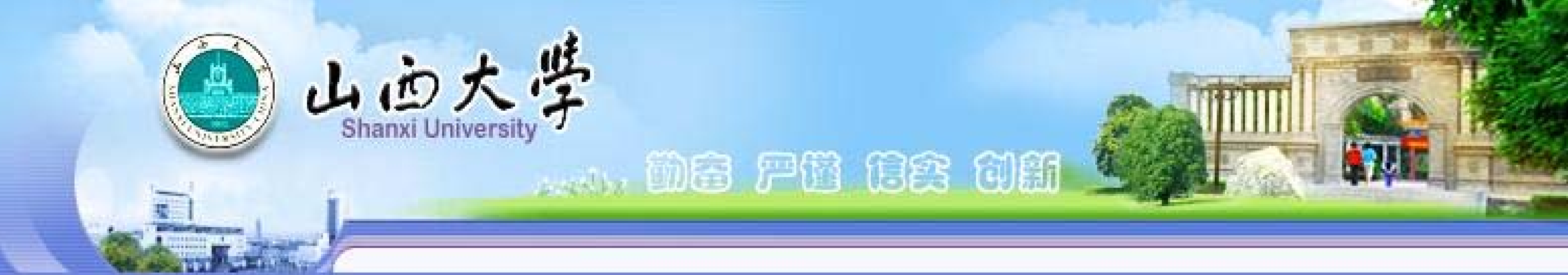


Nonlinear Quantum Physics in Bose Einstein Condensates

Wei-Dong Li

Institute of Theoretical Physics, Shanxi University

Outline

- Introduction
  - BECs presents one wonderful example on Mean field Theory
  - BECs with External potential: Single, double well and periodical potential
  - Developing Nonlinear quantum theory
  - Analytical Stationary solution for GPE
- One novel orthogonal basis for inhomogeneous BECs
- Nonlinear tunneling correction on BJJ
- Nonlinear Bloch functions and Wannier functions
- Weak force detector and quantum steps
- Conclusions

Introduction

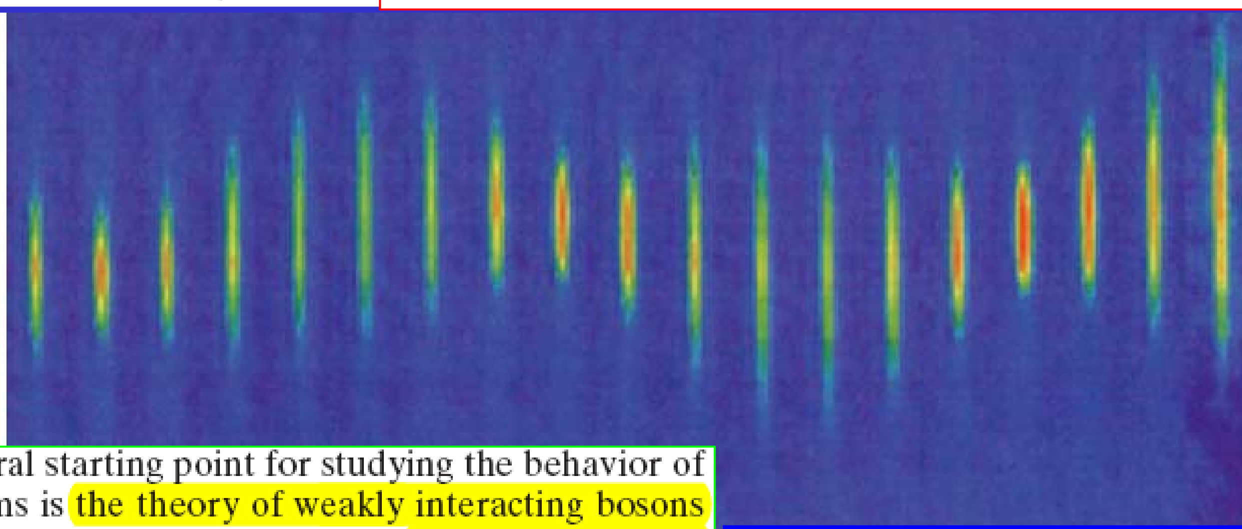
insight review articles

Bose–Einstein condensation of atomic gases

James R. Anglin & Wolfgang Ketterle

Research Laboratory for Electronics, MIT-Harva
Cambridge, Massachusetts 02139, USA

atoms. Condensates have become an ultralow-temperature laboratory for atom optics, collisional physics and many-body physics, encompassing phonons, superfluidity, quantized vortices, Josephson junctions and quantum phase transitions.



The natural starting point for studying the behavior of these systems is the theory of weakly interacting bosons which, for inhomogeneous systems, takes the form of the Gross-Pitaevskii theory. This is a mean-field ap-

Reviews of Modern Physics, Vol. 71, No. 3, April 1999

Introduction

Gross-Pitaevskii Equation

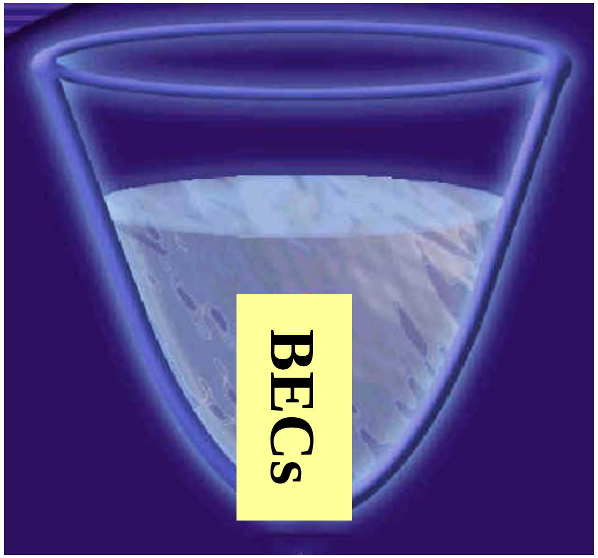
$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\partial_x^2\psi(x,t)+V_0(x)\psi(x,t)+g|\psi(x,t)|^2\psi(x,t)$$

$g=0$

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\partial_x^2\psi(x,t)+V_0(x)\psi(x,t)$$

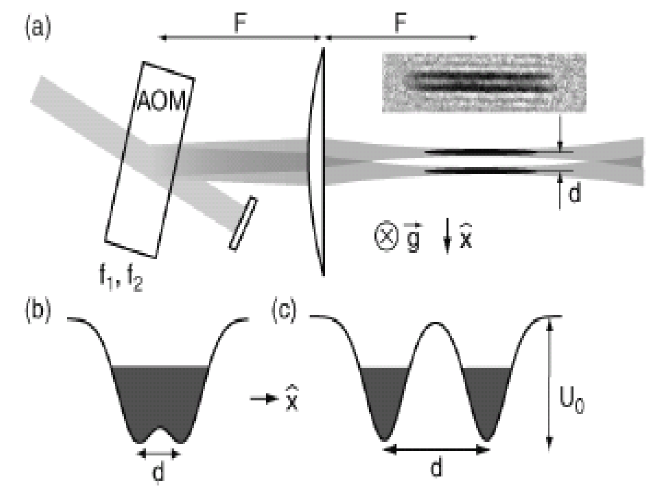
Schrödinger Equation **Basic principle, Dynamics, Time-dependent Theory**

Introduction: Single well and double wells

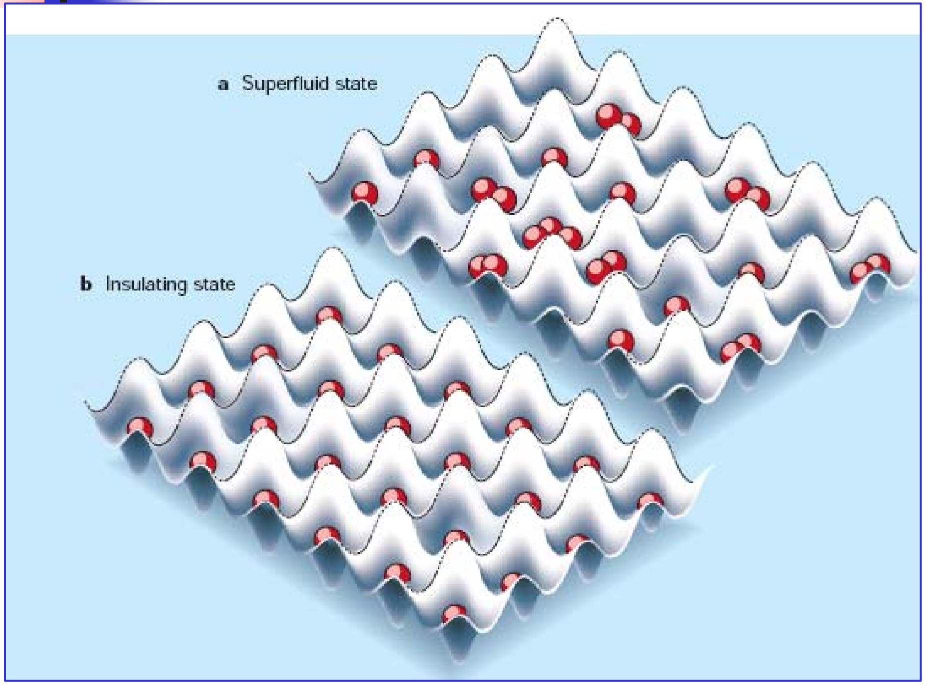


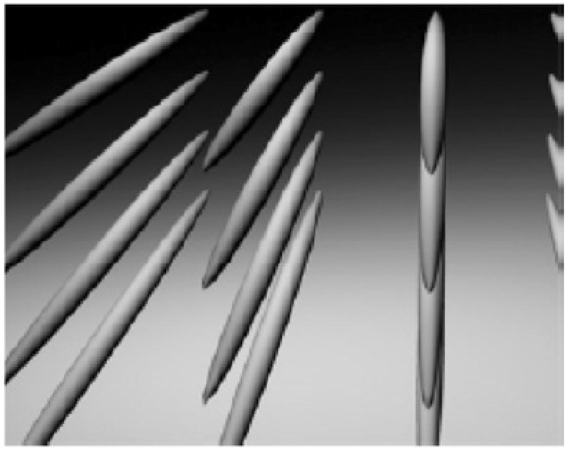
R. G. Hulet, Phys. Rev. Lett. **78** (1997)
Nature, 408, (2000)

