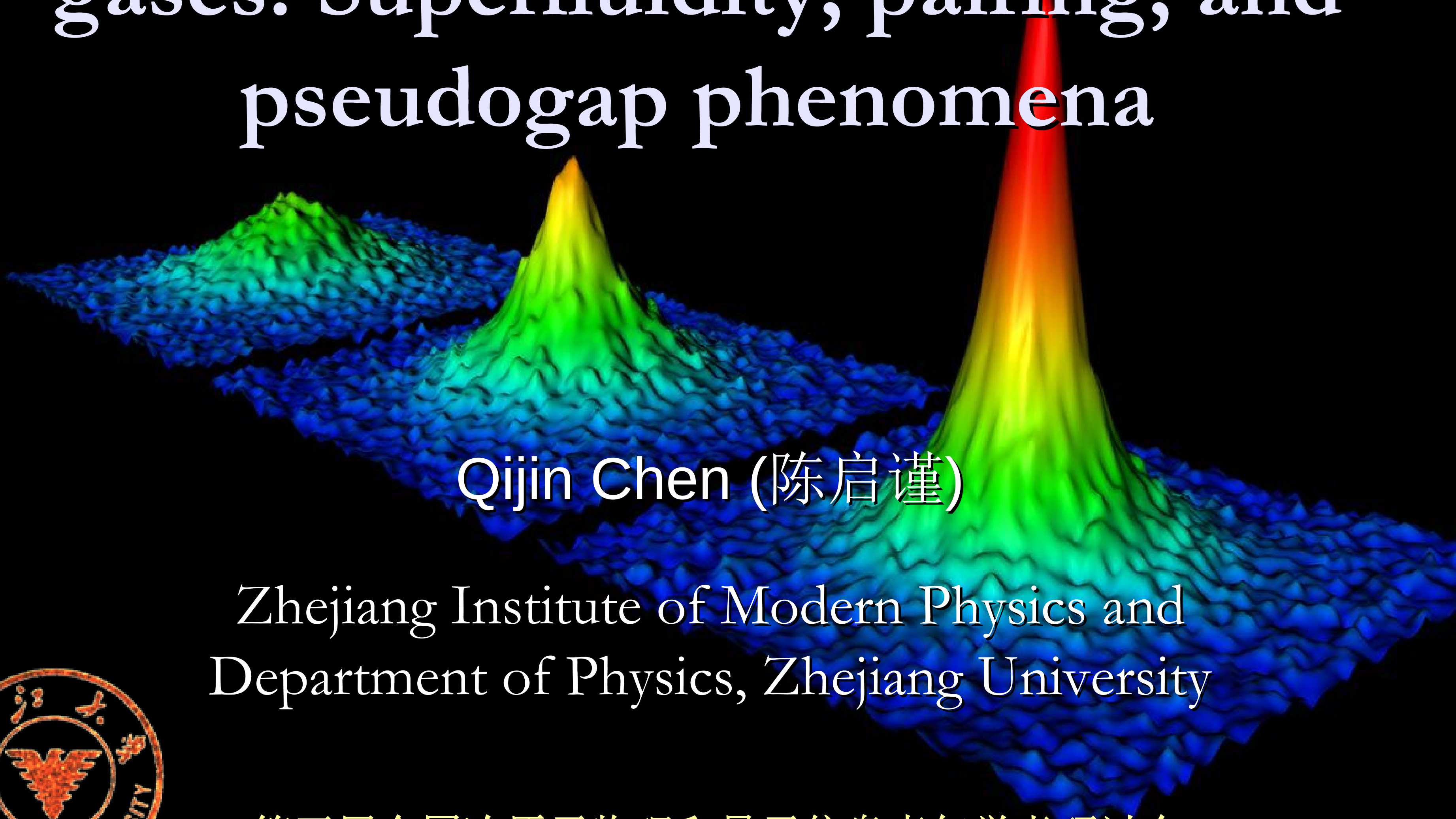
Strongly interacting atomic Fermi gases: Superfluidity, pairing, and pseudogap phenomena



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第五届全国冷原子物理和量子信息青年学者研讨会, **Aug 1, 2011**

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Outline

- Introduction
  - Recent progress in fermionic superfluidity in Fermi gases with tunable interactions
  - Overview of BCS-BEC crossover and pseudogap phenomena
- Theoretical formalism -- Pairing fluctuation theory
  - Homogeneous case
  - Local density approximation in a trap
- Comparing theory and experiment – Evidence for noncondensed pairs and pseudogap in Fermi gases
  - Equal spin mixture (Cuprates and Fermi gases)
  - Momentum resolved radio-frequency spectroscopy
  - Population imbalance
  - Probing homogeneous spectral function
- Summary

Phys. Rep. 412, 1-88 (2005)

Science 307, 1296 (2005)

Phys. Rev. Lett. 102, 190402 (2009).

