext}\left(x\right)$$

Breathing mode

$$\omega^2 = -2 \frac{\left\langle x^2 \right\rangle}{\frac{d\left\langle x^2 \right\rangle}{d\omega_x^2}} \qquad \left\langle x^2 \right\rangle = \frac{\int \rho\left(x\right) x^2 dx}{\int \rho\left(x\right) dx}$$

Comparison with experimental data

Breathing mode

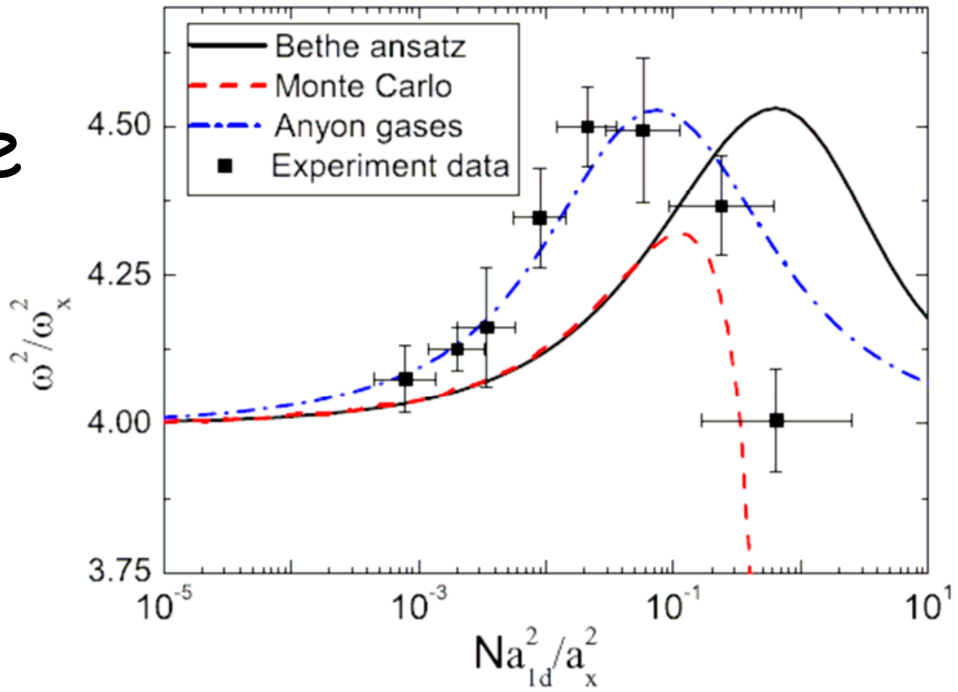


FIG. 4: (color online) Breathing mode of the STG gas. The experiment data with error bars and Monte Carlo result are reproduced from Fig. 3a of Ref. [10].

S Chen, L Guan, X Yin, Y Hao, and X-W Guan, Phys. Rev. A **81**, 031609(R) (2010)

Question: can the STG gas be the ground state of some systems?

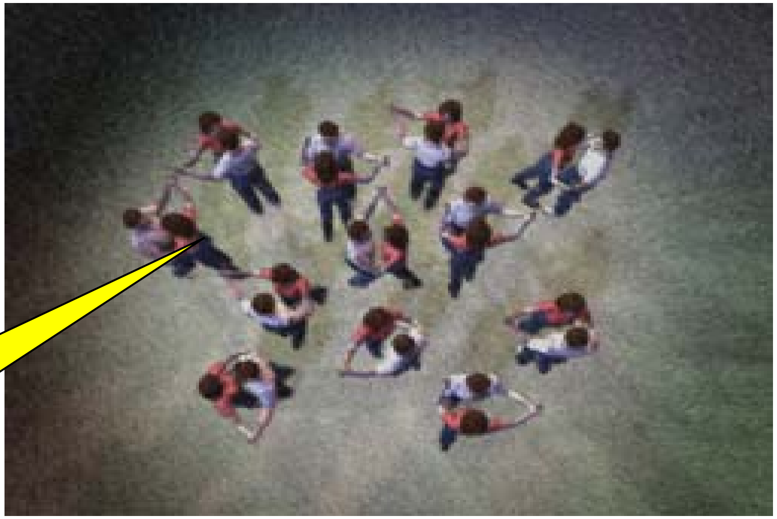
Attractive fermions and Super TG gas

Hamiltonian for 1D interacting fermions

$$H=-\frac{\hbar^2}{2m_F}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+g_F\sum_{i<j}\delta\left(x_i-x_j\right) \qquad c_F=\frac{m_F g_F}{\hbar^2}$$

Here we shall consider c_F -> $-\infty$

$$N_\uparrow=N_\downarrow=M=N/2$$



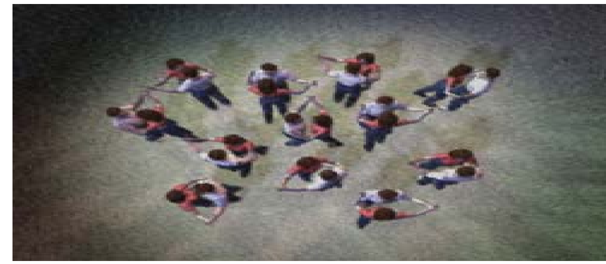
$$m_B = 2m_F$$

一对费米子形成了一个等效
的复合玻色子

binding energy $\epsilon_b = (\hbar^2/2m_F)c_F^2/2$

Question: can the Fermi pairs be described as an interacting Bose gas?

$$H = -\frac{\hbar^2}{2m_B}\sum_{i=1}^{N/2}\frac{\partial^2}{\partial x_i^2} + g_B\sum_{i<j}\delta\left(x_i - x_j\right)$$



$g_B = ?$

$\sim \pm g_F?$

$$g_F = -2\hbar^2/(m_F a_{1d}^F)$$

$$g_B = -2\hbar^2/(m_B a_{1d}^B)$$

To be + or -, this is a problem.