W. Ketterle Group, Science, 2004
Phys. Rev. Lett. 92, (2004)
B. V. Hall et al., Phys. Rev. Lett. 98, (2007)

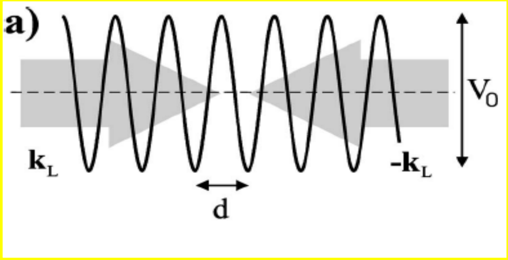


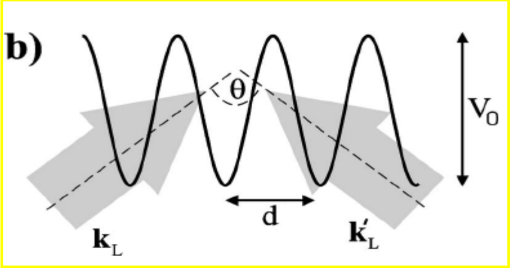
Introduction: Periodic Potential





Nature, 415, 39, 2002

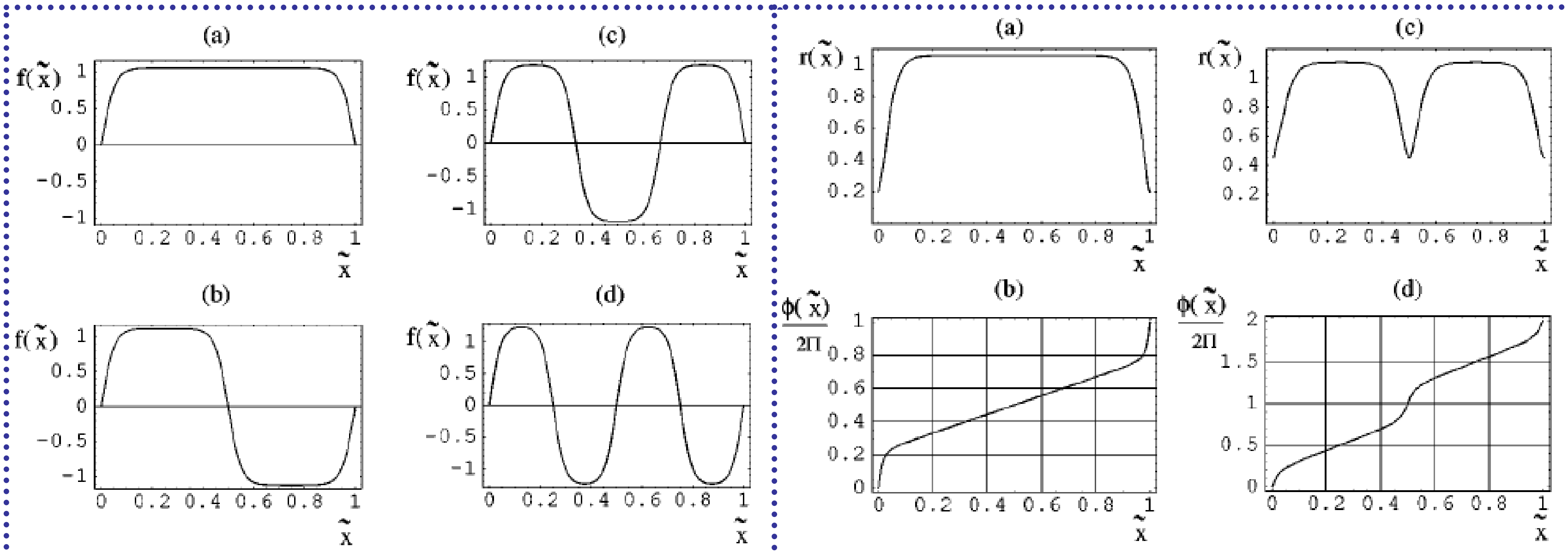




Rev. Mod. Phys. 78, 179 **(2006)**

Introduction: Box or ring

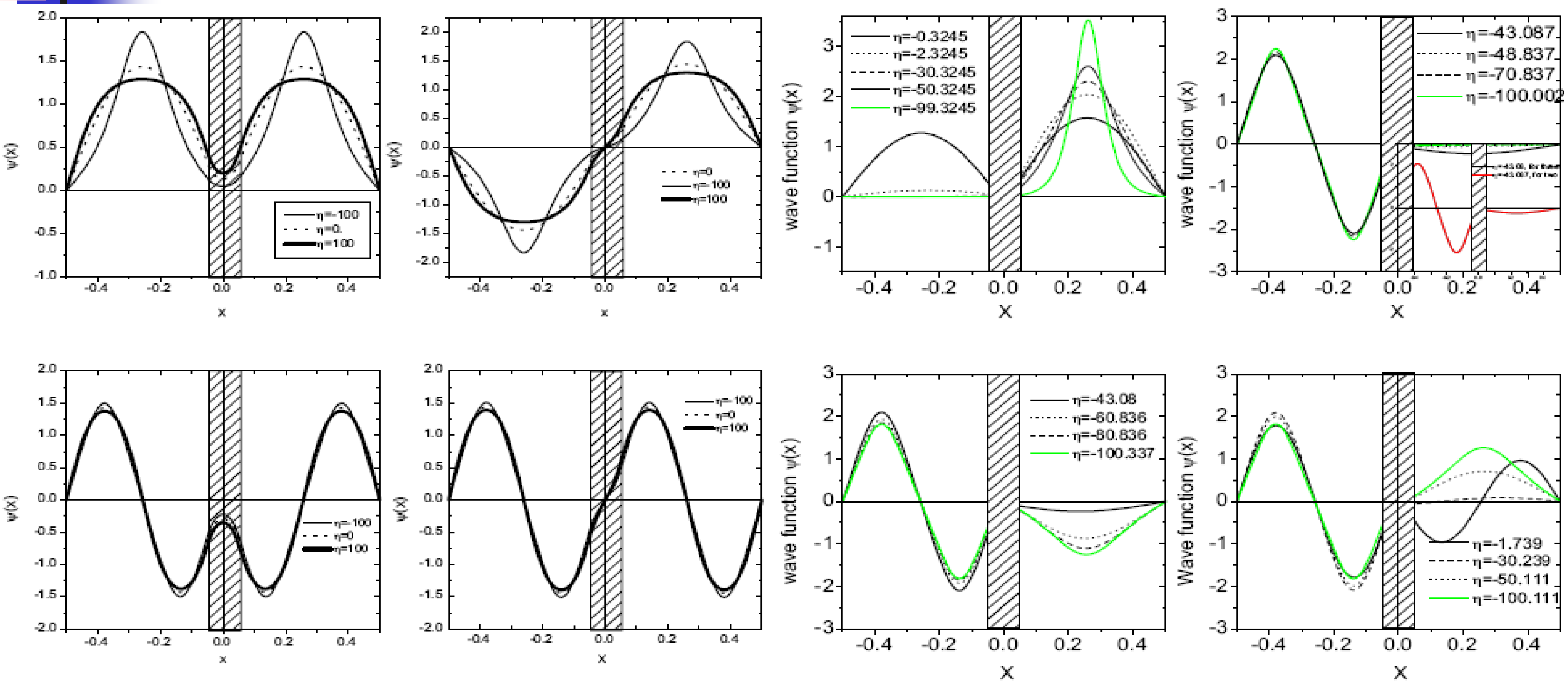
Developing Nonlinear Quantum Theory



Box boundary conditions **Periodic boundary conditions**

L. D. Carr, et. al. Phys. Rev. A. 62, 063610, 063611 (**1997**)

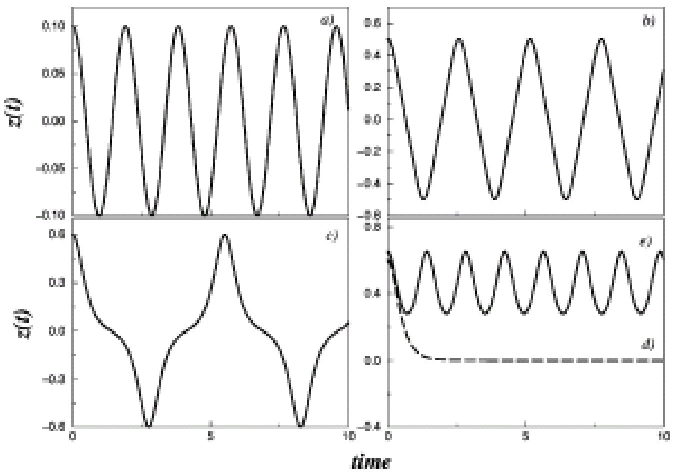
Introduction: Double wells



Symmetry-preserving solutions **Symmetry-breaking solutions**

Mahmud *et. al.* Phys. Rev. A 66, 063607 **(2002)**
WeiDong Li, Phys. Rev. A 74, 063612**, (2006)**

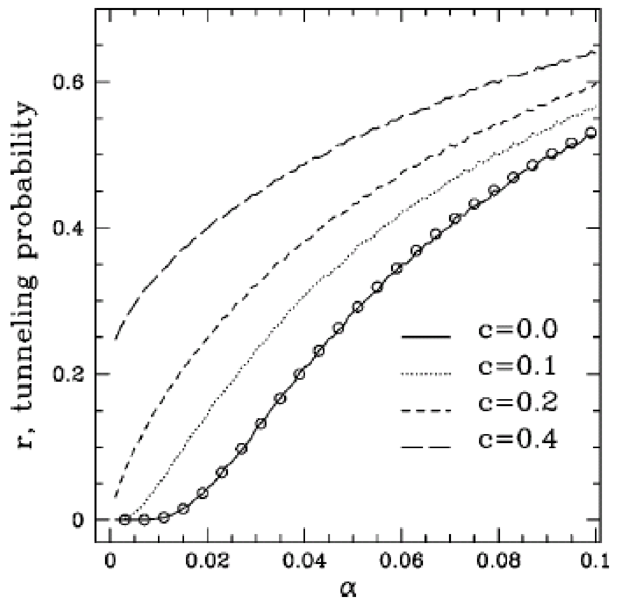
Introduction: Double wells



J. Javanainen, Phys. Rev. Lett. 57 3164 **(1986)**
A. Smerzi, Phys. Rev. A 59, 620, **(1999)**
Phys. Rev. Lett. 84, 4521 **(2000)**

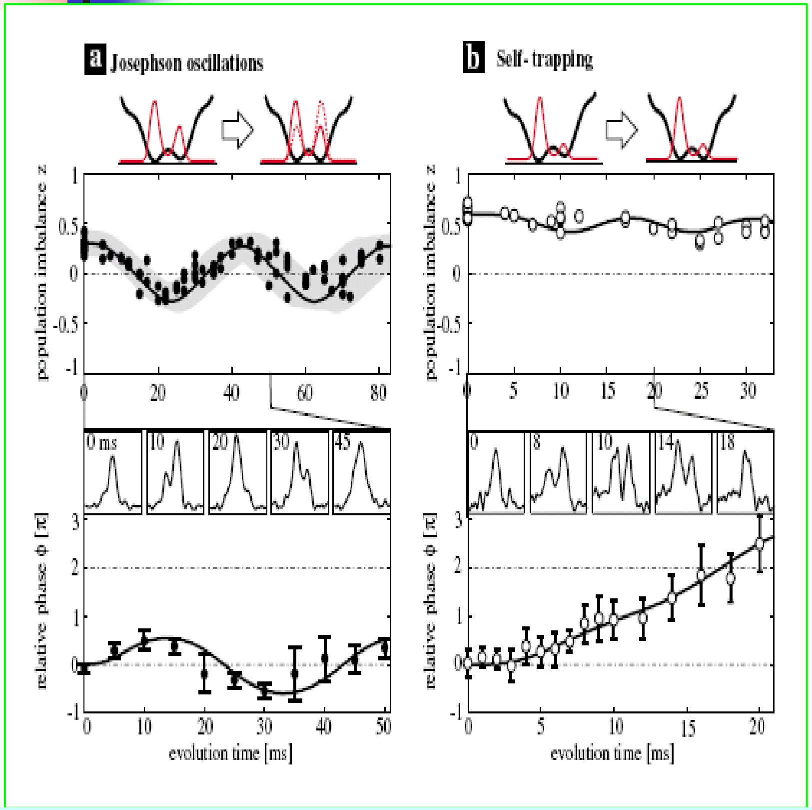
MQST

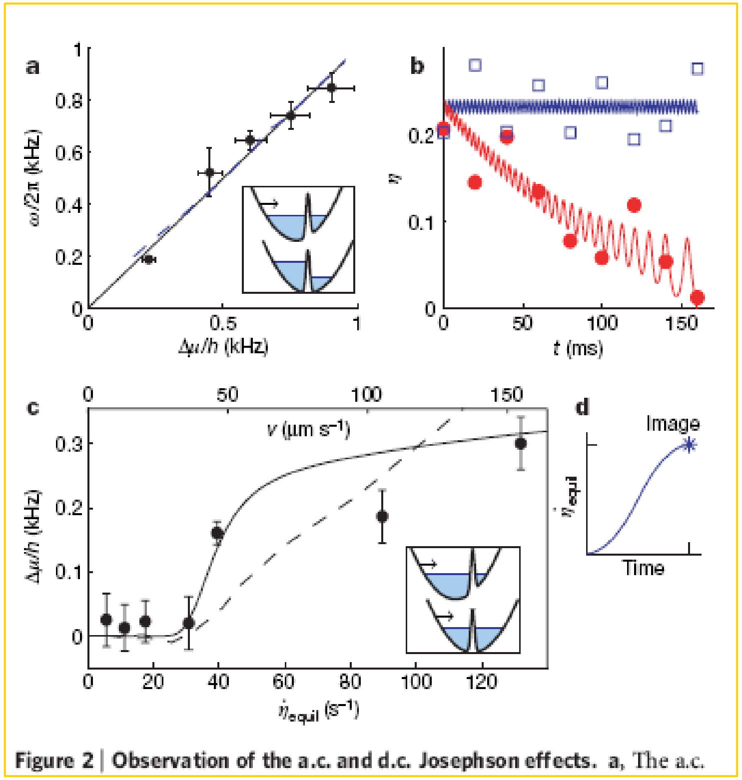
B. Wu, Phys. Rev. A, 61, 023402, **(2000)**



Nonlinear Landau-Zener

Introduction: Double wells



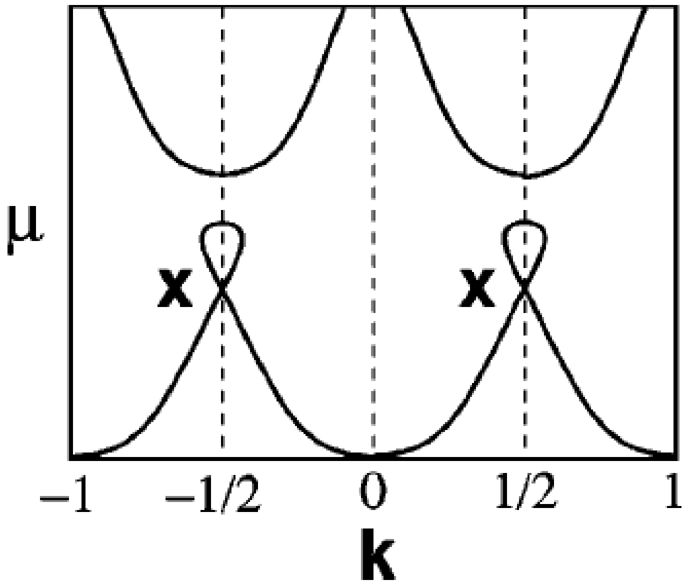


M. Albiez, et. al, PRL. 95, 010402 (2005)

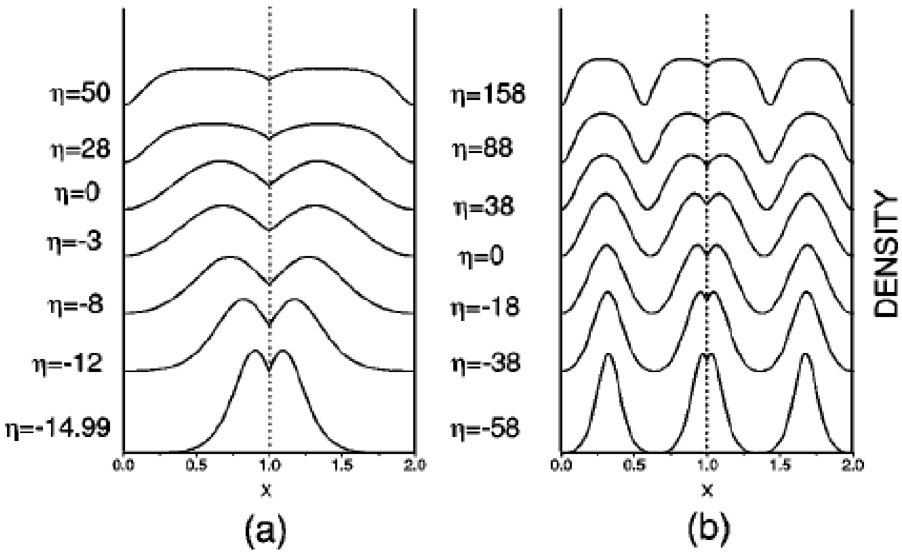
S. Levy, et. al, Nature, 449, 579 (2007)

Introduction: periodic potential

Loop structure



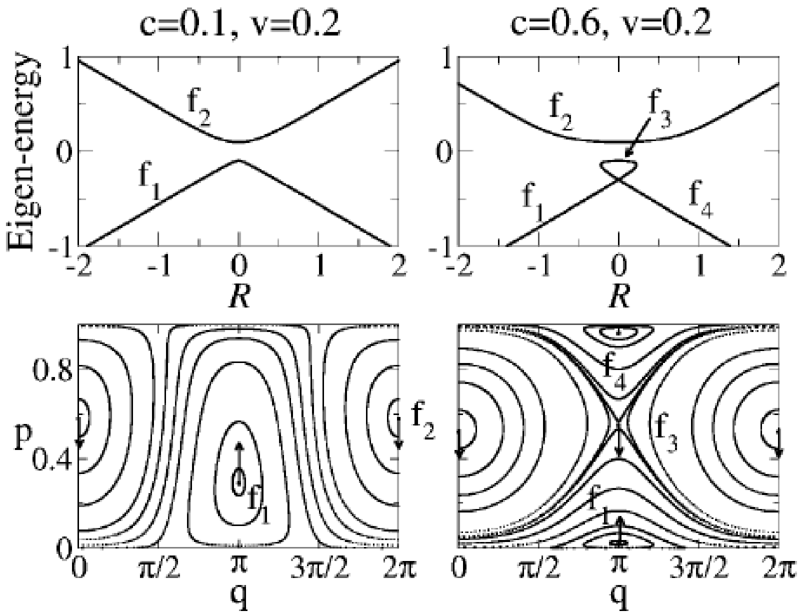
Non Bloch states



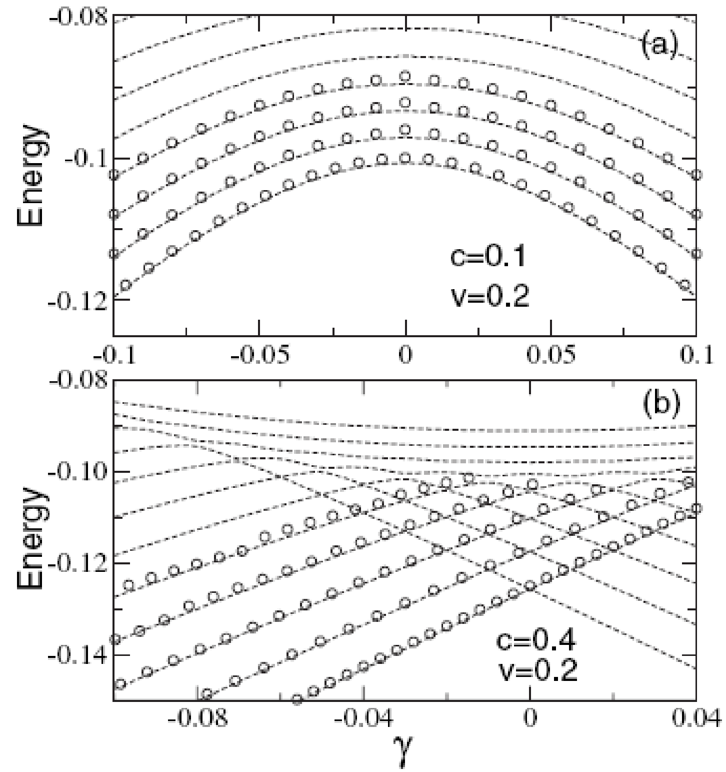
B. Wu, et. al., Phys. Rev. A, 61, 023402 **(2000);**
Phys. Rev. A, 65, 025601 **(2002)**
O. Zobay et. al., Phys. Rev. A, 64, 033603 **(2000);**
M. Machholm, et. al., Phys. Rev. A 67, 053613 **(2003);**
J. C. Bronski et. al., Phys. Rev. Lett. 86, 1402 **(2001);**