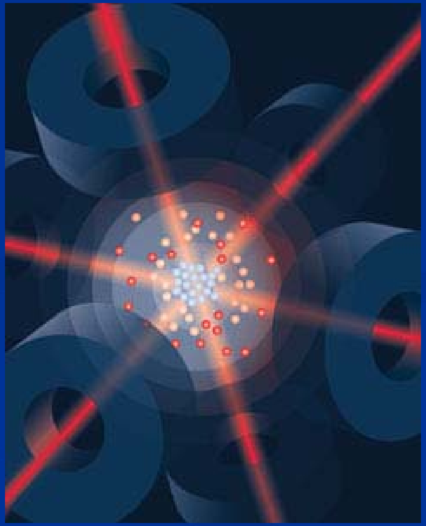
Part I Introduction

Breakthroughs in superfluidity

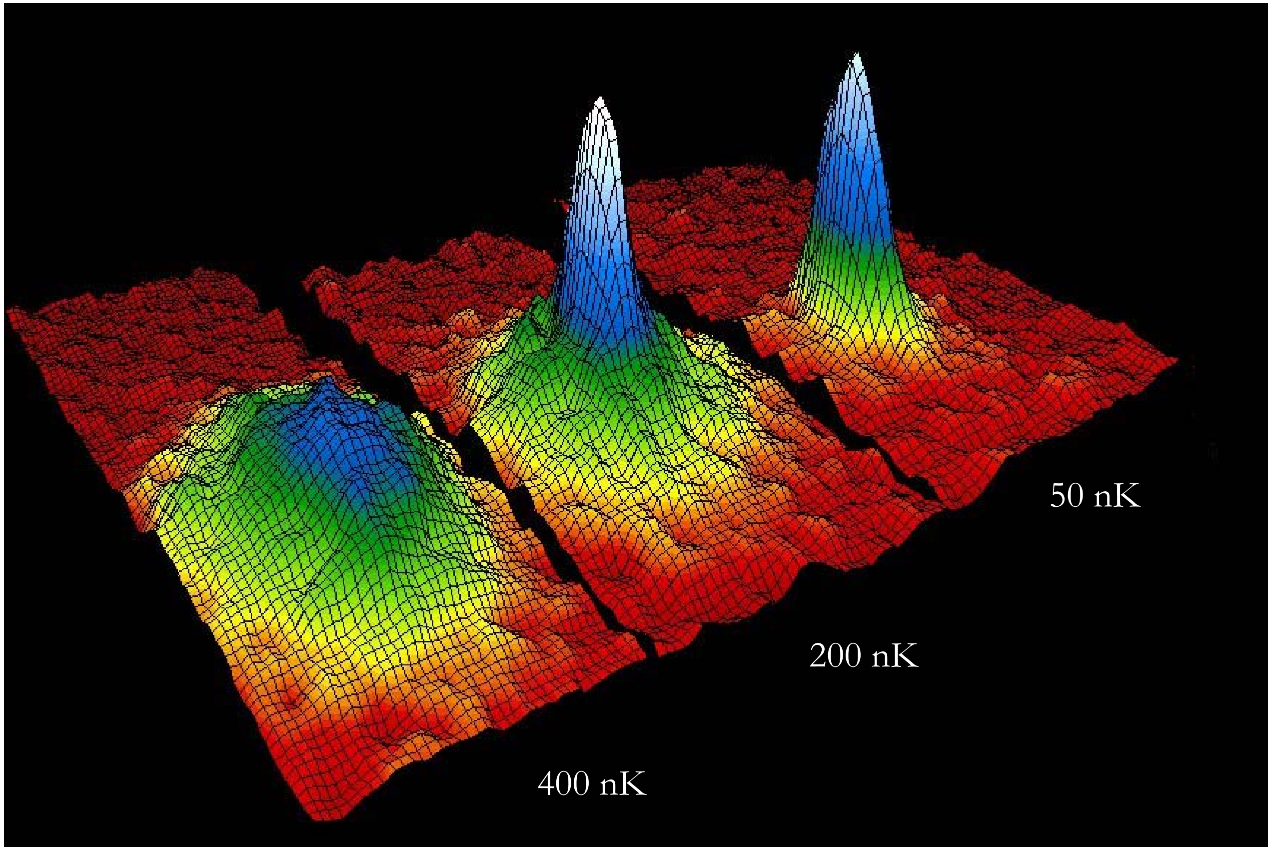
- Discovery of superconductivity, 1911 (Onnes)
- Prediction of Bose-Einstein condensation (1924)
- BCS theory of superconductivity, 1957
- Discovery of superfluid ^{3}He, 1972
- High Tc superconductors, 1986 (Bednorz and Muller)
- BEC in dilute gases of alkali atoms (1995)
- **Superfluidity in atomic Fermi gases (2003)**

What are Trapped Fermi Gases

- Most studied: ^{40}K and ^{6}Li
- Confined in magnetic and optical traps
- Atomic number $N=10^5$-10^6
- Fermi temperature $E_F \sim 1\ \mu K$
- Cooled down to $T \sim 10$-100 nK
- Two spin mixtures – spin "up" and "down"
- Tunable population imbalance
- Tunable mass ratio and dimensionality
- Optical lattices