Intuitively, we may think that gB is repulsive because an attractive Bose gas is not a stable gas and collapses into a cluster state!

$$a_{1D}^B = \frac{1}{2} a_{1D}^F$$

Four-body problem solved by Moral et al PRL 2004

Attractive fermions vs Bose gas

Energy

$$E_0^F = E + M\varepsilon_b \approx \frac{\hbar^2}{2m_F}\frac{1}{6}M\left(M^2-1\right)\frac{\pi^2}{L^2}\left(1-\frac{M}{L|c_F|}\right)^{-2}$$

Warning! Mismatch with the energy of TG gas!

$$E_{TG} \approx \frac{\hbar^2}{2m_B}\frac{1}{3}N_B\left(N_B^2-1\right)\frac{\pi^2}{L^2}\left(1+\frac{2N_B}{L|c_B|}\right)^{-2}$$

$$\boxed{c_B = 2c_F,\, N_B = M = N/2,\, m_B = 2m_F}$$

$$E_{STG} \approx \frac{\hbar^2}{2m_B}\frac{1}{3}N_B\left(N_B^2-1\right)\frac{\pi^2}{L^2}\left(1-\frac{2N_B}{L|c_B|}\right)^{-2}$$

Match with the energy of the STG gas!

S Chen, X-W Guan, X Yin, L Guan, M Batchelor, Phys. Rev. A **81**, 031608(R) (2010)

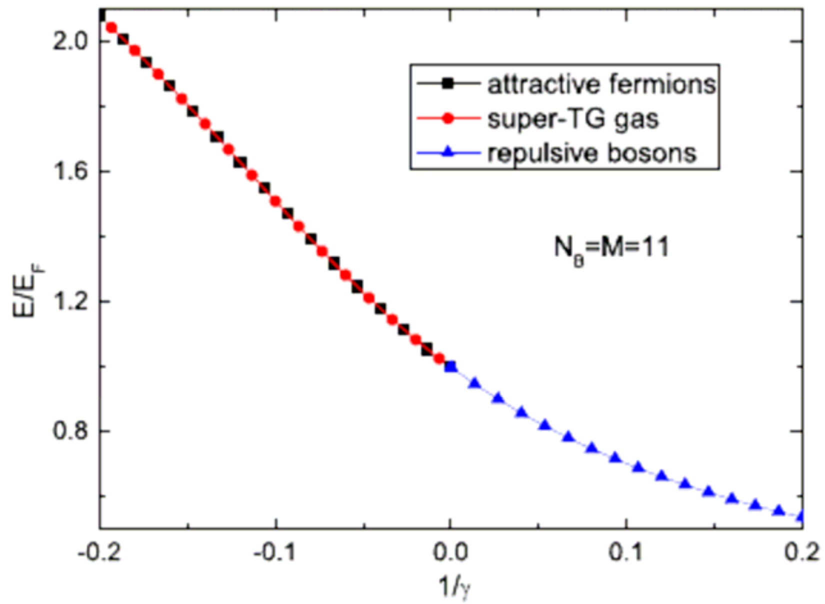
Comparsion with the boson gas

$$m_B = 2m_F$$

1D scattering length

$$a_{1D}^B = \frac{1}{2} a_{1D}^F$$

强吸引相互作用的一维费米气体的**基态**等效地有一个强吸引相互作用的一维玻色气体的**激发态**（**STG gas**）描述。



GS of attractive fermions		sTG phase of attractive bosons
$c_F << -1$	$\longleftrightarrow$	$c_B = 2c_F, \quad N_B = M = N/2 \quad \text{and} \quad m_B = 2m_F$

S Chen, X-W Guan, X Yin, L Guan, M Batchelor, Phys. Rev. A **81**, 031608(R) (2010) arXiv:0912.1633

Wave functions of the super Tonks-Girardeau gas and the trapped 1D hard sphere Bose gas

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(Dated: May 26, 2010)

a_{1D} the frequency decreased. These results agree with our theorem that the sTG state has the same ground state energy as a system of hard spheres, increasing with density [28, 29], so that the frequency in the sTG state

[29] A similar calculation for an untrapped system on a ring was carried out by S. Chen *et al.*, Phys. Rev. A **81**, 031609 (2010). The energy E_{TG} following Eq. (5) of that paper is not a hard sphere energy, but instead the energy of point particles with $\lambda \gg 1$ (strong Lieb-Liniger repulsion).

arXiv:1004.2925

Two super Tonks-Girardeau states of a trapped 1D spinor Fermi gas

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College of Optical Sciences, University of Arizona, Tucson, AZ 85721, USA
(Dated: May 24, 2010)

A generalization of the mapping to a 1D spinor Fermi gas is relevant to a recent prediction of a super Tonks-Girardeau (sTG) state in such a system [12], and will be employed herein. The model consists of an ultracold

[12] S. Chen, X.-W. Guan, X. Yin, L. Guan, and M.T. Batchelor, Phys. Rev. A **81**, 031608(R) (2010). E_{TG} in their

previously-discussed exact $N = 2$ solution. One expects such a Bose-like sTG state similar to that of [23] to exist also in the present case of harmonic trapping. It is al-

[23] S. Chen, L. Guan, X. Yin, Y. Hao, and X.-W. Guan, Phys. Rev. A **81**, 031609(R) (2010).

2-component Fermi gas in tightly confined optical trap

Transverse motion frozen → effective 1D interacting quantum mixture (FF)

trap potential

S-wave scattering between atoms with different spins

$$\left[-\sum_{i=1}^{N}\left[\frac{\partial^2}{\partial x_i^2}+V(x_i)\right]+2c\sum_{1\le i<j\le N}\delta\left(x_i-x_j\right)\right]\Psi=E\Psi$$

V(x)=0, Yang model (integrable)

V(x)=0, c= - ∞, bosonic STG gas of fermion-pairs

c=+∞, 任意V(x), Fermi TG gas

L Guan, **S Chen**, Y Wang and Z Q Ma, Phys. Rev. Lett, 102 160402 (2009)

c -> - ∞, Fermi STG gas ?

arXiv:1005.0461v3

Super-Tonks-Girardeau gas of spin-1/2 interacting fermions

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[1]*Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*

(Dated: July 28, 2010)