WeiDong Li, et. al., Phys. Rev. E, 70, 016605 **(2004);**
M. Machholm, et. al., Phys. Rev. A 67, 053613 **(2003);**

Introduction: Nonlinear Adiabatic Theory and so on



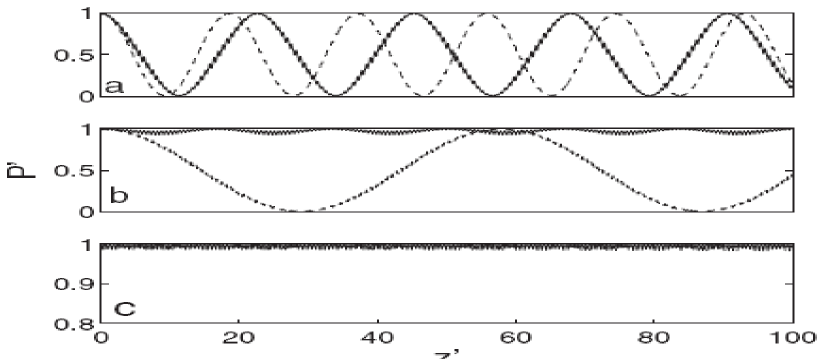
Adiabatic evolution

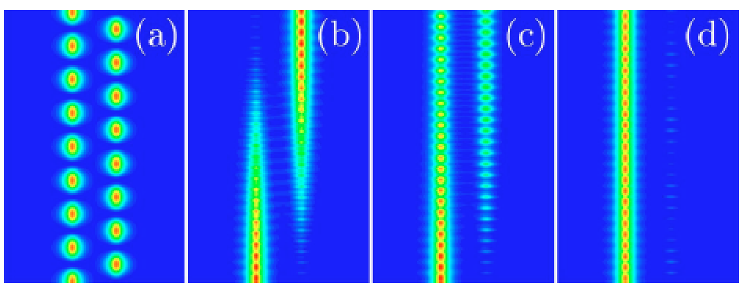


Commutability: Semi-classical Adiabatic limit

J. Liu, et. al., Phys. Rev. Lett. 90, 170404 **(2003)**
B. Wu, et. al., Phys. Rev. Lett. 94, 140402 **(2005)**
Phys. Rev. Lett. 86, 020405 **(2006)**

Introduction: time-dependent system

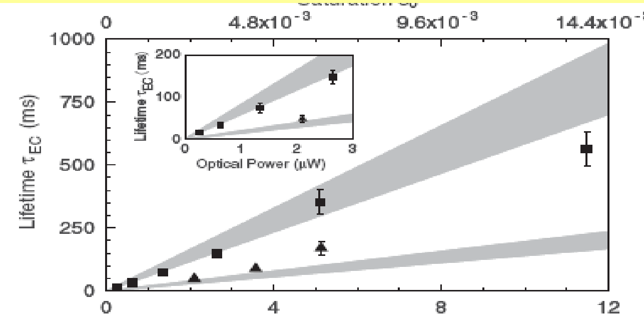


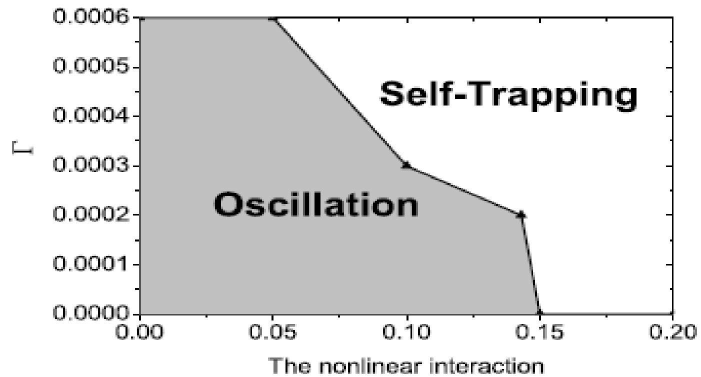


Nonlinear coherent destruction

B. Wu, et. al., Phys. Rev. A. 76, 051802 (R) **(2007)**
A. Szameit, et. Al., Opt. Lett. 34, 2700, **(2009)**
D. Ye, et. al., Phys. Rev. A, 77, 013402 **(2008)**

Nonlinear Quantum Zeno effect





E. W. Streed, et. al., Phys. Rev. Lett 97, 260402 **(2006)**
WeiDong Li, et. Al., Phys. Rev. A, 70, 016605 **(2004)**

Introduction: Nonlinear effect on QM

Nonlinear interaction

- **Novel stationary solutions**
- **Novel dynamics behaviors**
- **Positive effect for the time-dependent perturbation**
- **Broken some Quantum principles**
- **……**

GPE is also a very important starting point to understand the many-body Quantum Theory

Introduction:

Analytical Stationary solution for GPE

Gross-Pitaevskii Equation (GPE): $$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\partial_x^2\psi + V_0\psi + \eta|\psi|^2\psi$$

$$\psi(x) = \sqrt{\rho(x)}e^{-i\left(\Theta(x)-\frac{\mu}{\hbar}t\right)}$$

V_0 : **constant**.

$$\eta = \begin{cases} N_0 g_0 \\ g_0 \end{cases}$$

Hydrodynamic equations

A. V. Gurevich, Sov. Phys. JETP 65 (5), 1987

$$\left(\partial_x\rho(x)\right)^2 = 2\eta\rho^3(x) + 4(V_0-\mu)\rho^2(x) - \beta\rho(x) - 4\alpha^2$$

$$\Theta(x) = \int\frac{\alpha}{\rho(x)}dx + \phi$$

$$v(x) = \partial_x\Theta(x) = \frac{\alpha}{\rho(x)}$$ Velocity filed

α Current

Introduction:

Analytical Stationary solution for GPE

$$\rho_1(x)=\frac{A}{8k_1^2}-\left(\frac{A}{8k_1^2}-\frac{128k_1^4\alpha^2}{A(16k_1^4+A\eta)}\right)SN^2(k_1x+\delta_1,n_1) \qquad \mu>V_0$$

$$\mu=k_1^2+\left(\frac{A}{8k_1^2}+\frac{64k_1^4\alpha^2}{A(16k_1^4+A\eta)}\right)\eta$$

$$\rho_2(x)=\frac{B}{8k_2^2}+\left(\frac{B}{8k_2^2}+\frac{128k_2^4\alpha^2}{B(16k_2^4-B\eta)}\right)SC^2(k_2x+\delta_2,n_2) \qquad \mu<V_0$$

$$\mu=V_0-k_2^2+\left(\frac{B}{8k_2^2}+\frac{64k_2^4\alpha^2}{B(-16k_2^4+B\eta)}\right)\eta$$

$$\rho_2(x)=\frac{-B}{8k_2^2}+\left(\frac{k_2^2}{\eta}+\frac{B}{8k_2^2}-\frac{k_2\sqrt{Bk_2^2-16\alpha^2\eta}}{\sqrt{B}\eta}\right)NC^2(k_2x+\delta_2,n_2)$$

$$\mu=V_0-\frac{B}{8k_2^4}\eta-\frac{k_2\sqrt{Bk_2^2-16\alpha^2\eta}}{\sqrt{B}}$$

$$\rho_1(x)=\frac{A}{8k_1^2}-\left(\frac{A}{8k_1^2}-\frac{8\alpha^2}{A}\right)\sin^2(k_1x+\delta_1)$$

$$\mu=k_1^2$$

Linear Case:

$$\rho_2(x)=\frac{B}{8k_2^2}+\left(\frac{B}{8k_2^2}+\frac{8\alpha^2}{B}\right)\sinh^2(k_2x+\delta_2)$$

$$\rho_2(x)=\frac{-B}{8k_2^2}+\frac{B}{8k_2^2}\cosh^2(k_2x+\delta_2)$$

$$\mu=V_0-k_2^2$$

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- **One novel orthogonal basis for inhomogeneous BECs**
- Nonlinear tunneling correction on BJJ
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- Conclusions

Orthogonal basis for inhomogeneous BECs

Stationary solutions

$$\psi(x) = b \times SN\left(k(x+L/2), n\right), \qquad n = \frac{b^2}{2k^2}\eta;$$

$$\psi(x) = b \times CN\left(k(x+L/2), n\right), \qquad n = -\frac{b^2}{2k^2}\eta,$$

$$\mu = k^2 + \frac{b^2}{2}\eta$$

Boundary conditions

$$\psi(L/2) = 0, \qquad \psi(-L/2) = 0,$$

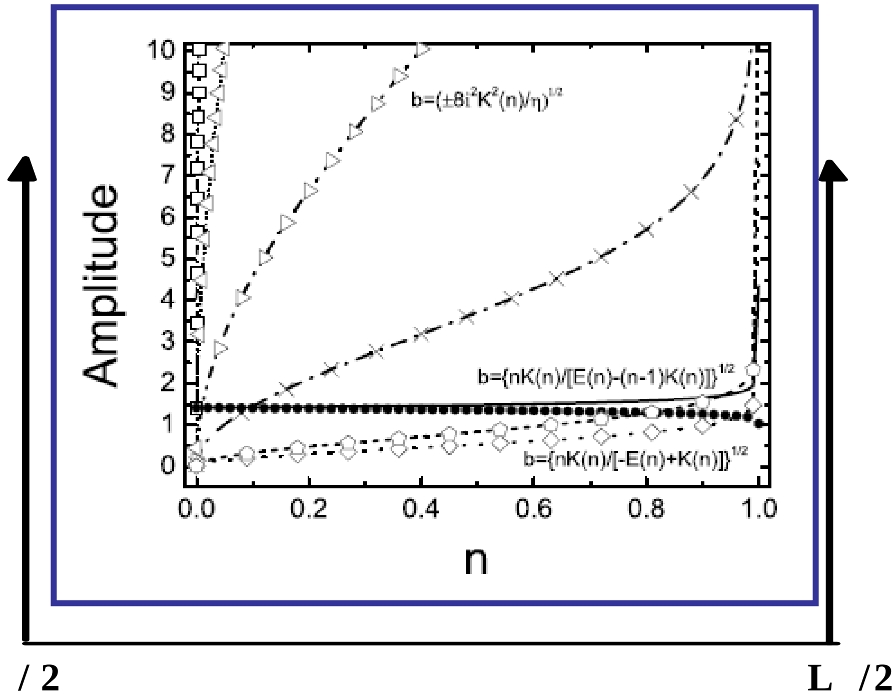
$$\int_{-L/2}^{L/2} |\psi(x)|^2 dx = 1.$$

$\eta < 0$

$$k = 2iK(n),$$
$$\delta = K(n)(i+1), \quad i = 1, 2, 3, 4, \ldots,$$
$$b = \sqrt{nK(n)/(E(n) + (n-1)K(n))};$$

$\eta > 0$

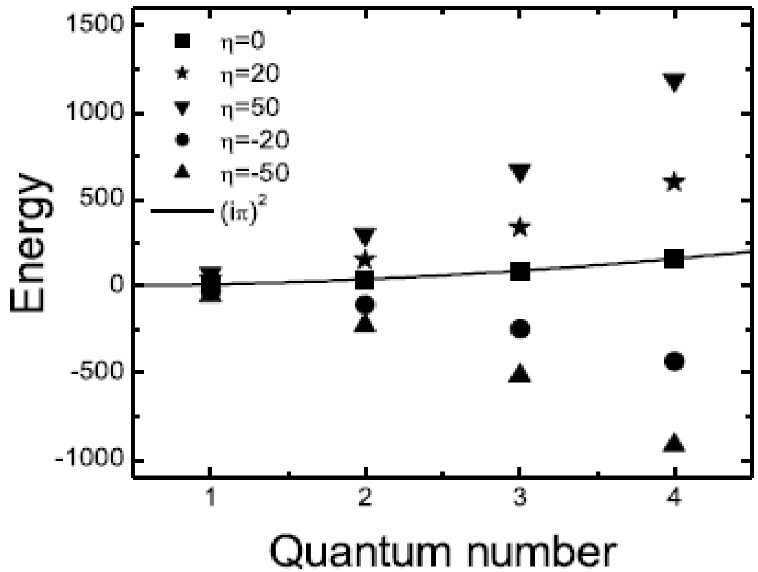
$$k = 2iK(n),$$
$$\delta = iK(n), \quad i = 1, 2, 3, 4, \ldots,$$
$$b = \sqrt{nK(n)/(-E(n) + K(n))},$$