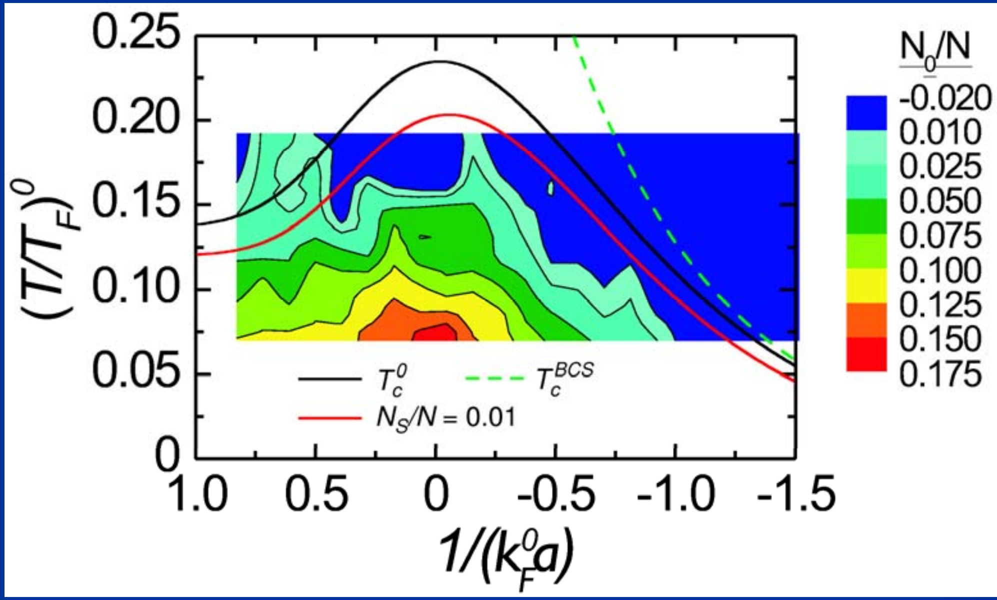
Momentum distribution of a BEC



http://www.colorado.edu/physics/2000/bec/index.html

Superfluidity from weak to strong coupling: early 2004

Phase Diagram from JILA

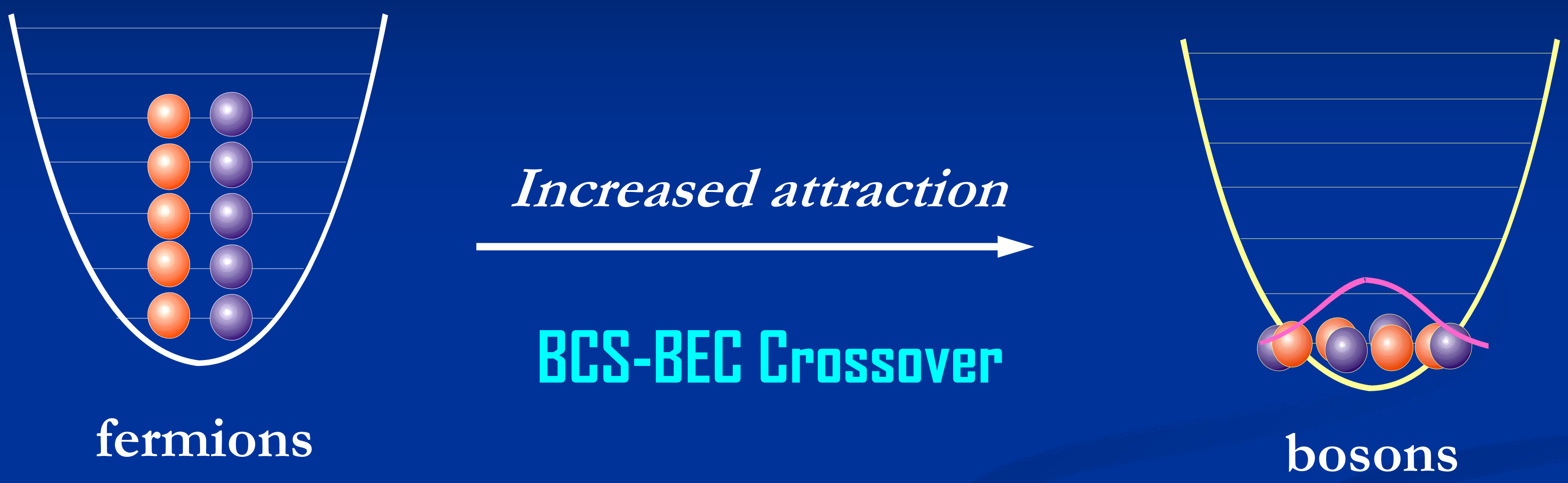


Jin et al, PRL 92, 2004.

Lines from our collaboration

PRA 73, 041601(R) (2006)