The Fermi gases confined in tight one-dimensional waveguides would form two-particle bound states of atoms in the presence of a strongly attractive interaction. Based on the exact solution of the one-dimensional spin-1/2 interacting Fermi gas, we demonstrate that a stable excited state with no pairing between attractive fermionic atoms can be realized by a sudden switch of interaction from strongly repulsive regime to the strongly attractive regime. Such a state is an exact fermionic analog of the experimentally observed super-Tonks-Girardeau state of bosonic Cesium atoms [Science **325**, 1224 (2009)] and should be possible to be observed by the experiment. The frequency of lowest breathing mode of the fermionic super-Tonks-Girardeau gas is calculated as a function of the interaction strength, which could be used as a detectable signature for the experimental observation.

$$\left[-\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+2c\sum_{1\le i<j\le N}\delta\left(x_i-x_j\right)\right]\Psi=E\Psi$$

Fermi STG gas：stable lowest gas-like excited state with no pairing for c<0.

c<0, GS : Fermi pairs

Step1: realization Fermi TG gas with c -> ∞

Step2: sudden swift of interaction from c -> ∞ to c -> -∞

Bethe ansatz solution of Yang model

Eigenstates take the BA wavefunction

$$|S, S_Z\rangle = |N/2 - M, N/2 - M\rangle \quad M = N_\downarrow$$

Behte-ansatz equations：

$$k_j L = 2\pi I_j - 2\sum_{\alpha=1}^{M} \tan^{-1}\left(\frac{k_j - \Lambda_\alpha}{c/2}\right)$$

$$\sum_{j=1}^{N} 2\tan^{-1}\left(\frac{\Lambda_\alpha - k_j}{c/2}\right) = 2\pi J_\alpha + 2\sum_{\beta=1}^{M} \tan^{-1}\left(\frac{\Lambda_\alpha - \Lambda_\beta}{c}\right)$$

Eigenenergy：

$$E = \frac{\hbar^2}{2m}\sum_j^N k_j^2$$

Limit of strong coupling

Behte-ansatz equations reduce to:

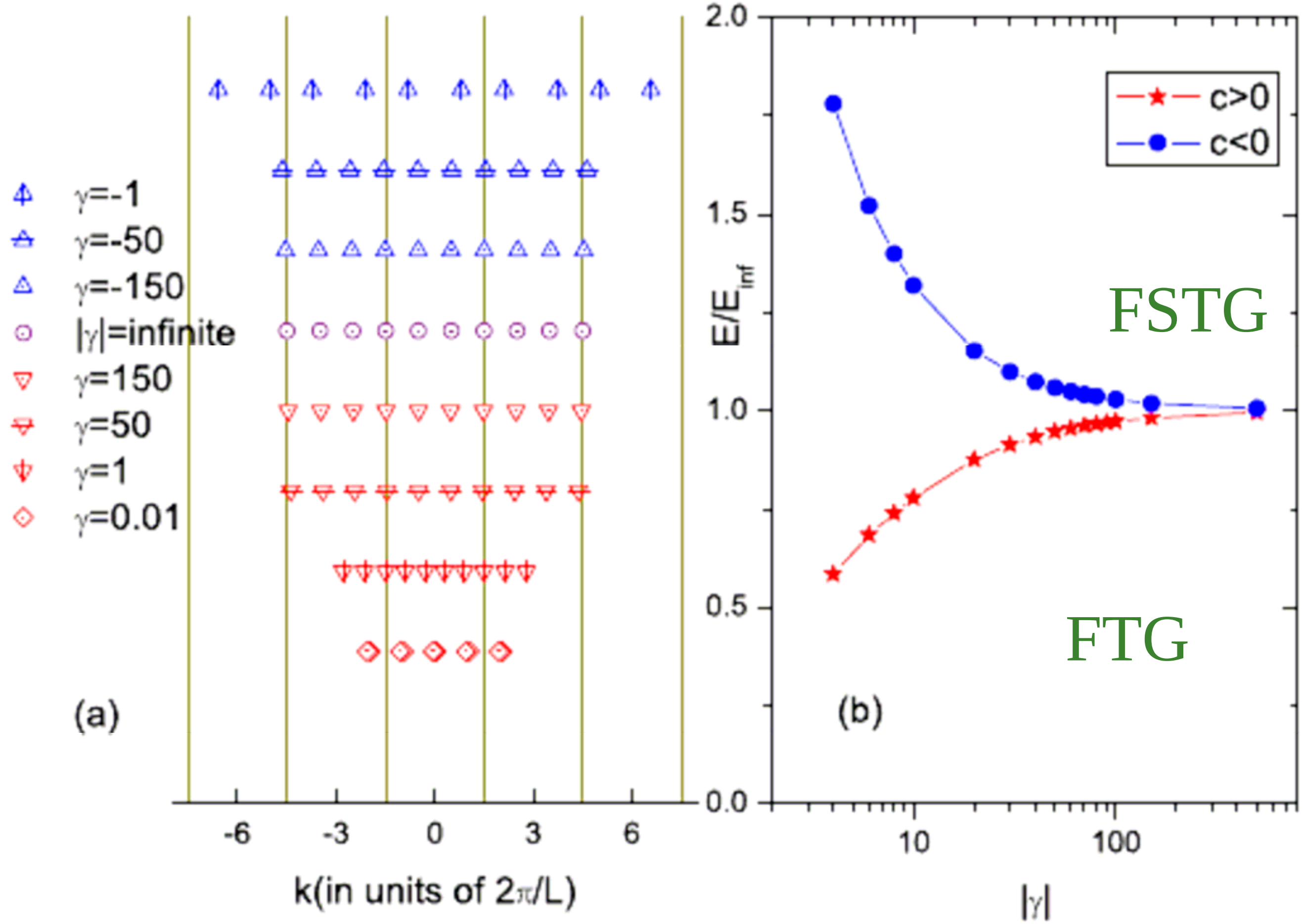
c -> ∞ $$k_j L = 2\pi I_j - \zeta \frac{k_j}{|c|} + O(|c|^{-3})$$ FTG gas

C= ∞, -> distribution of polarized Fermi gas

c -> -∞ $$k_j L = 2\pi I_j + \zeta \frac{k_j}{|c|} + O(|c|^{-3})$$ FSTG gas

with $\zeta = \sum_{\alpha=1}^{M} \frac{1}{(\Lambda_\alpha / c)^2 + 1/4}$ determined by solving:

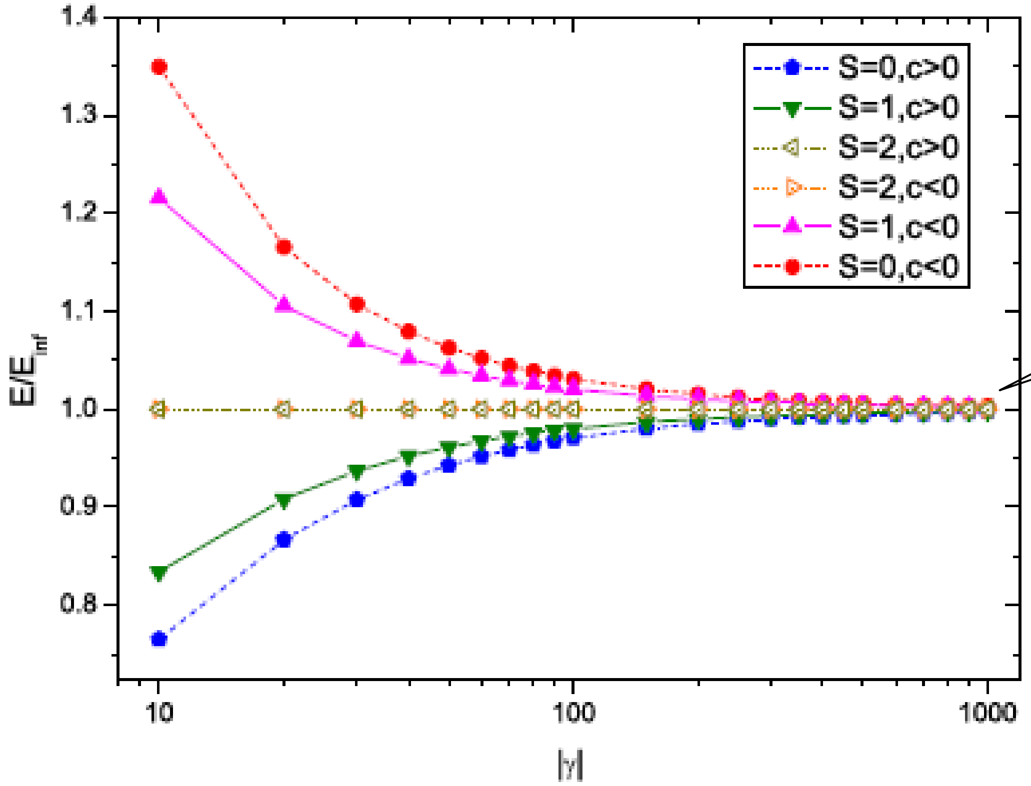
$$2N \tan^{-1}\left(\frac{\Lambda_\alpha}{c/2}\right) = 2\pi J_\alpha + 2\sum_{\beta=1}^{M} \tan^{-1}\left(\frac{\Lambda_\alpha - \Lambda_\beta}{c}\right)$$



$N = 10$ and $M = 5$

$$E_{FTG} = \frac{\hbar^2}{2m}\frac{\pi^2}{3L^2}N(N^2-1)(1+\frac{\zeta}{L|c|})^{-2}+O(|c|^{-3})$$

$$E_{FSTG} = \frac{\hbar^2}{2m}\frac{\pi^2}{3L^2}N(N^2-1)(1-\frac{\zeta}{L|c|})^{-2}+O(|c|^{-3}).$$

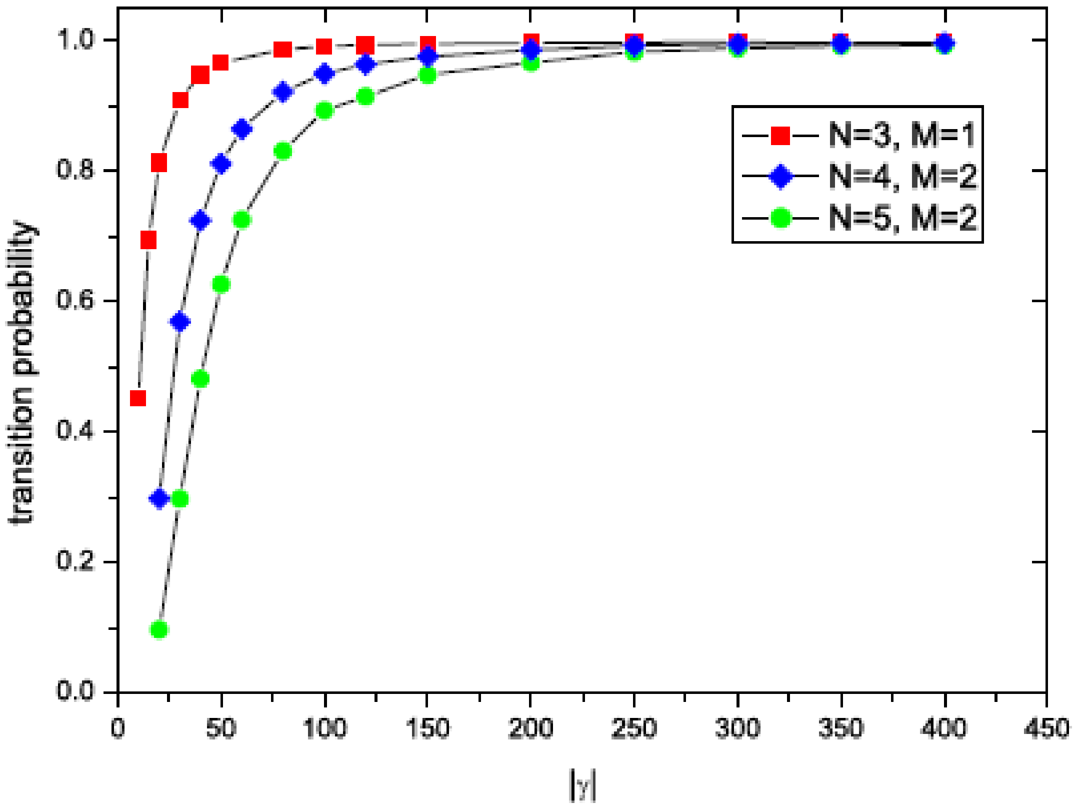


Unitary limit: highly degenerate!