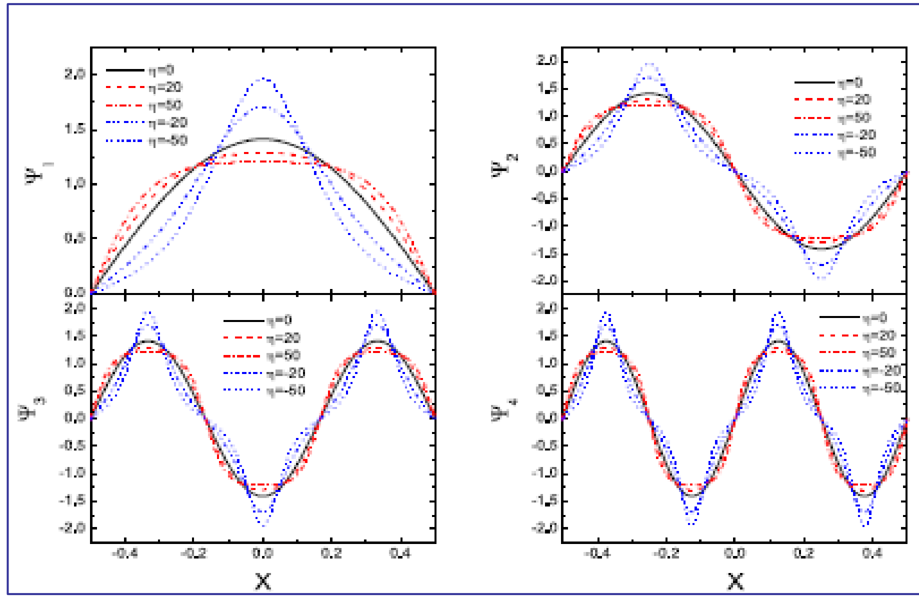


One-to-one correspondence

Phys. Lett. A 373 2764, **2009**

Orthogonal basis for inhomogeneous BECs





$$\mathrm{CN}(kx+\delta,n)=\cos\big(\mathrm{AM}(kx+\delta,n)\big),$$

$$\mathrm{SN}(kx+\delta,n)=\sin\big(\mathrm{AM}(kx+\delta,n)\big),$$

$$\alpha_j(z)=\cos[(2j+1)\pi z]\ \text{ or }\ \alpha_j(z)=\sin[2j\pi z],$$

$$\int_{-0.5}^{0.5}\alpha_i(z)\alpha_j(z)w(x)\,dx=c_{ij}\delta_{ij},\qquad z=\mathrm{AM}(kx+\delta,n)/\pi$$

where $w(x)=k\,\mathrm{DN}(kx+\delta,n)/\pi$, c_{ij} is the normalized constant and δ_{ij} just the Kronecker delta function. Therefore, we arrive at a gen-

Phys. Lett. A 373 2764, **2009**

Dynamical Stability analysis

Introducing one small derivation from object state

$$\Psi = \Psi_0 + \delta\psi$$

$$i\partial_t \begin{pmatrix} \delta\psi_\perp(\vec{r},t) \\ \delta\psi_\perp^\dagger(\vec{r},t) \end{pmatrix} = L \begin{pmatrix} \delta\psi_\perp(\vec{r},t) \\ \delta\psi_\perp^\dagger(\vec{r},t) \end{pmatrix}$$

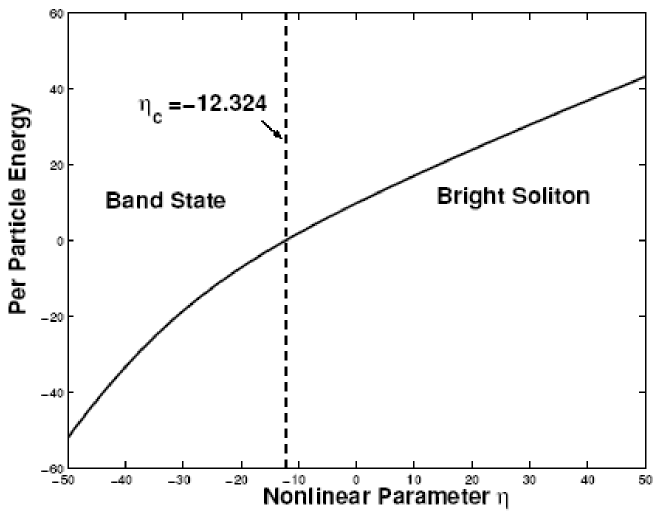
$$\hat{L} = \begin{pmatrix} \hat{A} & \hat{B} \\ -\hat{B} & -\hat{A} \end{pmatrix}$$

$$\hat{A} = \hat{Q}\,(H_{\mathrm{GP}} + \eta\Psi_0^2)\,\hat{Q} \quad \text{and} \quad \hat{B} = \hat{Q}\,\eta\Psi_0^2\,\hat{Q}$$

$$\hat{Q} = 1 - |\Psi_0\rangle\langle\Psi_0|,$$

$$H_{\mathrm{GP}} = -\partial^2/(\partial x^2) + \eta|\Psi_0|^2 - \mu$$

Dynamical Stability analysis of Ground state



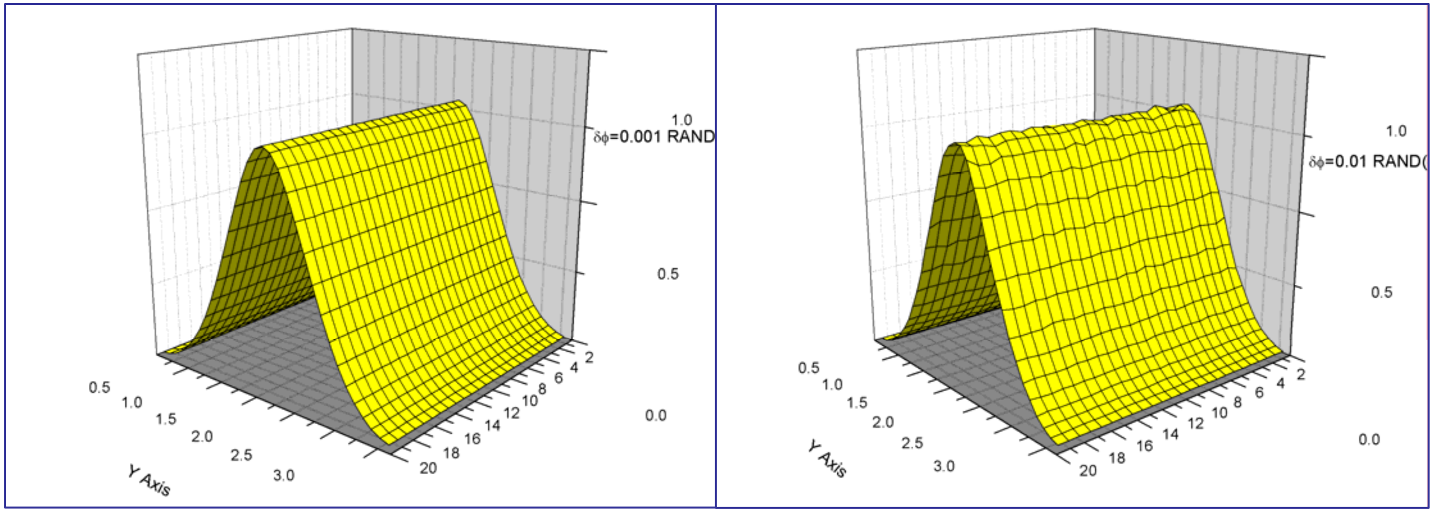
- Ground state is stable both for positive and negative interaction
- modify safely from positive to negative
- Gapless from positive to negative
- when smaller than the critical value, it is under bound state.
- This new method can be extended to excited state

R. Kanamoto, et. al, PRA 67, 013608 **(2003);**
A. Parola, et. al, PRA 72, 063612**, (2005).**

Phys. Lett. A 373 2764, **2009**

Dynamical Stability analysis of Ground state

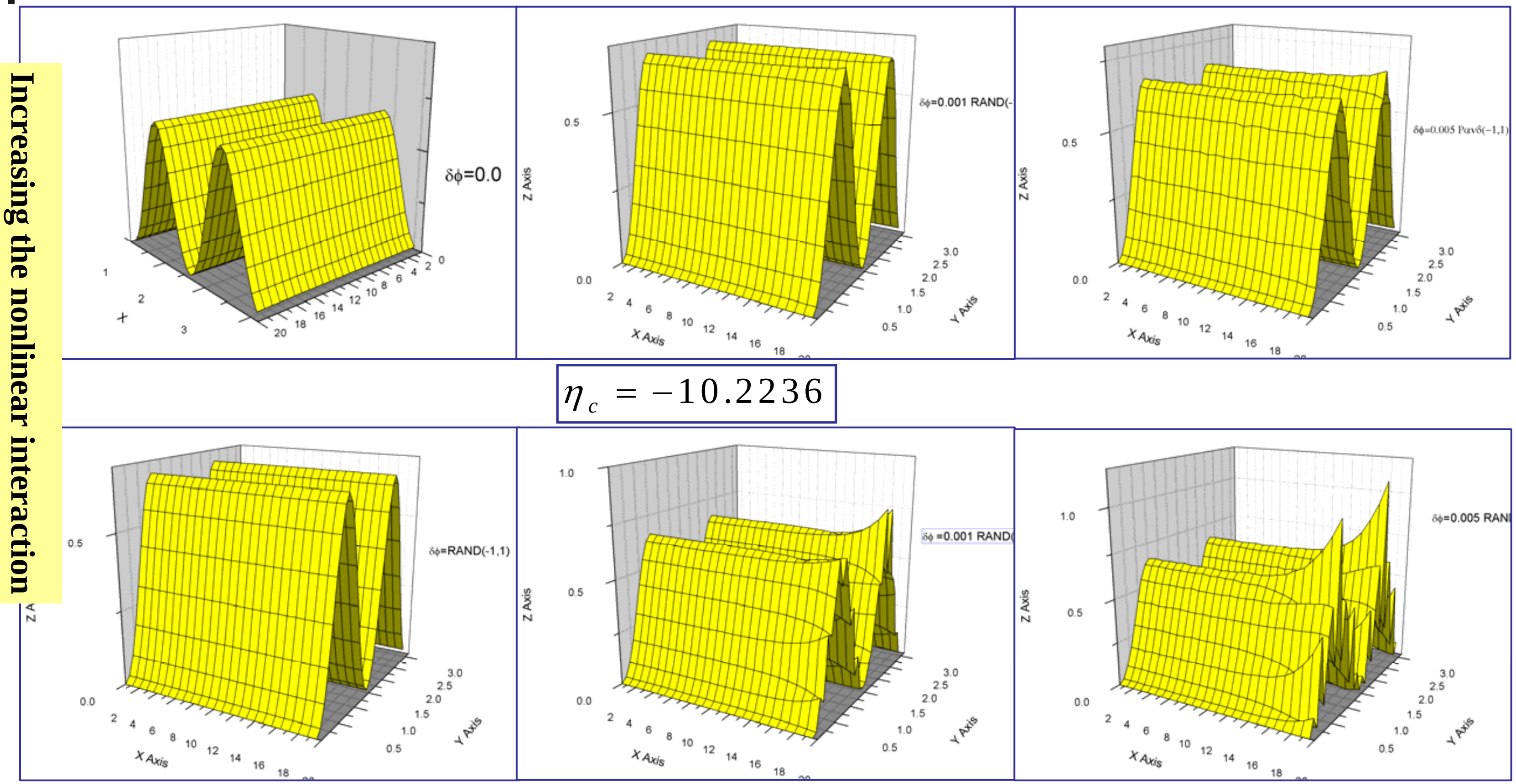
Numerical simulation supports our analytical calculation



Phys. Lett. A 373 2764, **2009**

Dynamical Stability analysis of first excited state

Increasing the nonlinear interaction



$\eta_c = -10.2236$

Phys. Lett. A 373 2764, **2009**

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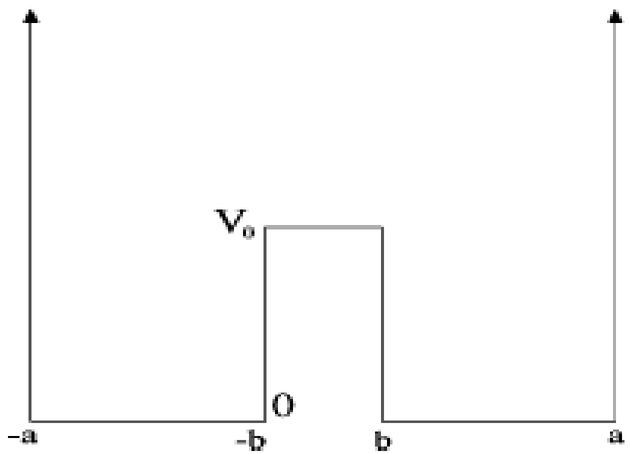
Nonlinear Correction for BJJ

Two modes approximation

$$\Psi(r,t) = \psi_1(t)\Phi_1(r) + \psi_2(t)\Phi_2(r).$$

Boson Josephson-Junction model (BJJ)

$$i\hbar\frac{\partial\psi_1}{\partial t} = \left(E_1^0 + U_1 N_1\right)\psi_1 - \kappa\psi_2,$$
$$i\hbar\frac{\partial\psi_2}{\partial t} = \left(E_2^0 + U_2 N_2\right)\psi_2 - \kappa\psi_1,$$



$$E_0 = \int \frac{\hbar^2}{2m}|\nabla\Phi|^2 + |\Phi|^2 V_{ext}(r)dr. \qquad U = g_0 \int |\Phi|^4 dr$$

$$\kappa \approx -\int\left[\Phi_1\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}\right)\Phi_2\right]dr.$$

A. Smerzi, *et al.*, *PRL*. **79**, 4950 (1997); S. Raghavan, *et al.*, *PRA* **59**, 620 (1999)

Nonlinear Correction for BJJ

neglect

$$\chi_2 = \eta\int \Phi_1^2(x)\Phi_2^2(x)\,dx$$

Keep the higher order terms

$$i\frac{\partial}{\partial t}\psi_1(t) = \left(E_1^0 + n_1U_1 + 2\Re\left(\psi_1^*(t)\psi_2(t)\right)\chi_1^1\right)\psi_1(t) - \left(\kappa + n_2\chi_1^2 + n_1\chi_1^1\right)\psi_2(t),$$

$$i\frac{\partial}{\partial t}\psi_2(t) = \left(E_2^0 + n_2U_2 + 2\Re\left(\psi_1^*(t)\psi_2(t)\right)\chi_1^2\right)\psi_2(t) - \left(\kappa + n_1\chi_1^1 + n_2\chi_1^2\right)\psi_1(t),$$