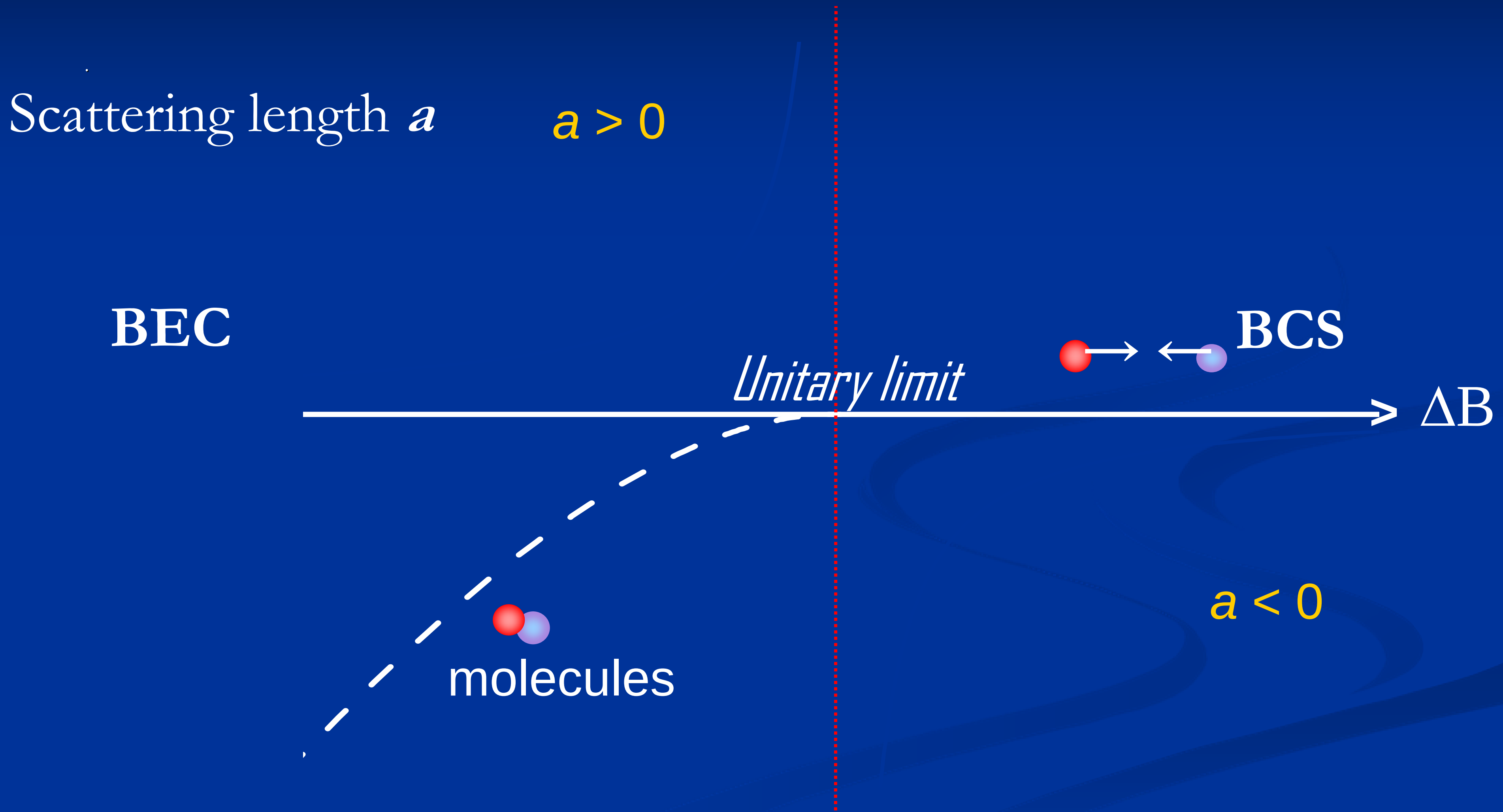
Essence of Fermionic Superfluidity



Attractive interactions turn fermions into "composite bosons" (or Cooper pairs or pairons).

These are then driven by statistics to Bose condense.

Remarkable Tuning Capability in Cold Gases via Feshbach Resonance

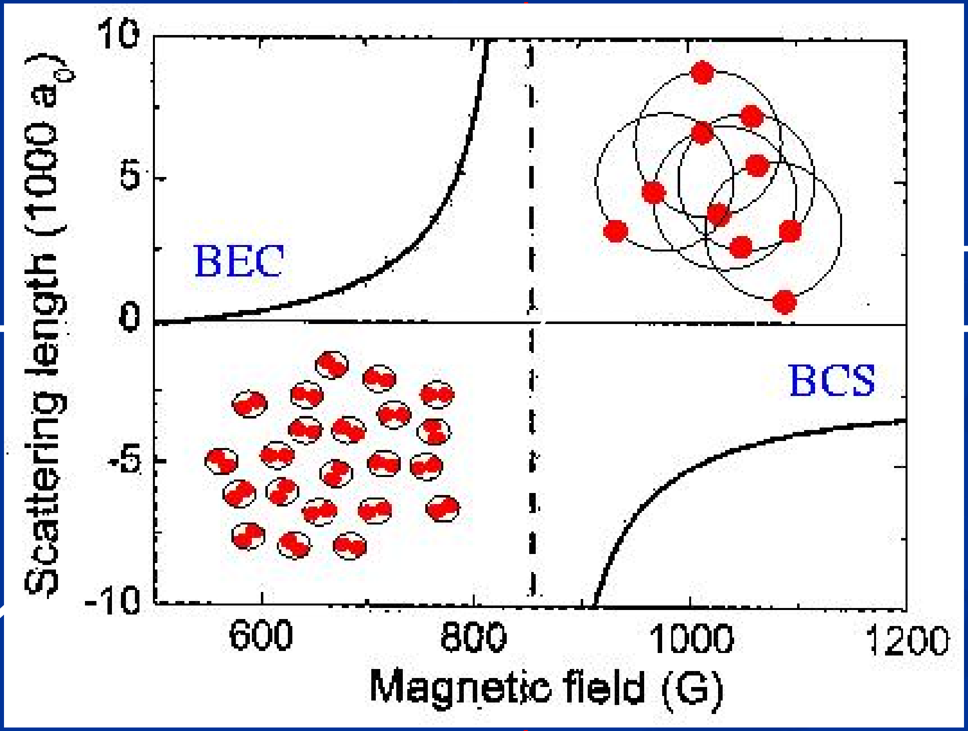


Remarkable Tuning Capability in Cold Gases via Feshbach Resonance

Scattering length ***a***

$a > 0$

BEC



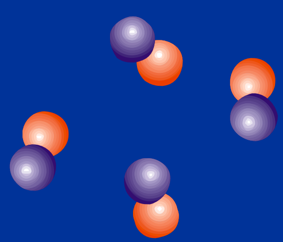
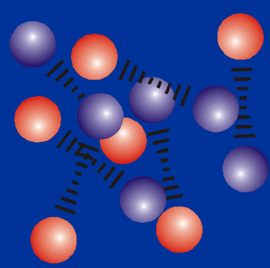
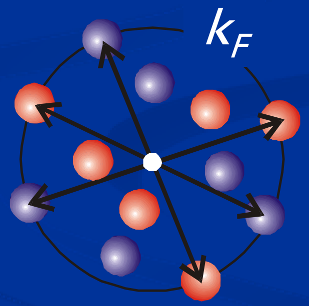
BCS