N=4, M=2

FIG. 4: Energy vs γ for states with different total spin S.

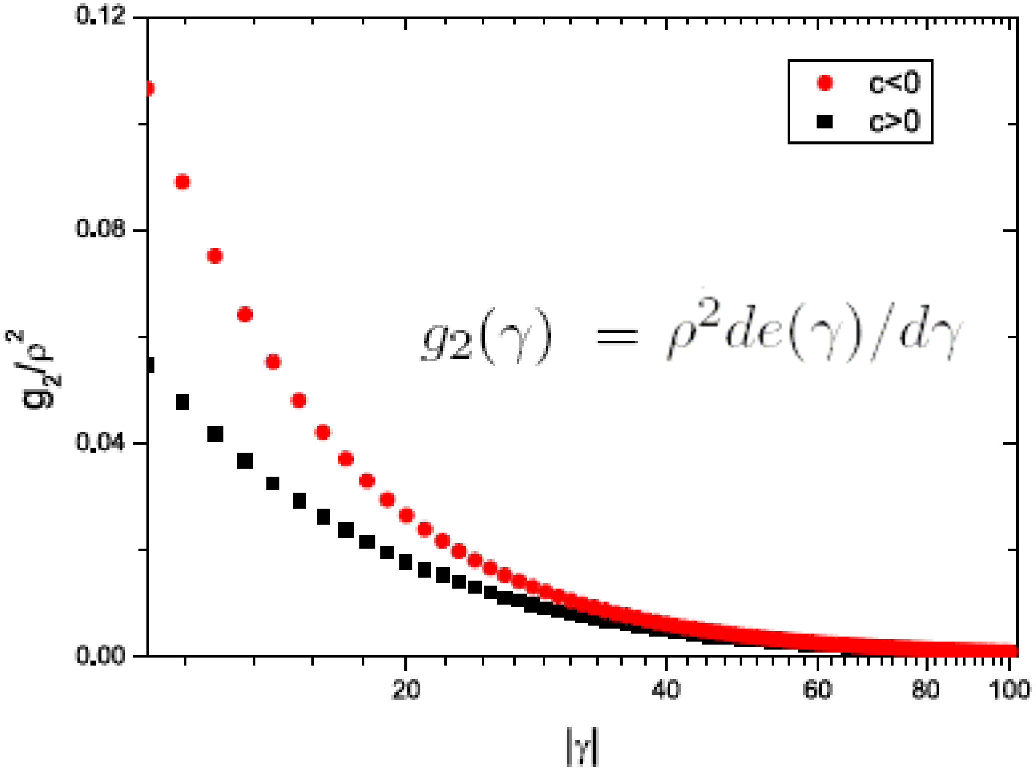
Transition probabilities from FTG to FSTG gas



In the strong coupling limit

$$P(N, M, \gamma) = 1 - a(N, M)/\gamma^2$$

Local two-particle correlation function



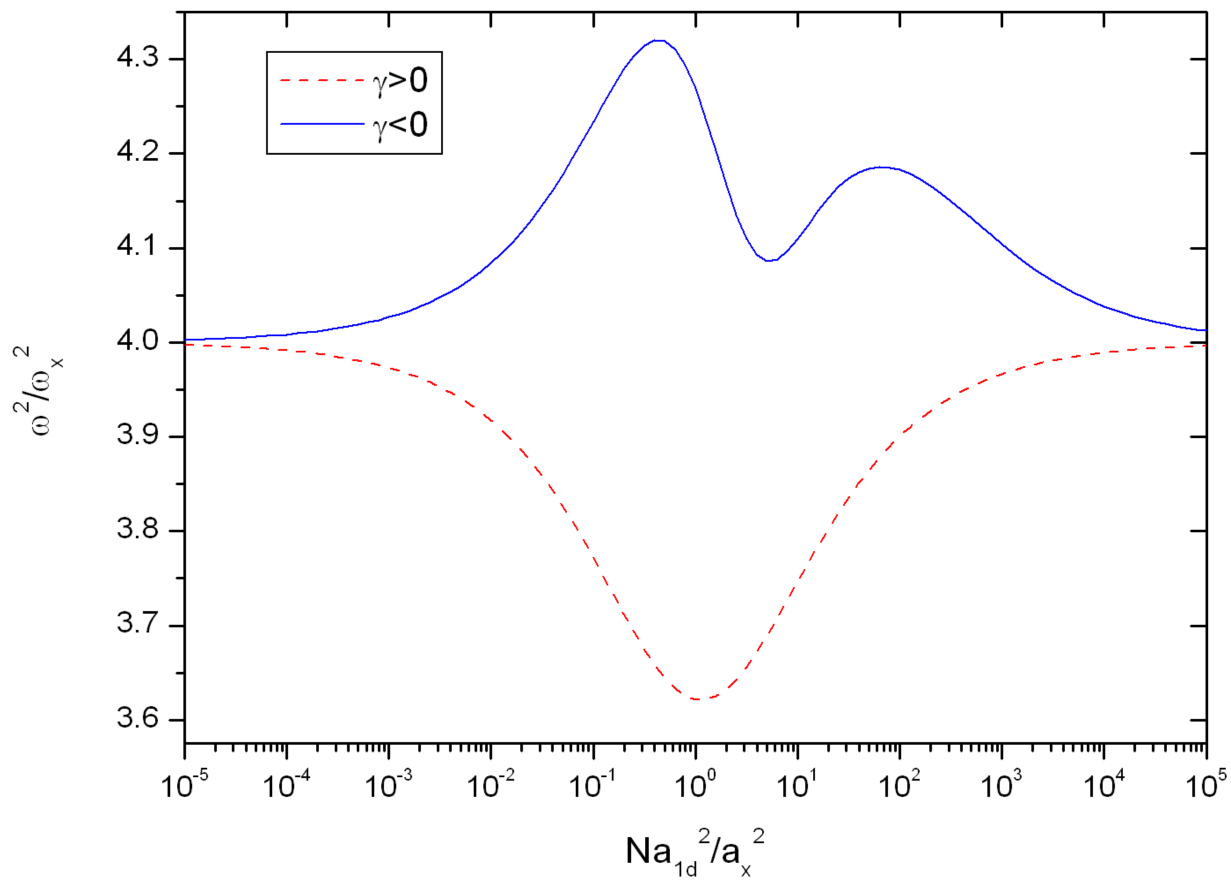
$$g_2(\gamma) = \rho^2 de(\gamma)/d\gamma$$