Where

$$E_i^0 = \int \Phi_i(x)\left(-\frac{\partial^2}{\partial x^2} + V(x)\right)\Phi_i(x)\,dx, \qquad \kappa = -\int \Phi_1(x)\left(-\frac{\partial^2}{\partial x^2} + V(x)\right)\Phi_2(x)\,dx,$$

$$U_i = \eta\int \left|\Phi_i(x)\right|^4 dx,$$

Nonlinear tunneling

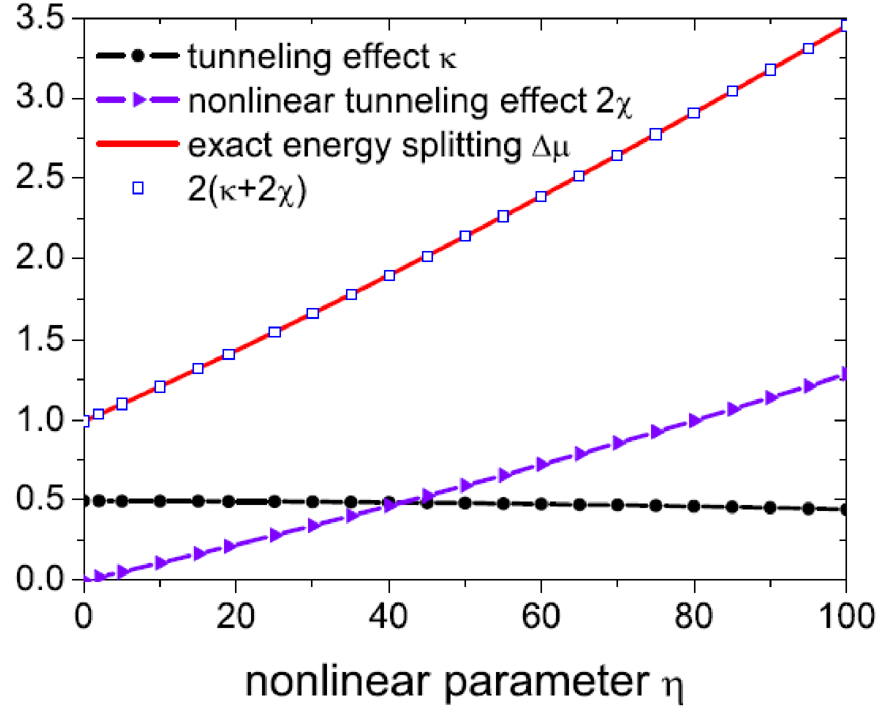
$$\chi_1^1 = -\eta\int \Phi_1^3(x)\Phi_2(x)\,dx, \quad \chi_1^2 = -\eta\int \Phi_1(x)\Phi_2^3(x)\,dx,$$

$$\chi_1^{1,2} = -\int \Phi_1(x)\left[\eta\Phi_{1,2}^2(x)\right]\Phi_2(x)\,dx$$

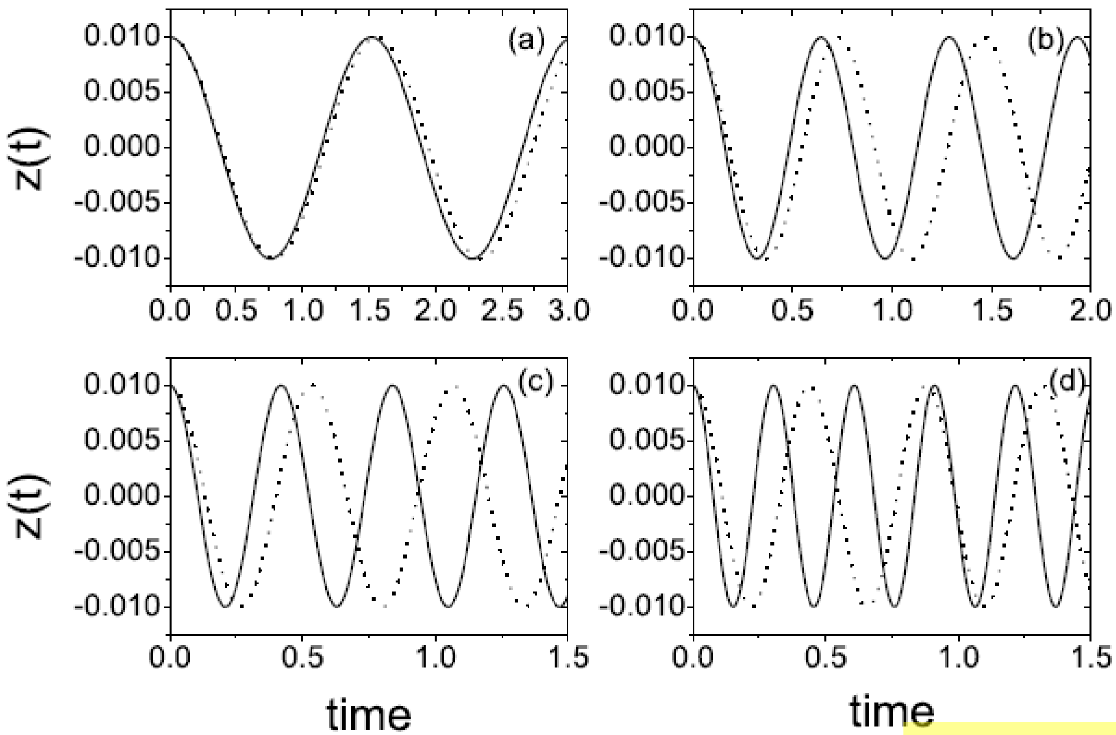
Nonlinear Correction for BJJ

TABLE I: The values of $\mathcal{K}$, $\chi_1^{1,2}$ and χ_2 for given η

η	$\mathcal{K}$	$\chi_1^{1,2}$	χ_2
10	0.494044	0.0544708	0.0035229
20	0.492262	0.111326	0.00532183
30	0.489441	0.170379	0.0109569
35	0.487659	0.200695	0.0129352
40	0.485633	0.231526	0.0149725
45	0.483365	0.262870	0.0170735
50	0.480852	0.294722	0.0192429
60	0.475089	0.359955	0.0238032
70	0.468318	0.427239	0.0286850
80	0.460511	0.496601	0.0339178



Nonlinear tunneling effect shifts the period of the oscillation modes



—— Including χ

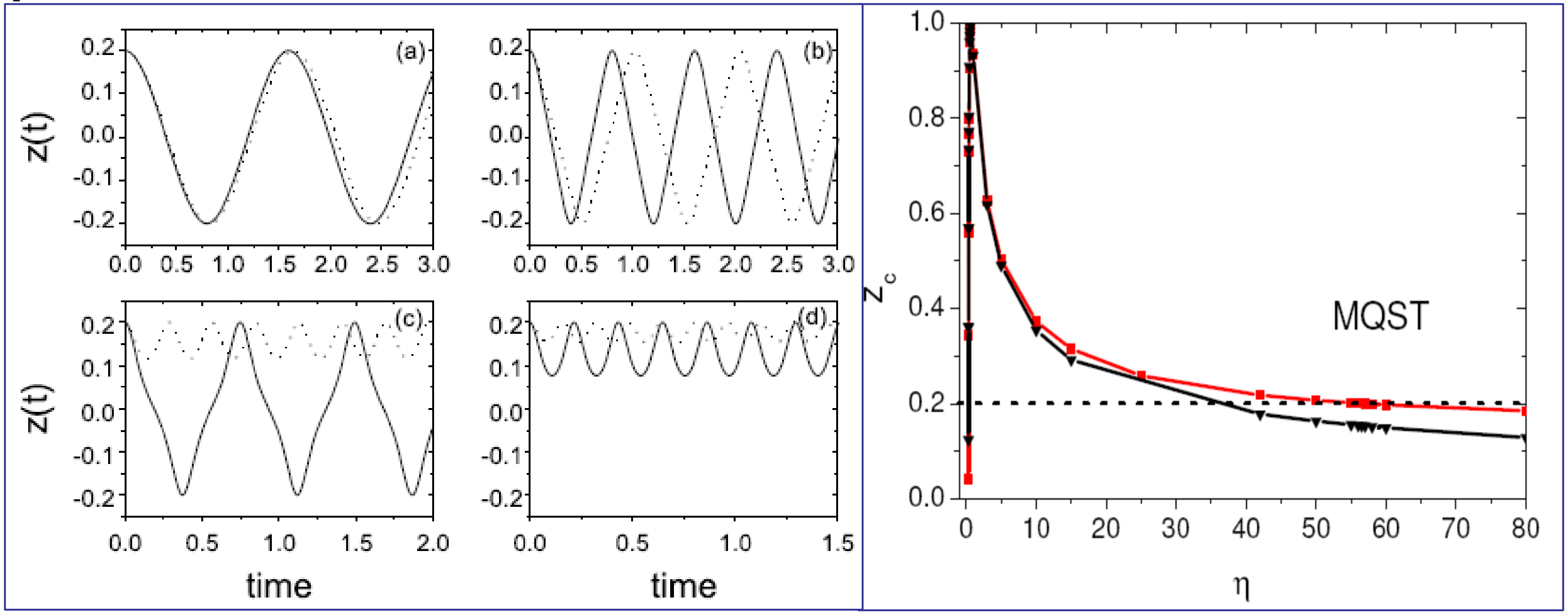
- - - - Without χ

Initial condition

$$\phi(0)=0, z(0)=0.01$$

<u>Xin-Yan Jia</u>, Wei-Dong Li and J. Q. Liang,
***Phys. Rev. A* 78 ,** 023613 **(2008).**

Nonlinear tunneling effect corrects the MQST conditions



Initial condition $\phi(0)=0, z(0)=0.2$

$(a)\ \eta=5, (b)\ \eta=25,\ (c)\ \eta=50, (d)\ \eta=80.$

<u>Xin-Yan Jia</u>, Wei-Dong Li and J. Q. Liang,
***Phys. Rev. A* 78 ,** 023613 **(2008).**

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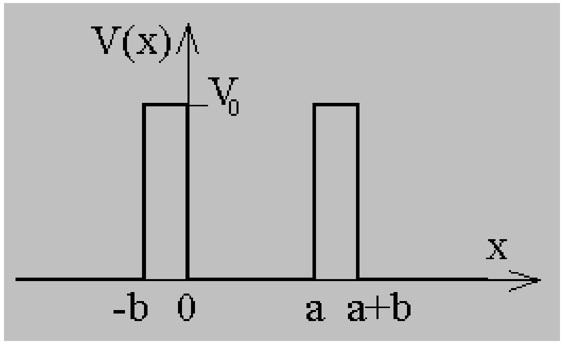
- Nonlinear tunneling correction on BJJ
- **Nonlinear Bloch functions and Wannier functions**
- Weak force detector and quantum steps
- Conclusions

On Periodic Square wells

The Kronig-Penny Potential



$$V(x)=\begin{cases}0 & 0<x<a\\ V_0 & a<x<a+b\end{cases}$$

Stationary Schrodinger equation

$$\mu\psi=-\frac{\hbar^2}{2m}\partial_x^2\psi+V(x)\psi$$

$$\psi(x)=\begin{cases}Ae^{iKx}+Be^{-iKx} & \text{in well}\\ Ce^{Qx}+De^{-Qx} & \text{within barrier}\end{cases}$$

$$\rho(x)=\begin{cases}\dfrac{A}{8K^2}-\left(\dfrac{A}{8K^2}-\dfrac{8\alpha^2}{A}\right)\sin^2(Kx+\delta)\\ \dfrac{B}{8Q^2}+\left(\dfrac{B}{8Q^2}+\dfrac{8\alpha^2}{B}\right)\sinh^2(Qx+\gamma)\end{cases}$$

Krong-Penny Models

Using the matching conditions:

$$\psi_1(x)|_{x=0} = \psi_2(x)|_{x=0}$$

$$\frac{\partial \psi_1(x)}{\partial x}|_{x=0} = \frac{\partial \psi_2(x)}{\partial x}|_{x=0}$$

$$\varepsilon_1 = \varepsilon_2, \qquad \int_{x=-b}^{x=0} \rho_2(x)dx + \int_{x=0}^{x=a} \rho_1(x)dx = 1$$

Bloch Theory

$$\psi_2(x)|_{x=-b} = \psi_1(x)|_{x=a}\ e^{iqT}$$

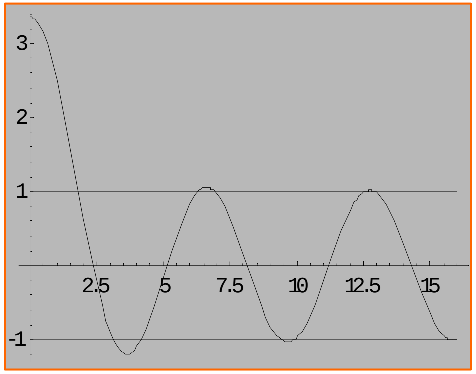
$$\frac{\partial \psi_2(x)}{\partial x}|_{x=-b} = \frac{\partial \psi_1(x)}{\partial x}|_{x=a}\ e^{iqT}$$

$$\frac{Q^2 - K^2}{2KQ}\sinh Qb \sin Ka + \cos Ka \cosh Qb = \cos qT$$

Delta potential

$$\lim_{\substack{b\to 0\\ V_0\to\infty}} V_0 b = P$$

$$\frac{P}{Ka}\sin Ka + \cos Ka = \cos qT$$



Nonlinear Bloch functions

Periodic boundary condition

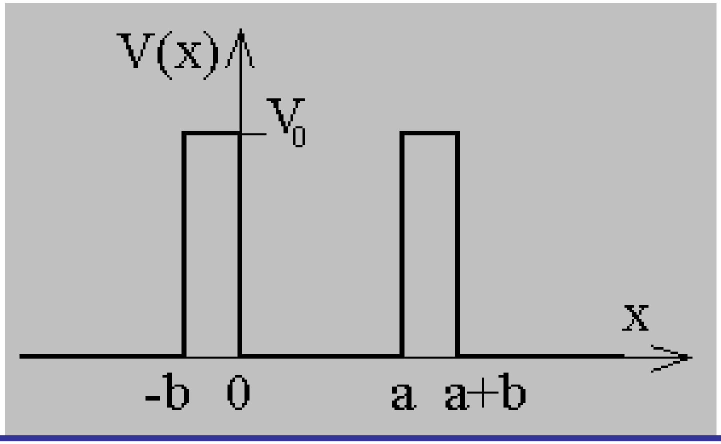
$$\rho_1(a)=\rho_2(a),\quad \partial_x\rho_1(a)=\partial_x\rho_2(a),$$

$$k=-\theta(x)=-\alpha\int_0^T\frac{dx}{\rho(x)}$$

$$\rho_1(0)=\rho_2(T),\quad \partial_x\rho_1(0)=\partial_x\rho_2(T),$$

$$\mu_1=\mu_2,$$

$$\int_0^T\left|\psi_k(x)\right|^2dx=\int_0^a\rho_1(x)\,dx+\int_a^T\rho_2(x)\,dx=1.$$

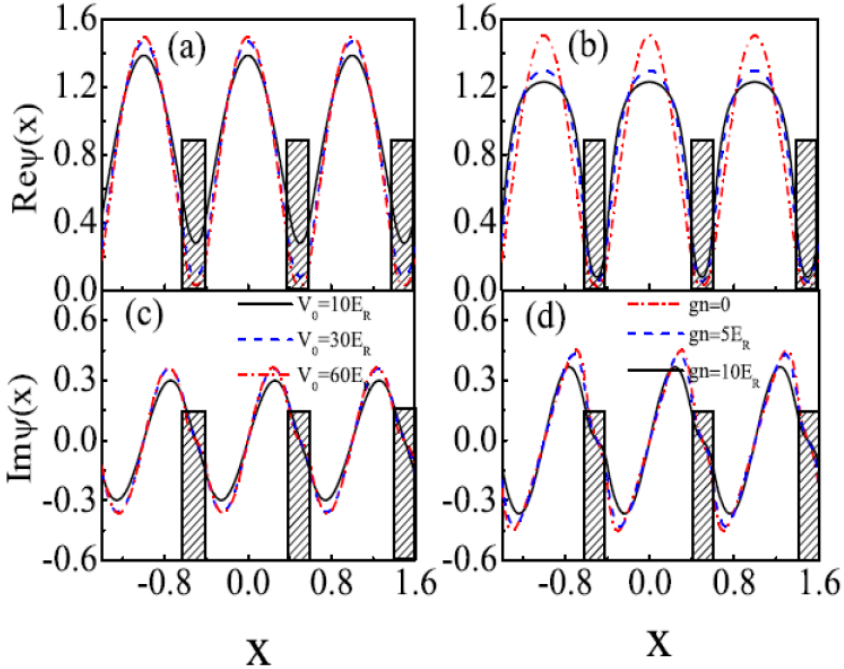


$$\rho_1(x)=\frac{A}{8k_1^2}-\left(\frac{A}{8k_1^2}-\frac{128k_1^4\alpha^2}{A\left(16k_1^4+A\eta\right)}\right)SN^2\left(k_1x+\delta_1,n_1\right)$$

$$\rho_2(x)=\frac{B}{8k_2^2}+\left(\frac{B}{8k_2^2}+\frac{128k_2^4\alpha^2}{B\left(16k_2^4-B\eta\right)}\right)SC^2\left(k_2x+\delta_2,n_2\right)$$