ΔB

$a < 0$

Crossover under control in cold Fermi atoms (1st time possible)

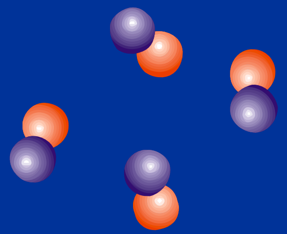
Magnetic Field

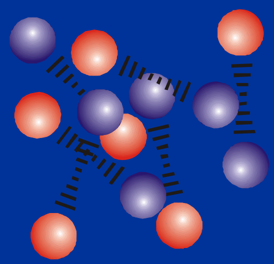
Molecules of fermionic atoms	hybridized Cooper pairs and molecules	Cooper pairs
		
BEC of bound molecules	**Pseudogap / unitary regime**	**BCS** superconductivity Cooper pairs: correlated momentum-space pairing

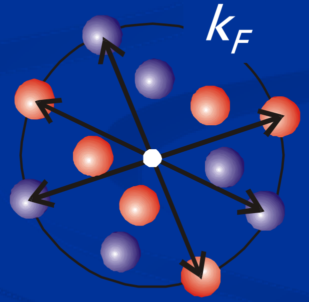
Crossover under control in cold Fermi atoms (<u>1st time possible</u>)

Magnetic Field →

Molecules of fermionic atoms	hybridized Cooper pairs and molecules	Cooper pairs







BEC of bound molecules

Pseudogap / unitary regime

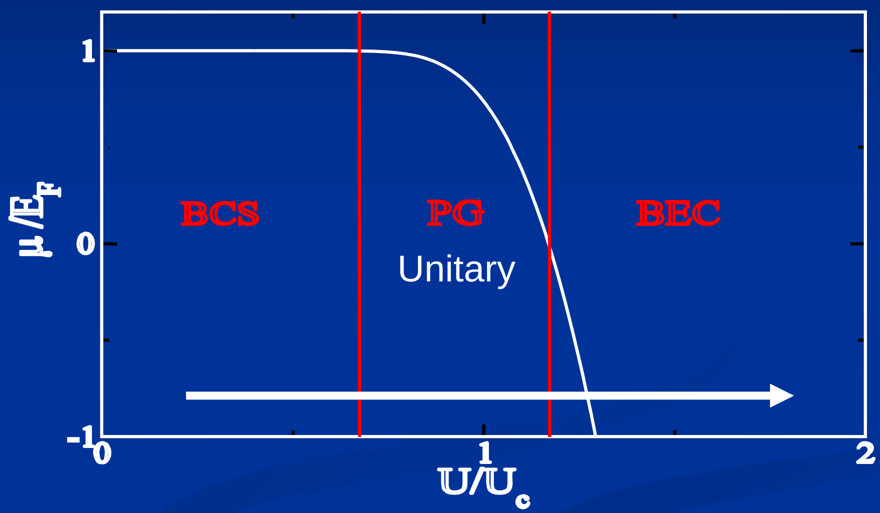
BCS superconductivity
Cooper pairs: correlated momentum-space pairing

BCS-BEC crossover in a nutshell

Zero T BCS-BEC crossover: Tuning the attractive interaction

- Change of character: fermionic → Bosonic

  (U_c – critical coupling)



- Use ground state BCS-Leggett crossover wave function:

$$\Psi_0 = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}})|0\rangle$$

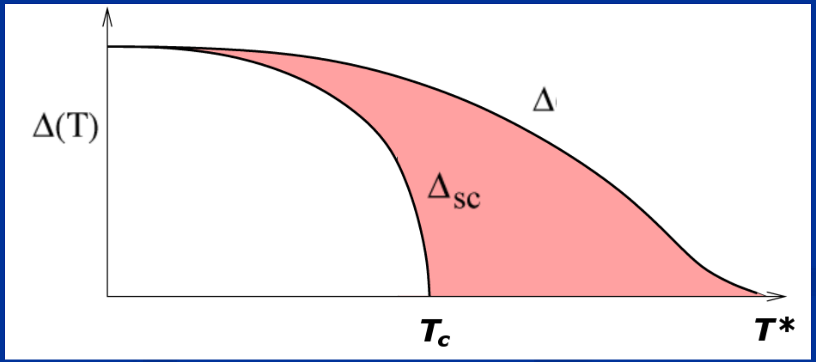
Eagles and Leggett

Thermal excitations

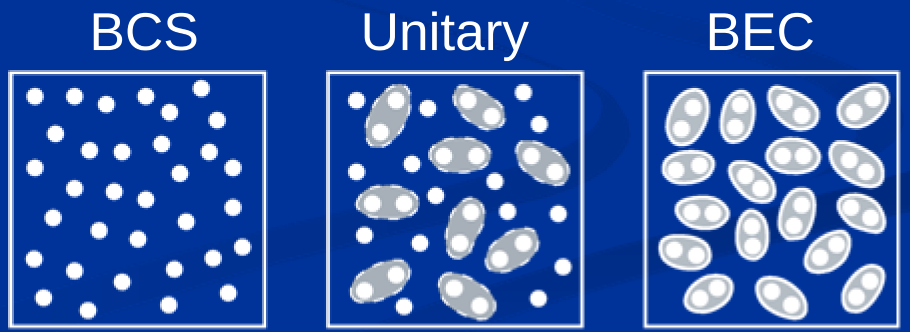
- Pairs form without condensation → pseudogap.
- $\Delta(T)$ is natural measure of bosonic degrees of freedom.