$$g_2 = \langle \psi_{\uparrow}^{+} \psi_{\downarrow}^{+} \psi_{\uparrow} \psi_{\downarrow} \rangle$$

$$g_2(\gamma)_{TG}/\rho^2 \approx (4\pi^2/3)(\ln 2|\gamma|^{-2} - 6(\ln 2)^2|\gamma|^{-3})$$

$$g_2(\gamma)_{STG}/\rho^2 \approx (4\pi^2/3)(\ln 2|\gamma|^{-2} + 6(\ln 2)^2|\gamma|^{-3})$$

Lowest Breathing mode of Fermi STG gas



Preparation of STG gas in optical Lattices

Bosons trapped in optical lattices are described by:

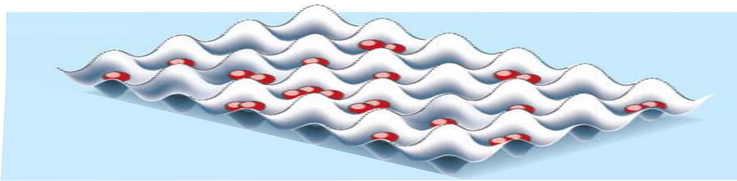
Bose Hubbard model

$$\hat{H} = -J \sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i \right) + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Tonks realized in 1d optical lattices
B. Paredes, et. al, **Nature** 429, 277 (2004).

$$\gamma = \frac{U}{J}$$

Quenstion 1: Is the STG gas also in 1d optical lattices?



Observation of repulsively bound atom pairs

Vol 441 | 15 June 2006 | doi:10.1038/nature04918 nature

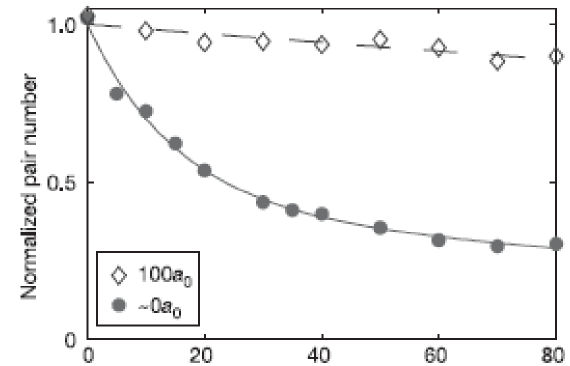
LETTERS

Repulsively bound atom pairs in an optical lattice

K. Winkler[1], G. Thalhammer[1], F. Lang[1], R. Grimm[1,3], J. Hecker Denschlag[1], A. J. Daley[2,3], A. Kantian[2,3], H. P. Büchler[2,3] & P. Zoller[2,3]

Throughout physics, stable composite objects are usually formed by way of attractive forces, which allow the constituents to lower their energy by binding together. Repulsive forces separate particles in free space. However, in a structured environment such as a periodic potential and in the absence of dissipation, stable composite objects can exist even for repulsive interactions. Here we report the observation of such an exotic bound state, which comprises a pair of ultracold rubidium atoms in an optical lattice. Consistent with our theoretical analysis, these repulsively bound pairs exhibit long lifetimes, even under conditions when they collide with one another. Signatures of the pairs are also recognized in the characteristic momentum distribution and through spectroscopic measurements. There is no analogue in traditional condensed matter systems of such repulsively bound pairs, owing to the presence of strong decay channels. Our results exemplify the strong correspondence between the optical lattice physics of ultracold bosonic atoms and the Bose–Hubbard model[1,2]—a link that is vital for future applications of these systems to the study of strongly correlated condensed matter and to quantum information.

c



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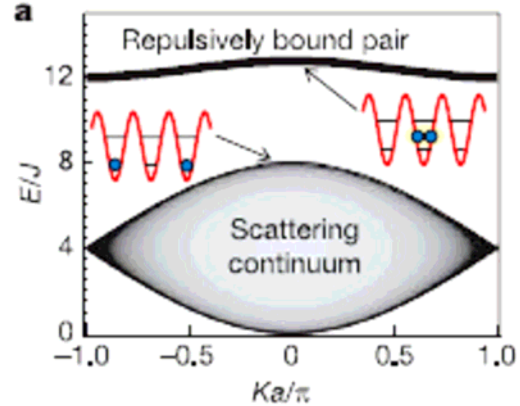


Another counter-intuition example!

Quenstion 2: Can the STG gas and repulsively bound state be understood in a same theoretical framework?

Bose Hubbard Models

$$\hat{H} = -J\sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i\right) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

The BHM is generally not exactly solvable!

But 2-particle problem of BHM is exact solvable by Bethe-ansatz method!

$$|\Psi\rangle = \sum_{x_1,x_2} \Psi(x_1,x_2)|x_1,x_2\rangle$$

$$\Psi(x_1,x_2) = A_{12}e^{i(k_1x_1+k_2x_2)} + A_{21}e^{i(k_2x_1+k_1x_2)}$$

$$\exp(ik_jL) = \frac{\sin k_l - \sin k_j - iU/2}{\sin k_l - \sin k_j + iU/2}. \qquad E = -2(\cos k_1 + \cos k_2)$$

Real solutions -> scattering states; complex solutions -> bound states!