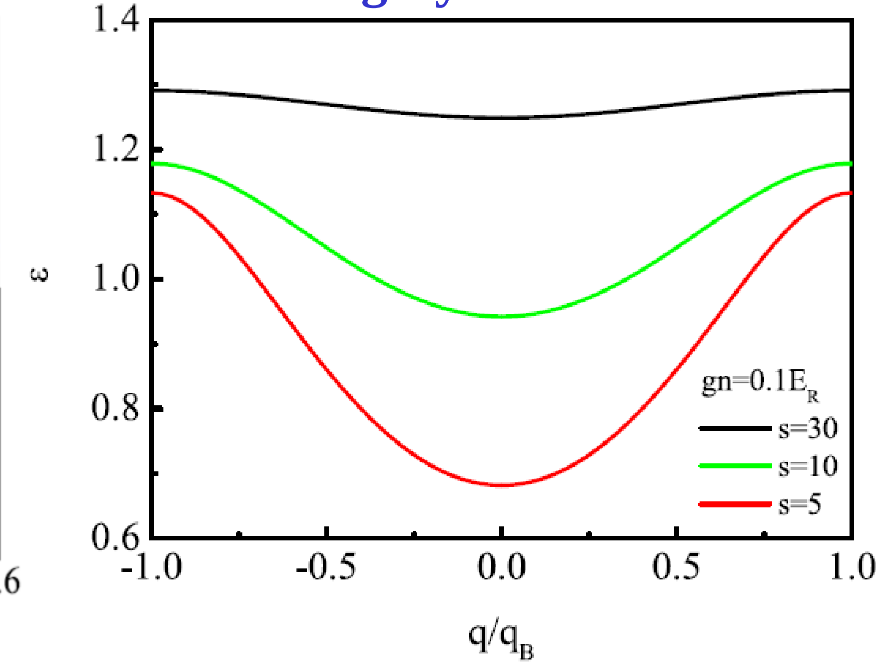
<u>Rui Xue</u>, Z. Liang and Wei-Dong Li,
J. Phys. B: at Mol. Opt. Phys. 42, 085302 **(2009).**

Nonlinear Bloch functions



$$a = 0.6 \quad b = 0.4 \quad k = 0.5\pi$$

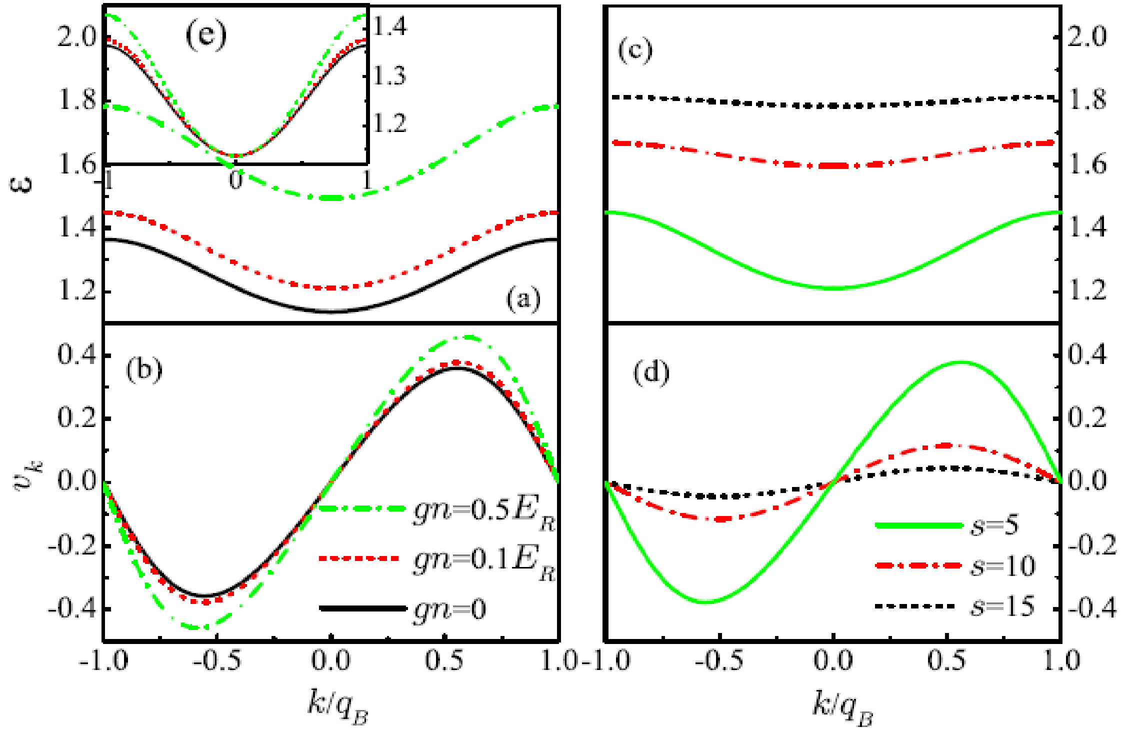
Engery Bloch



<u>Rui Xue</u>, Z. Liang and Wei-Dong Li,
J. Phys. B: at Mol. Opt. Phys. 42, 085302 **(2009).**

Nonlinear Bloch functions

Bloch energy band and group velocity



$$s = 5$$

$$gn = 0.1E_R$$

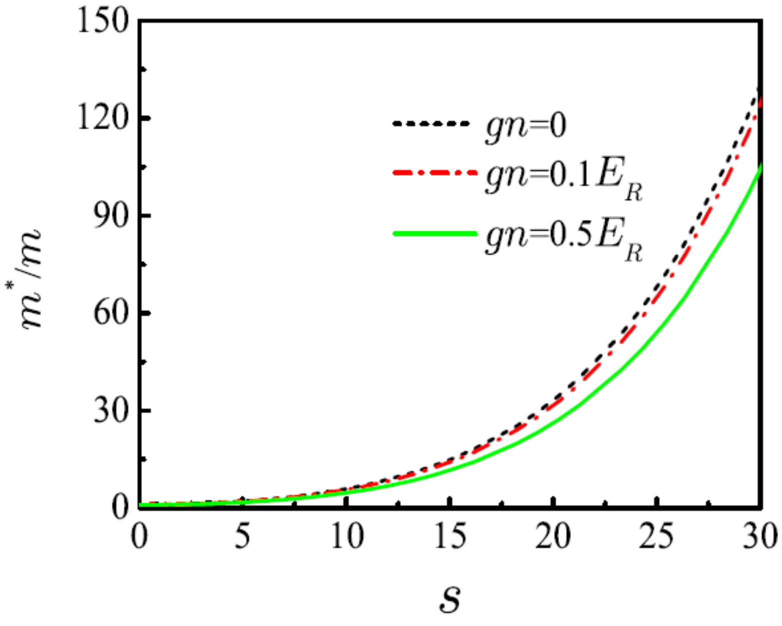
$$v_k = \partial \varepsilon / \partial k$$

J. Phys. B: 42, 085302 **(2009).**

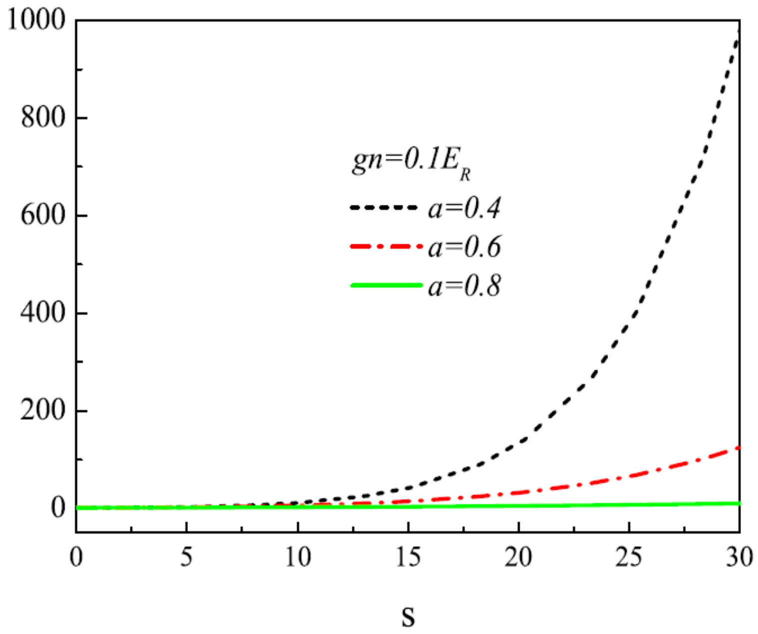
Nonlinear Bloch functions

Effective mass

$$\frac{1}{m^*} = \left.\frac{\partial^2 \varepsilon(k)}{\partial k^2}\right|_{k=0}$$



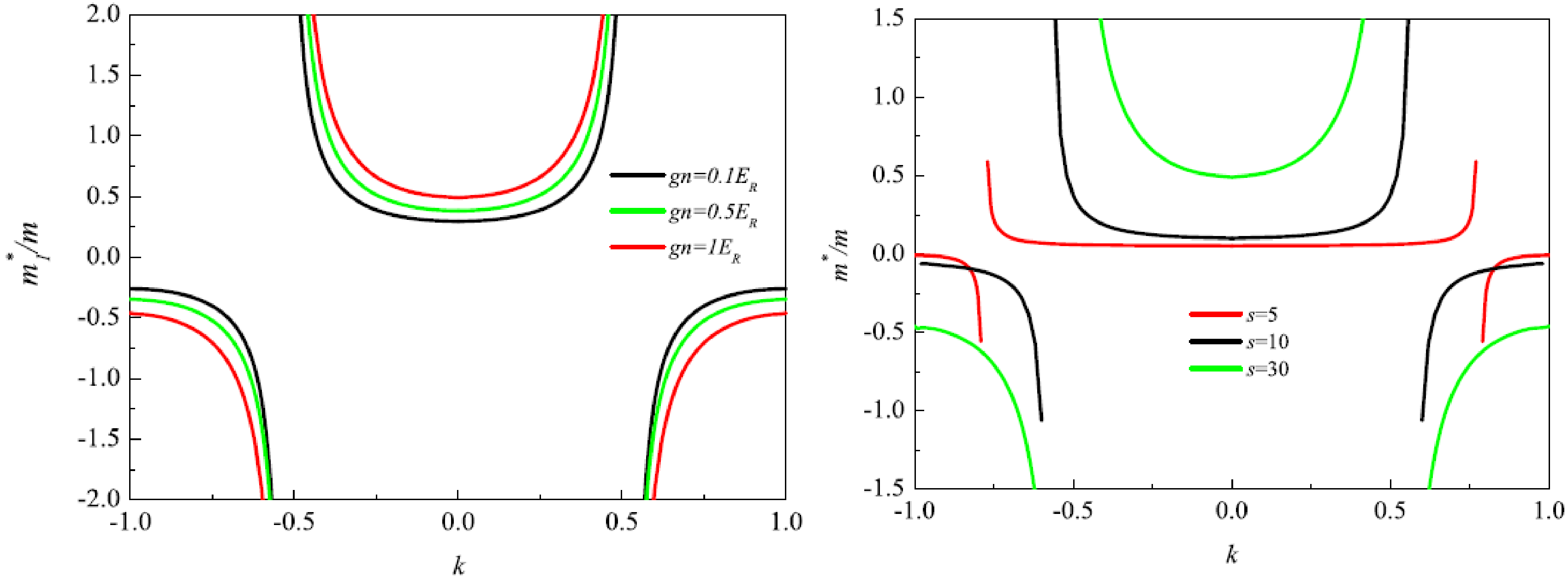
$a = 0.6 \quad b = 0.4$



J. Phys. B: 42, 085302 **(2009).**

Nonlinear Bloch functions

Effective mass



$s = 30$

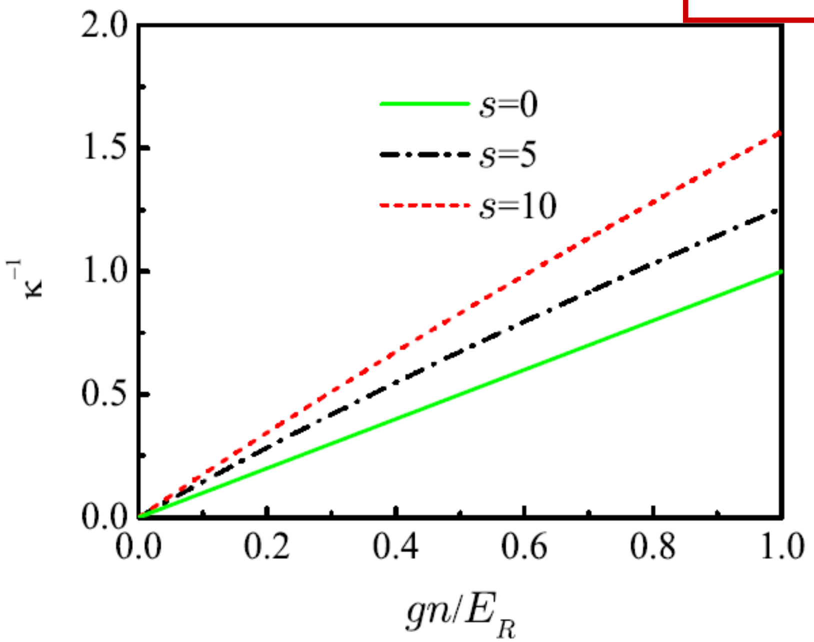
$$\frac{1}{m^*} = \frac{\partial^2 \varepsilon(k)}{\partial k^2}$$