Except in BCS

$$\Delta \neq \Delta_{sc} \qquad T_c \neq T^*.$$



Two types of excitations

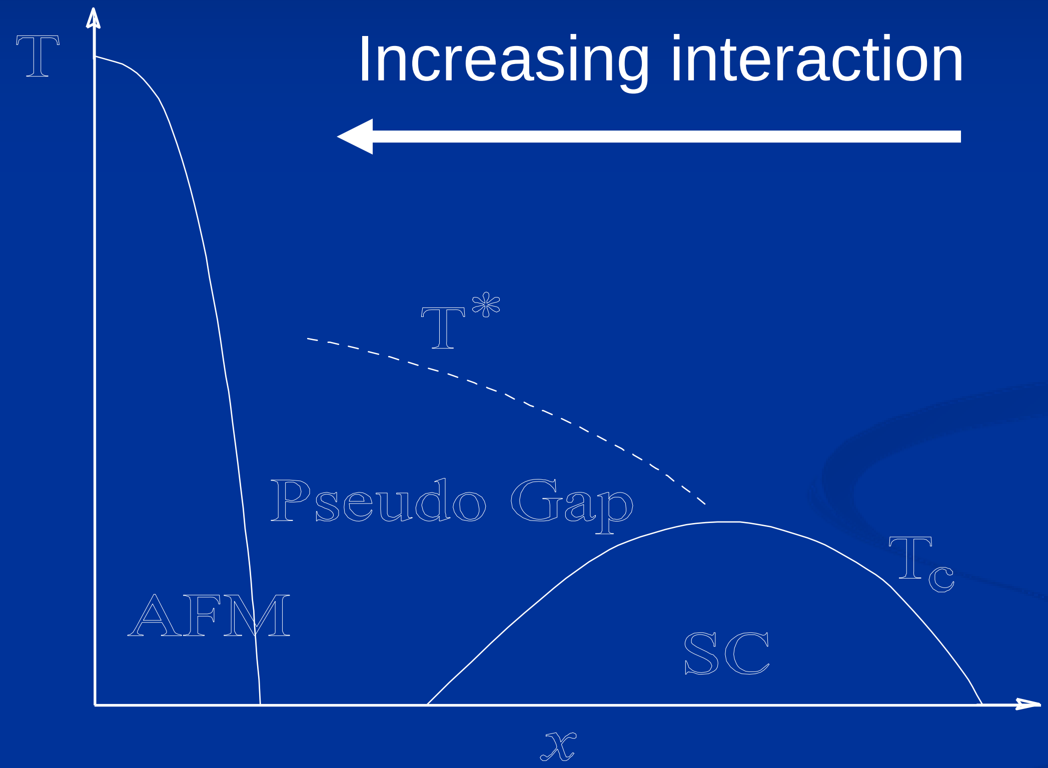


- Novel form of superfluidity
- Never seen before, except possibly in high Tc

- Nozieres Schmitt-Rink (NSR) presented scheme for computing Tc.
- T.D. Lee, Randeria, Chicago group, Strinati, Micnas, Haussman --- high Tc applications.

Behaviors of pseudogap

High Tc superconductors:
Tuning parameter: hole doping concentration



Cannot reach bosonic regime due to d-wave pairing

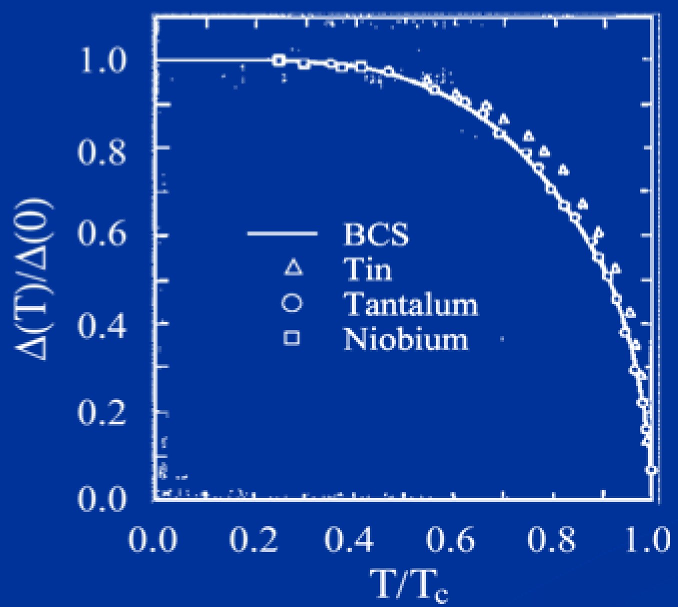
Pseudogap seen in high T_C superconductors!

Pseudogap (normal state gap) is very prominent.