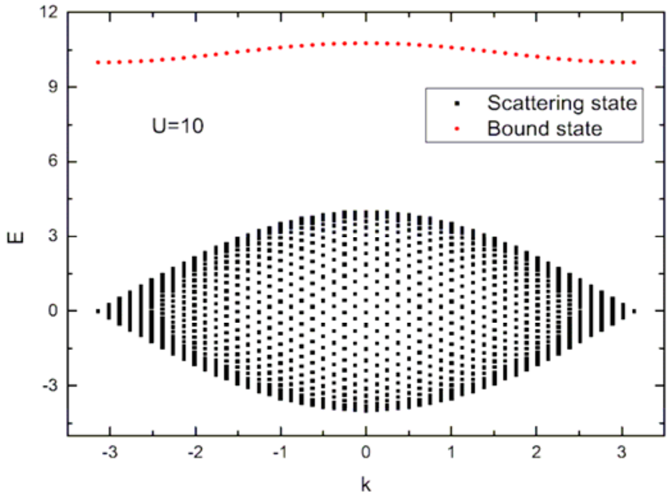
$U > 0$ *repulsive* GS: real solution

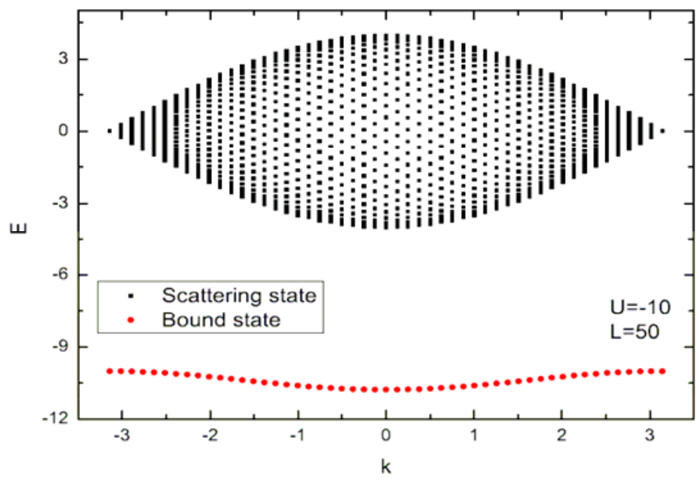
$U < 0$ *attractive* GS: complex solution $k_1 = k + i\Lambda \quad k_2 = k - i\Lambda$

Bose Hubbard Models

$$\hat{H} = -J\sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i\right) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

2-particle spectrum of Bose Hubbard model



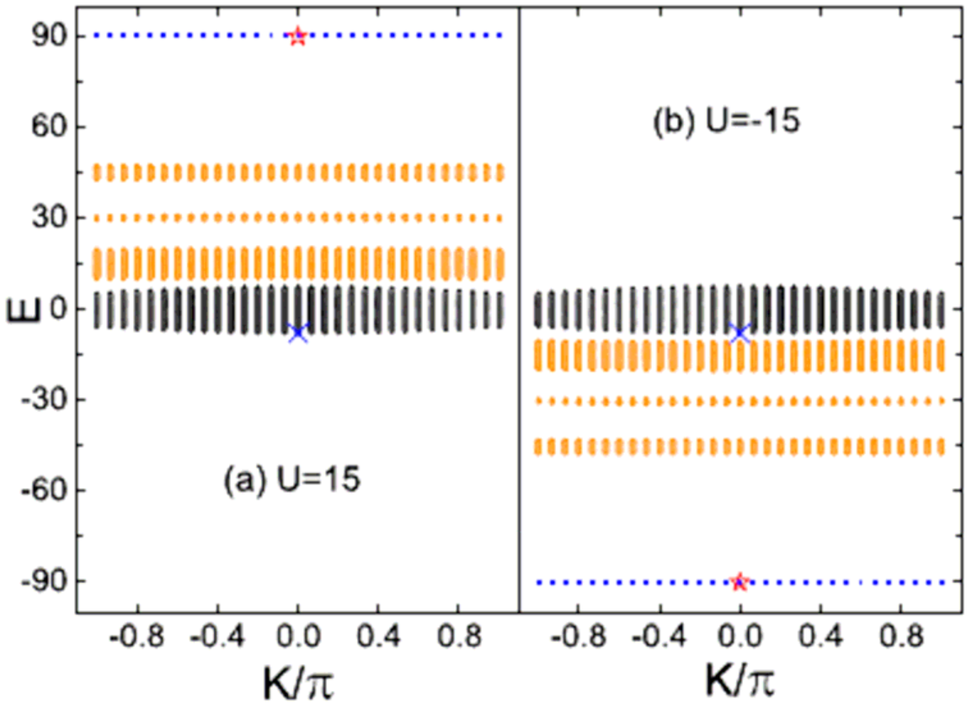


$U > 0$ *repulsive*

$U < 0$ *attractive*

Full spectrum of many-body systems

$$\hat{H} = -J\sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \hat{b}_{i+1}^\dagger \hat{b}_i\right) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$