$gn = 0.1E_R$

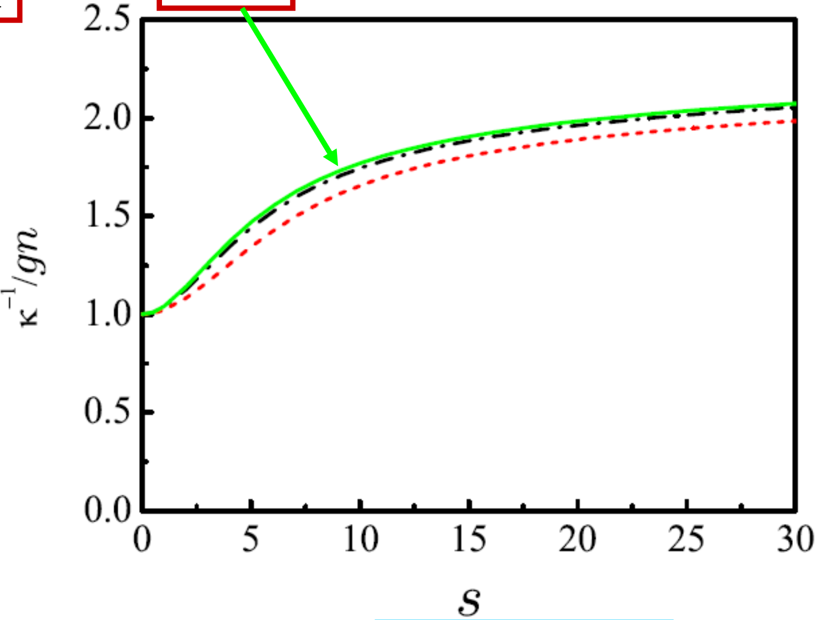
Nonlinear Bloch functions

Compressiblity

$$\kappa^{-1} = n\frac{\partial \mu}{\partial n}$$



$\tilde{g}/g$



$$\kappa^{-1} = \tilde{g}(s)n$$

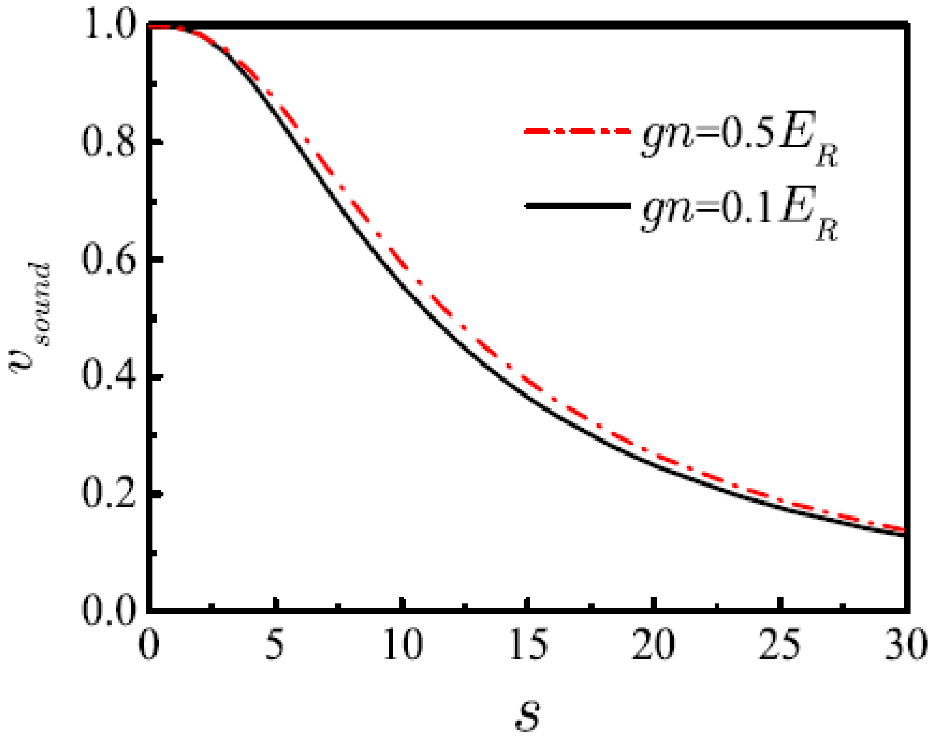
$$\tilde{g} = g\int_0^T \phi_{\eta=0}^4(x)\,dx$$

Eur.Phys. J. D (2003) **27** 247

Phys. Rev. A (2008) **78**, 023622

Nonlinear Bloch functions

Sound velocity



Eur. Phys. J. D (2003) **27**, 247

Phys. Rev. A (2004) **70**, 023609

$$v_{sound} = \frac{1}{\sqrt{\kappa m^*}}$$

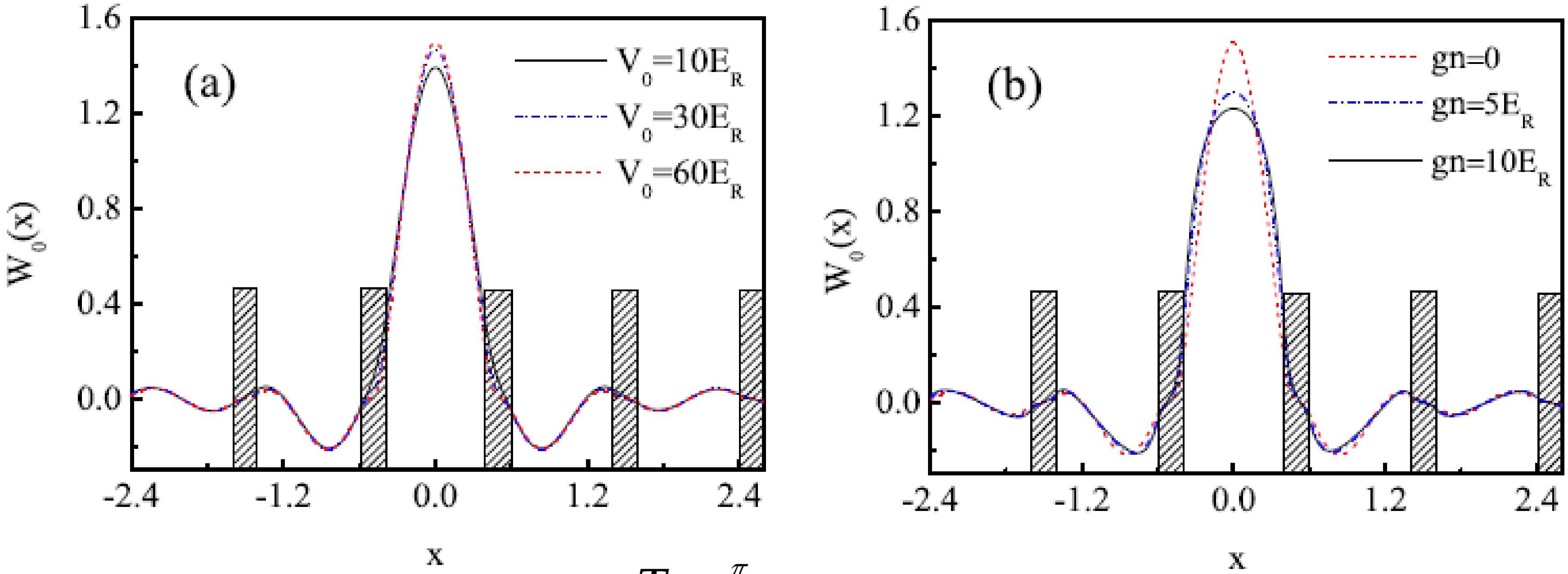
Nonlinear Wannier functions

$$W_l(x-x_i)=\frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}}\psi_{lk}(x)e^{-ikx_i}dk$$

- Symmetric
- Real
- Fall off exponential as $W_l(x)\sim\exp(-\overline{h}_l x)$

W. Kohn, *Phys. Rev.* (1959) **115**, 809

Nonlinear Wannier functions



$$W_l(x) = \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \sqrt{\rho(x)}e^{-i\theta(x)}e^{-iqx}dq$$

Chin. Phys. B: 18, 4130 **(2009).**

Nonlinear Wannier functions

Bose Harbard model

$$\psi(x) = \sum_i a_i W_0(x - x_i)$$

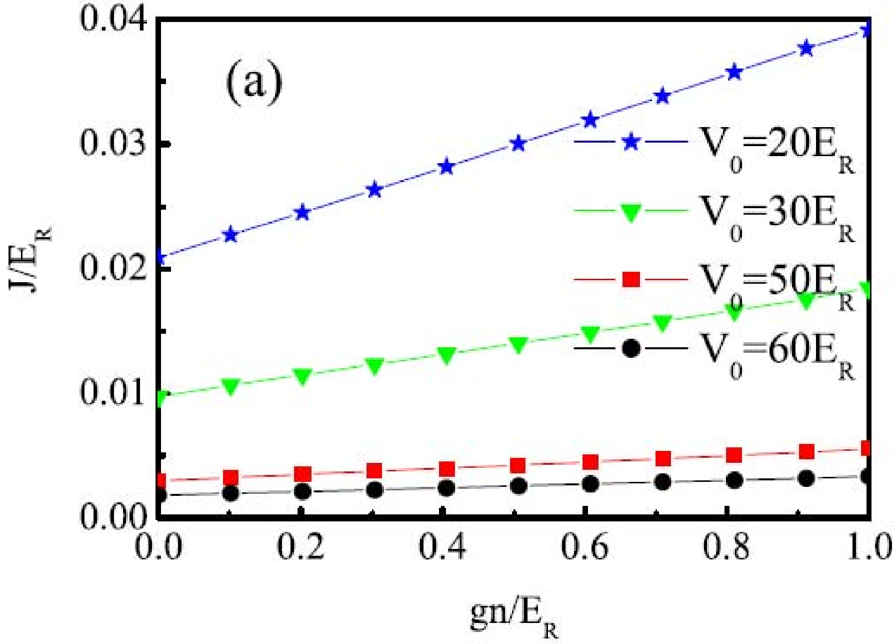
$$H = -J \sum_{<i,j>} a_i^+ a_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$$

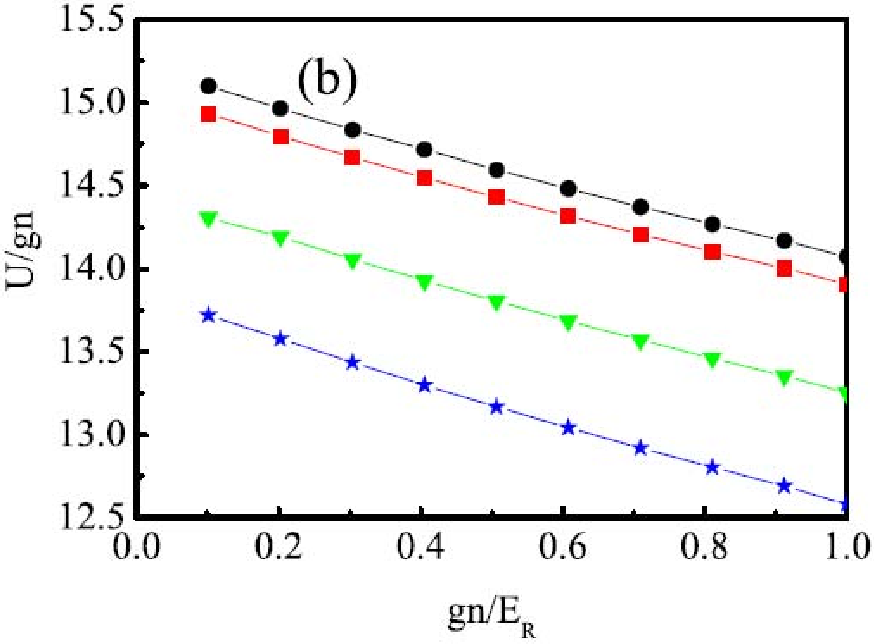
$$J = -\int dx W_0^*(x - x_i)\{-\frac{\hbar^2}{2m}\nabla^2 + V_{pot}(x)\} W_0(x - x_j)$$

$$U = \frac{4\pi a_s \hbar^2}{m} \int dx \left| W_0(x - x_i) \right|^4$$

U/J **very important parameters for system**

Nonlinear Wannier functions

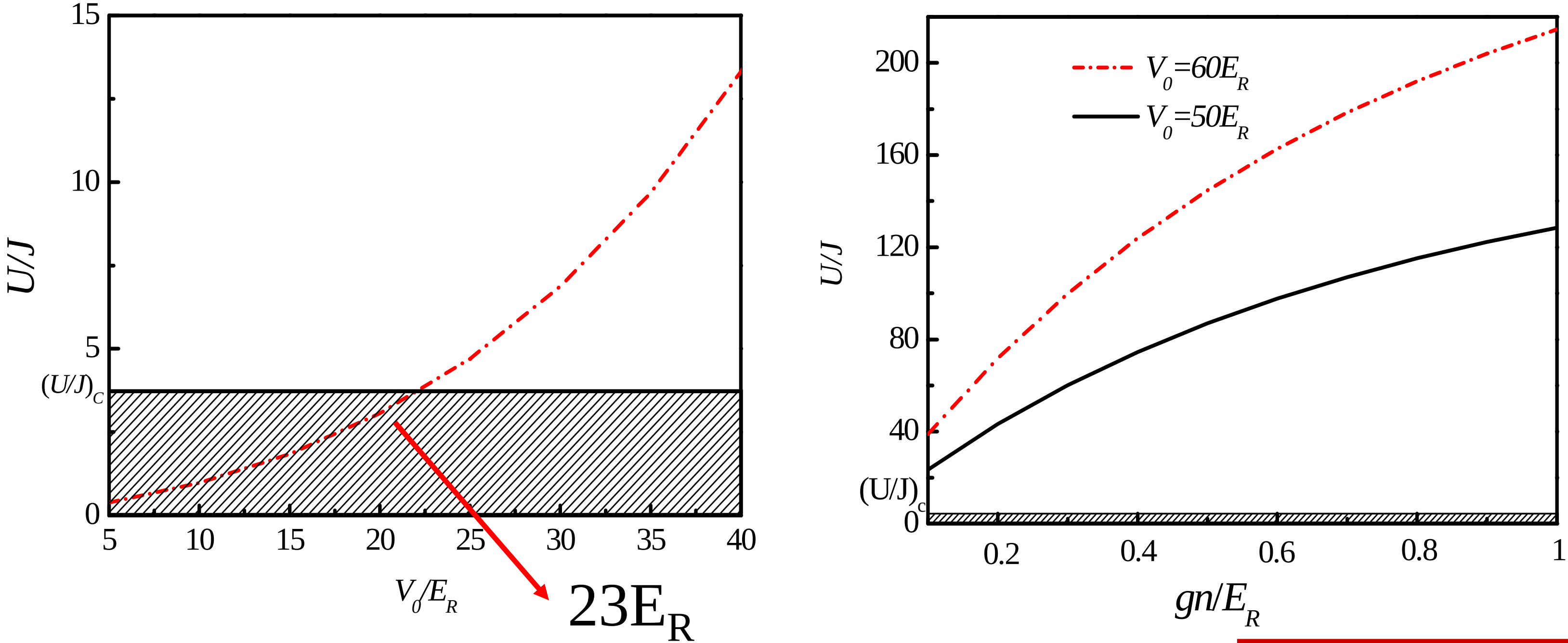




Monotone behavior with nonlinear parameters

Chin. Phys. B: 18, 4130 **(2009).**

Nonlinear Wannier functions



J. Opt. B: Quantum Semiclass. Opt. (2003) **5**, S9

Critical value $(U/J)_c = 3.84$

Chin. Phys. B: 18, 4130 **(2009).**

Outline

- Introduction
  - BECs presents one wonderful example on Mean field Theory
  - BECs with External potential: Single, double well and periodical potential
  - Developing Nonlinear quantum theory
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Weak force detector