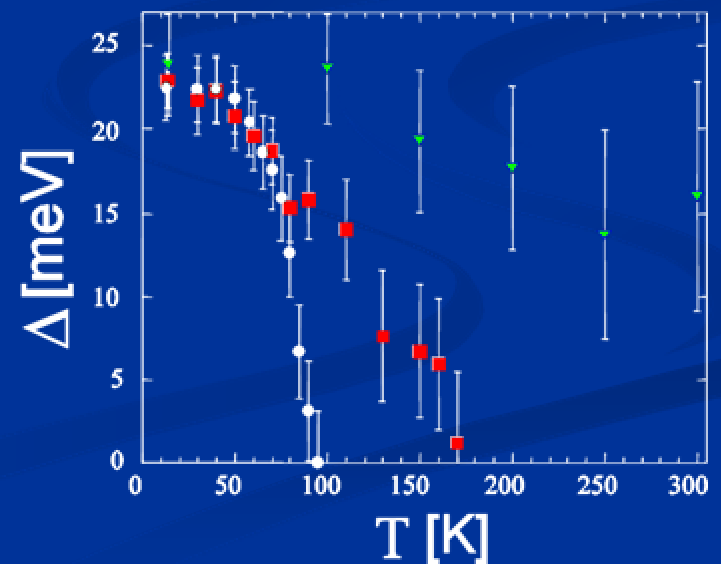
BCS-BEC crossover physics is a possible explanation.