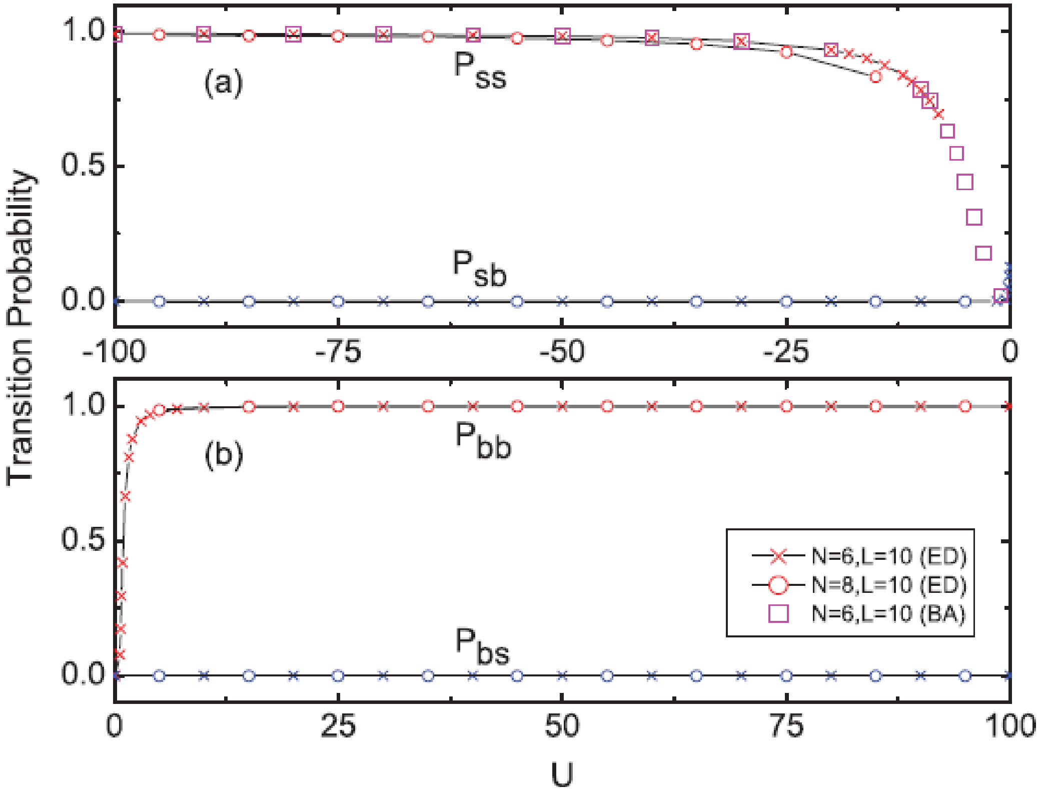
N=4,L=30,obtained by full ED

Dimension of Hilbert spaces:

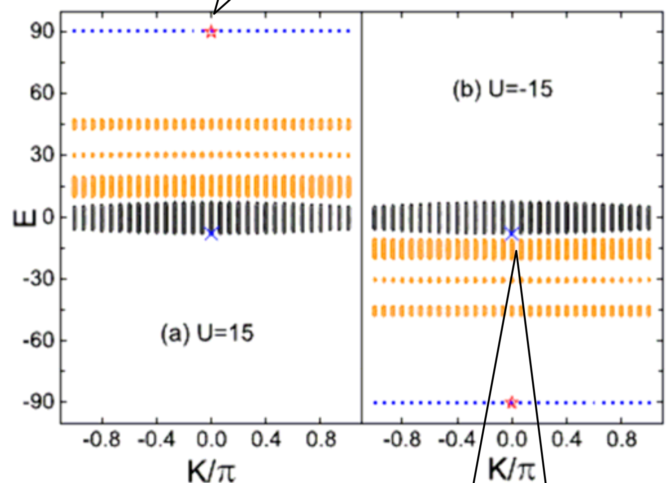
$$D = (N+L-1)!/[N!(L-1)!]$$

N>3, no analytically exact solution! But for large c, the spectrum can be well described by BA solutions.

Switch interaction from U to -U



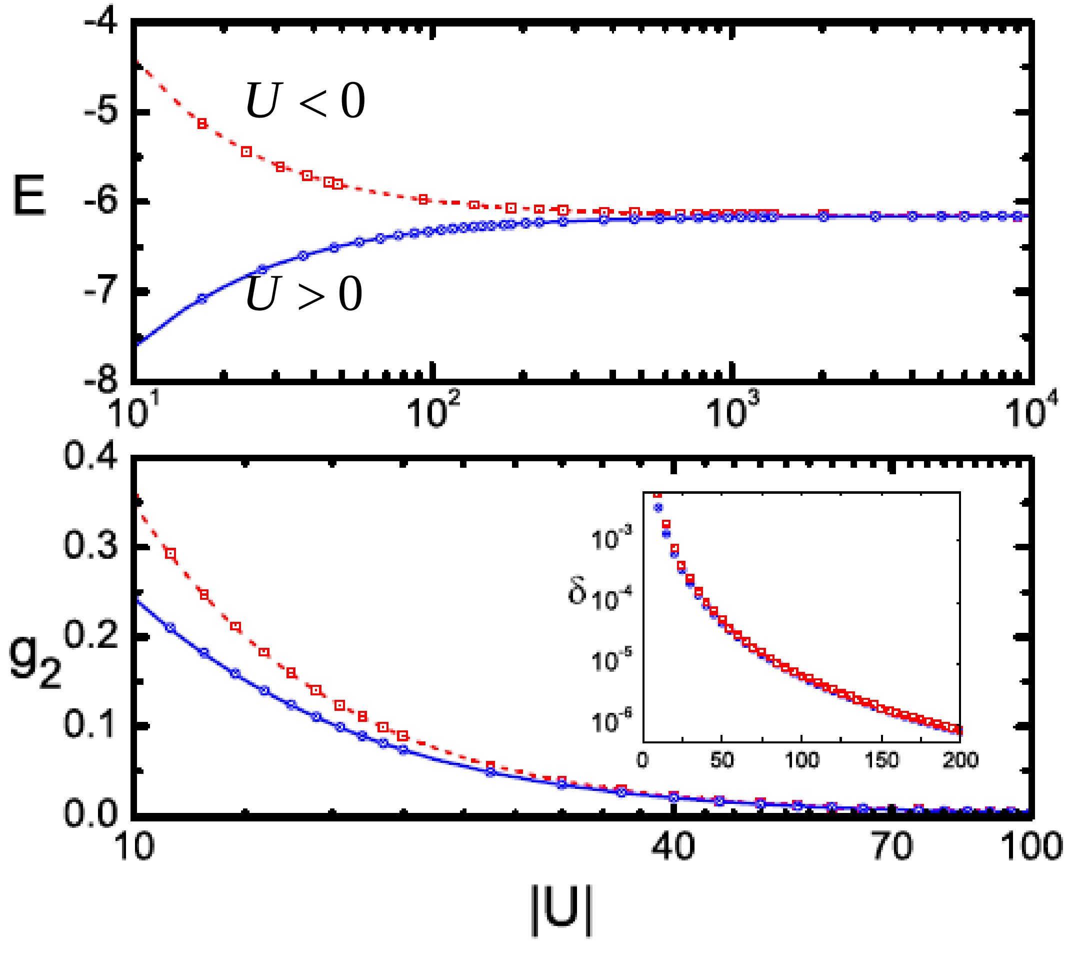
Repulsively bound state



STG gas

We provide an unified theoretical explanation to two seemingly unrelated experiment!

Properties of the lowest gas-like state



STG gas in optical lattices

summary

- Transition from a Tonks-Girardeau gas to a super-Tonks-Girardeau gas as an exact many-body dynamics problem
- Realization of effective super Tonks-Girardeau gases via strongly attractive one-dimensional Fermi gases
- Fermi STG gas
- Preparation of stable excited states in optical lattices

Thank you for your attention!