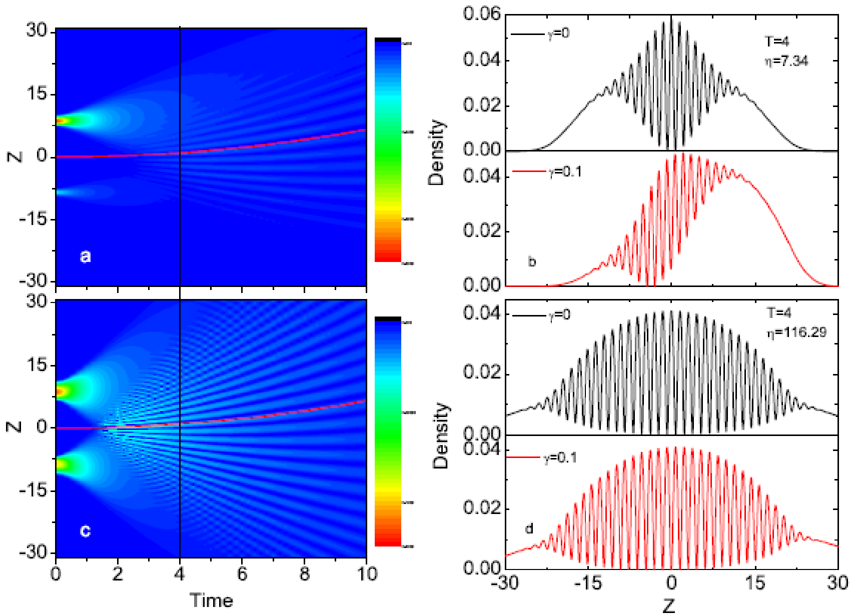


Figure 2: (Color on-line) The interference pattern of imbalance BECs for $\gamma = 0.1$. (a) $\eta = 7.34$ and (c) $\eta = 116.29$; (b) The section of (a) at $t = 4$ and (d) The section of (c) at the same time with (b). The red curve denotes the free-fall law.

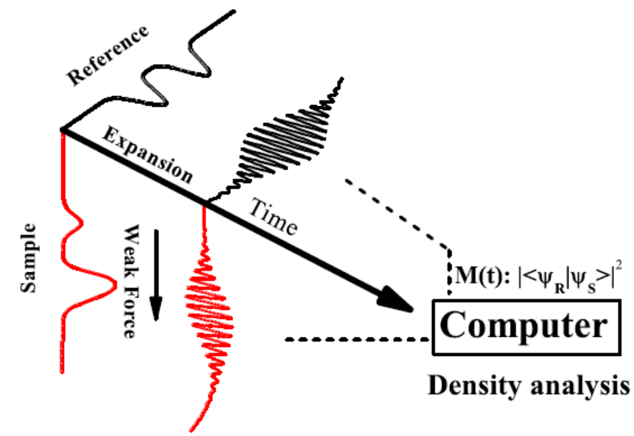
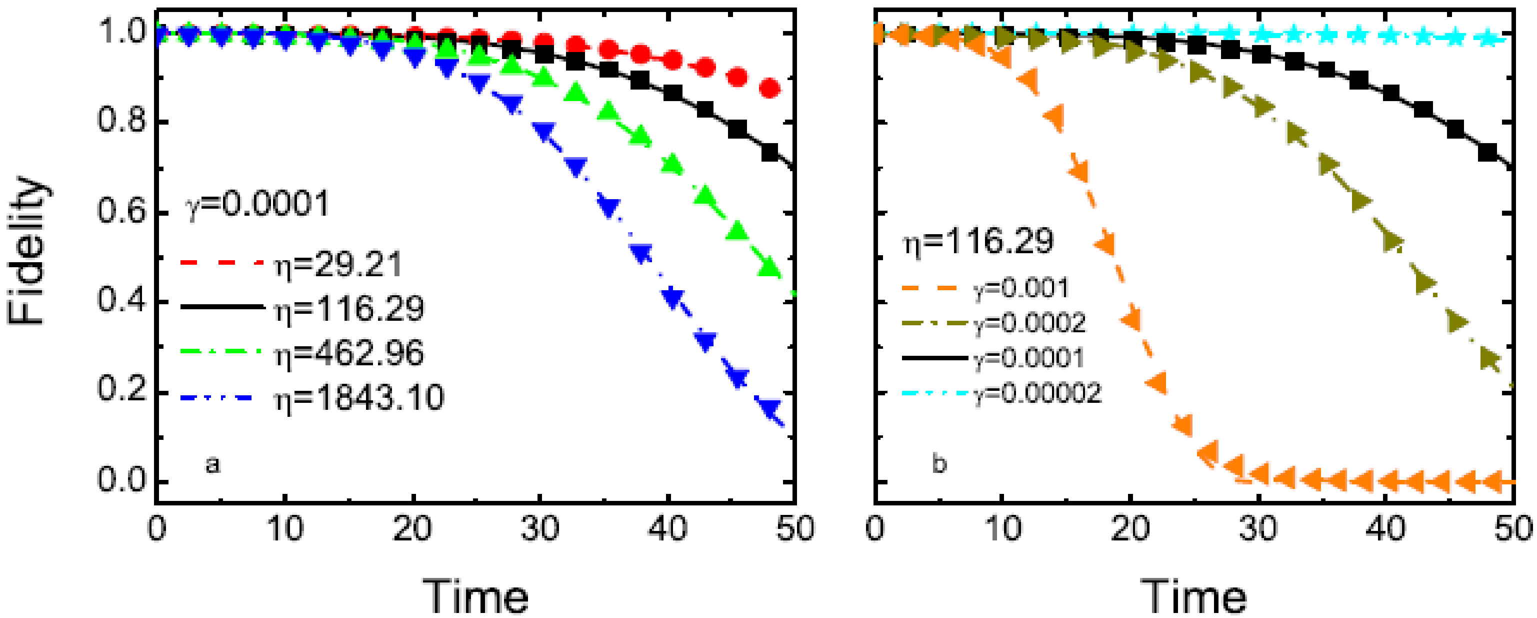


Figure 1: (Color on-line) The scheme for our Atom interferometer. The Loschmidt echo ($\mathcal{M}(t)$) (or Density analysis) between the reference fringes and sample fringes are measured.

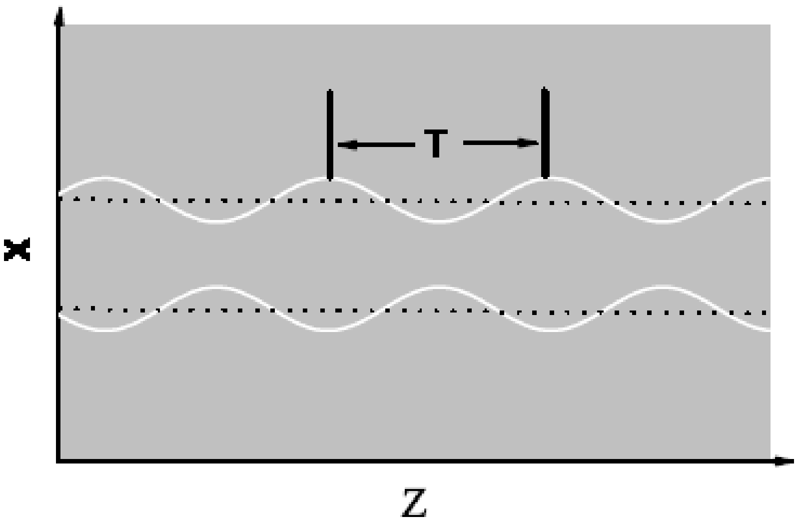
***Revised in* (2010)**

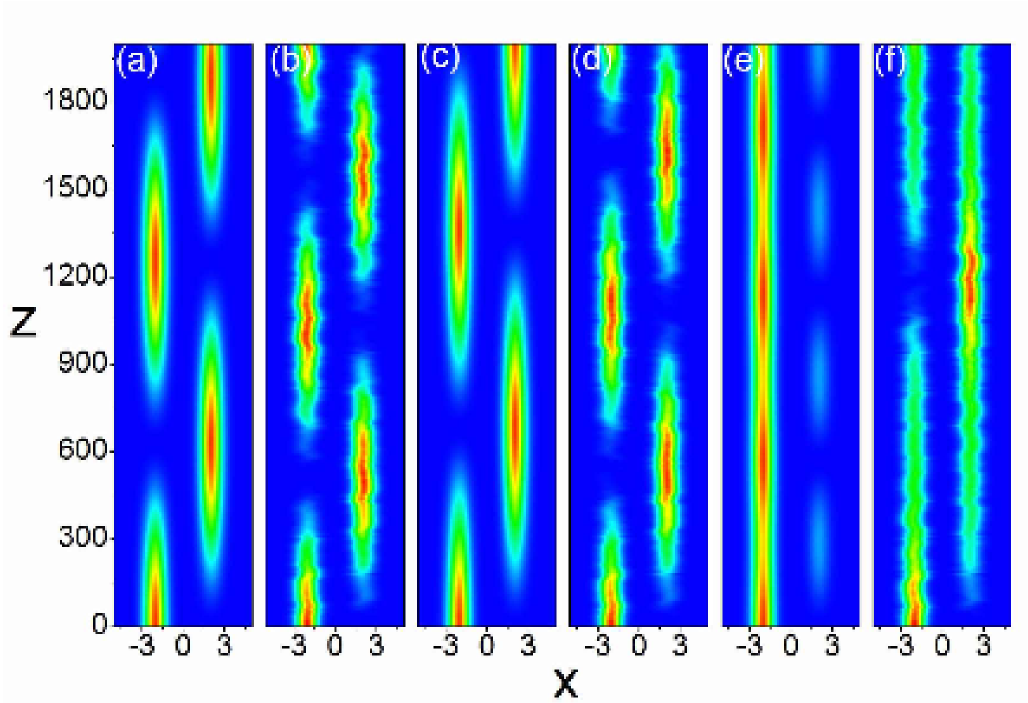
Weak force detector



***Revised in* (2010)**

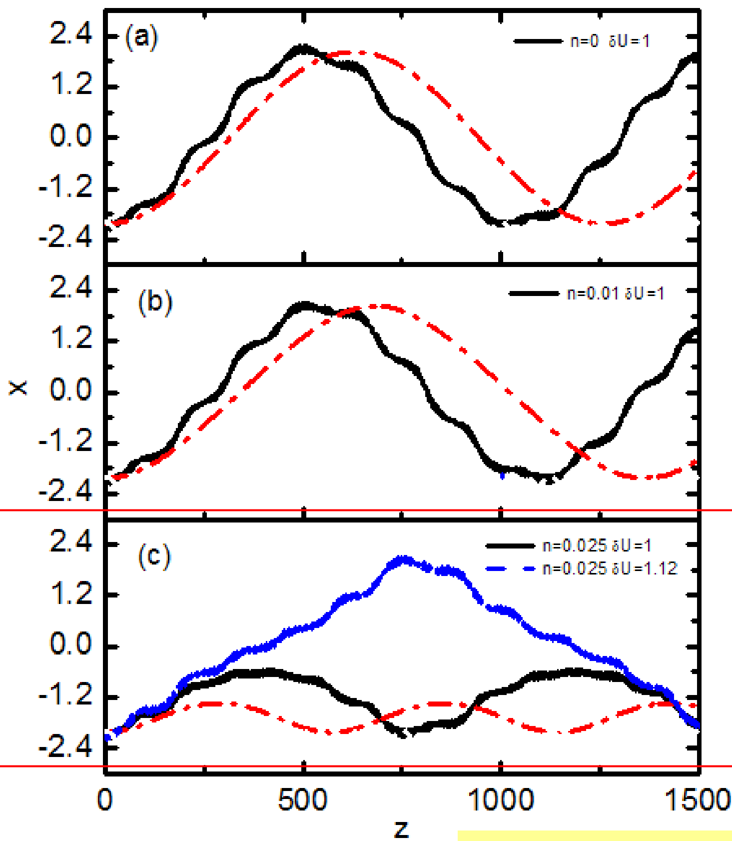
Quantum steps

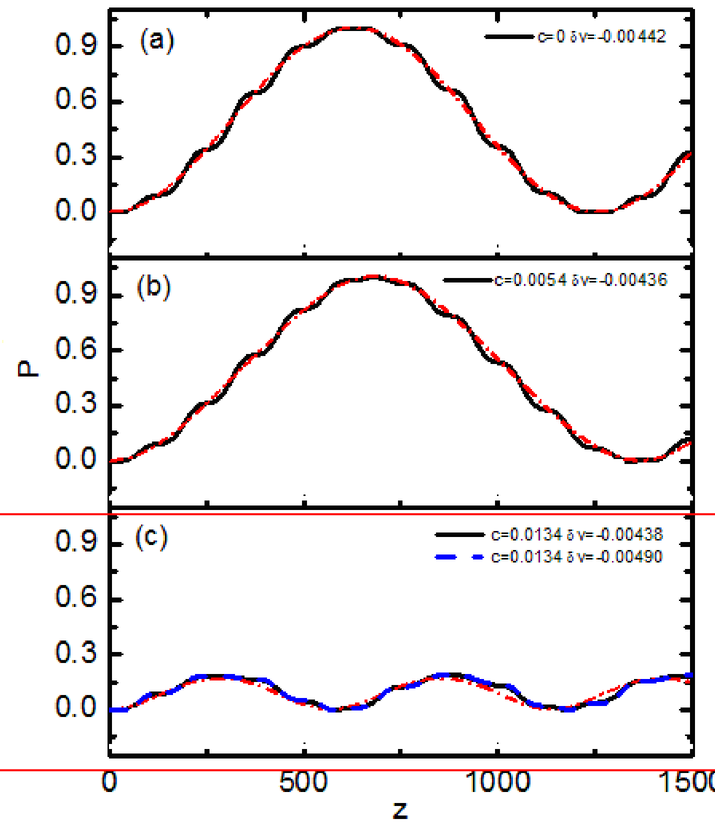




***Eur. J. Phys. D. Revised in* (2010)**

Quantum steps





***Eur. J. Phys. D. Revised in* (2010)**

Conclusion and Outlook

- **Orthogonal basis for confined BECs**
  - **The dynamical stability of first excited state**
- **Nonlinear Correction for the MQST**
  - **The nonlinear tunneling makes an important contribution in lager nonlinear case**
- **Nonlinear Bloch and Wannier functions**
  - **The nonlinear interaction modifies**
  - **The relative parameters are presented**
- Loop structure and the symmetry-breaking solutions
- On Asymmetrical double well
- Adiabatic Evolution on double well

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Thanks for your attentions