Introducing pseudogap into Fermi gases

BCS



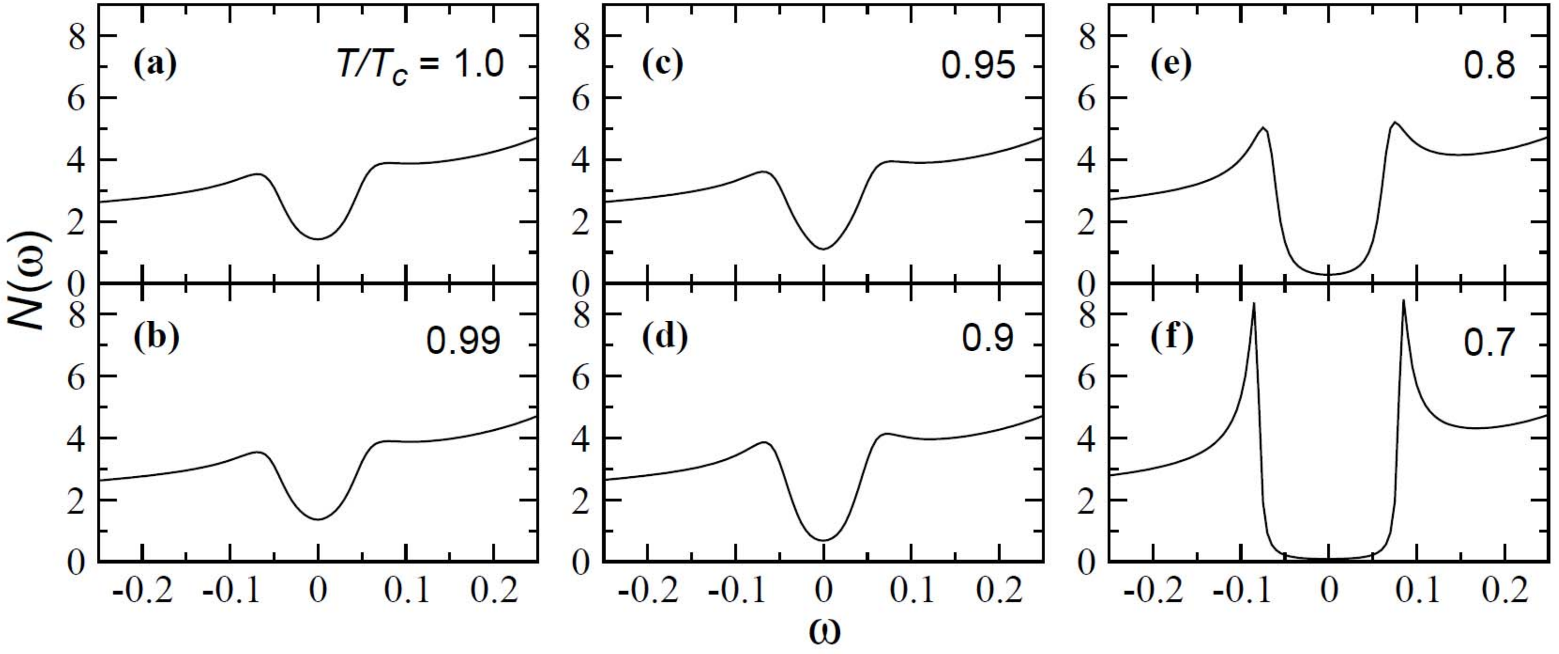
High Tc



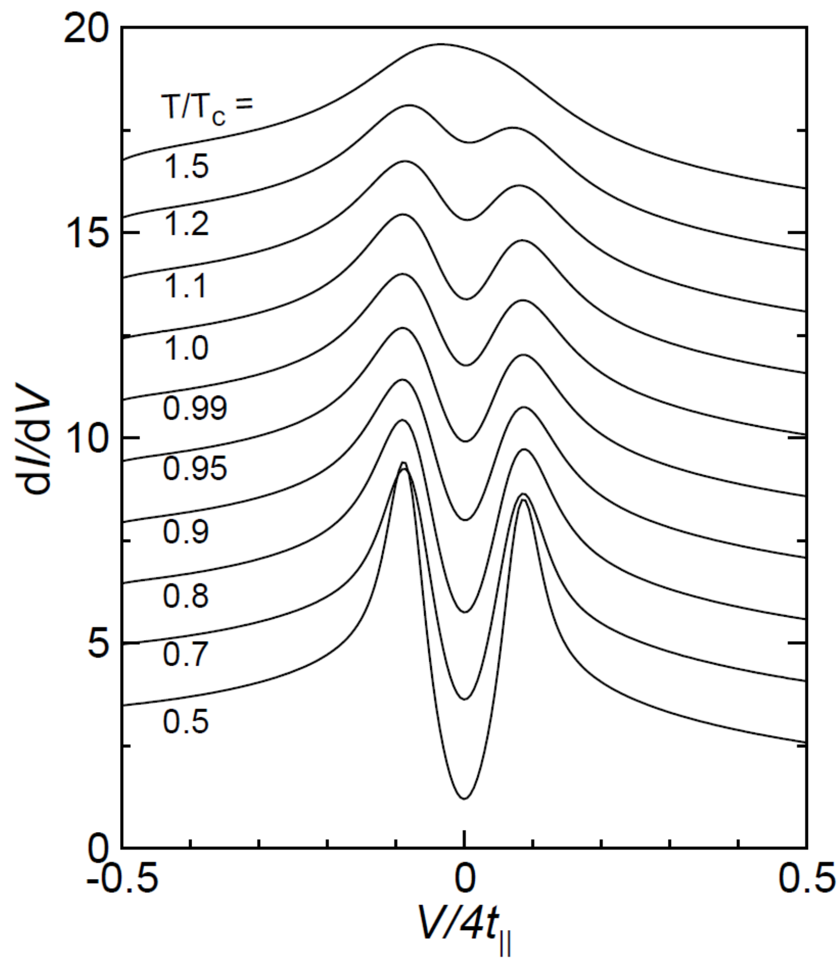
Ding et al, Nature 2006

What is a pseudogap, anyway?

Density of States (s-wave)



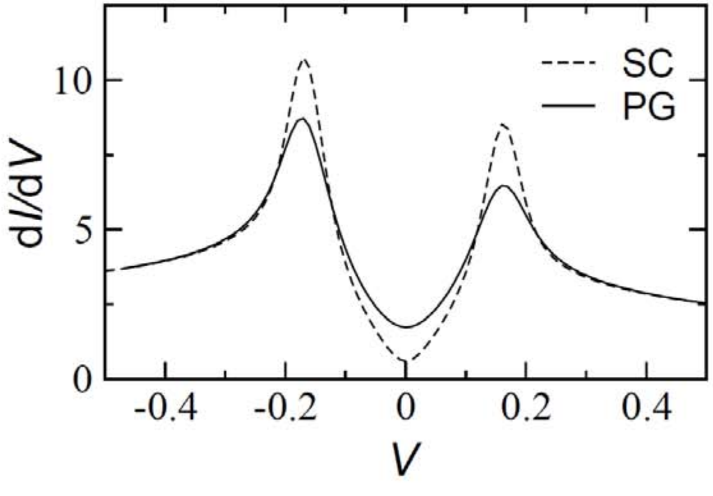
SIN tunneling

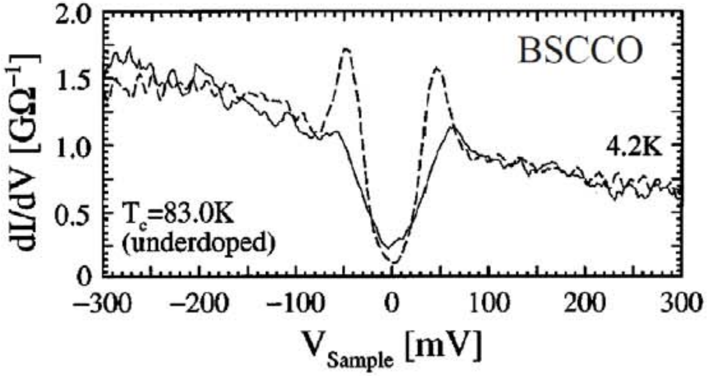


Optimal doping

Extrapolated Pseudogap (Normal) State Below T_c

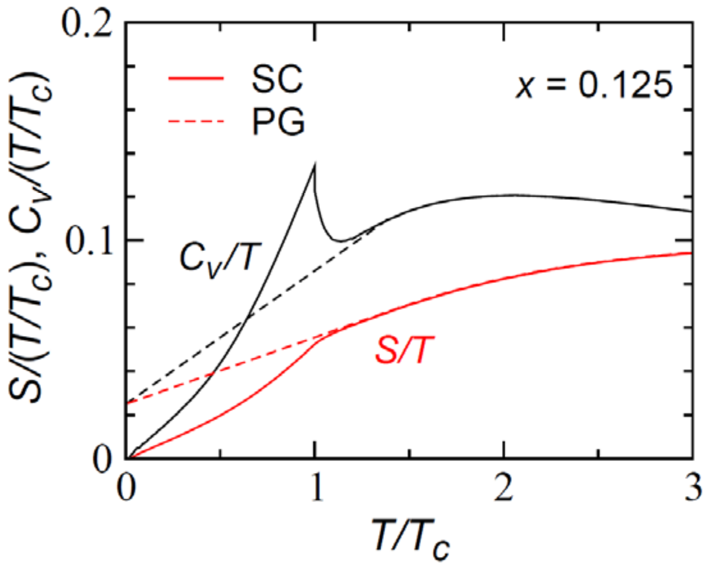
SIN Tunneling dI/dV characteristics

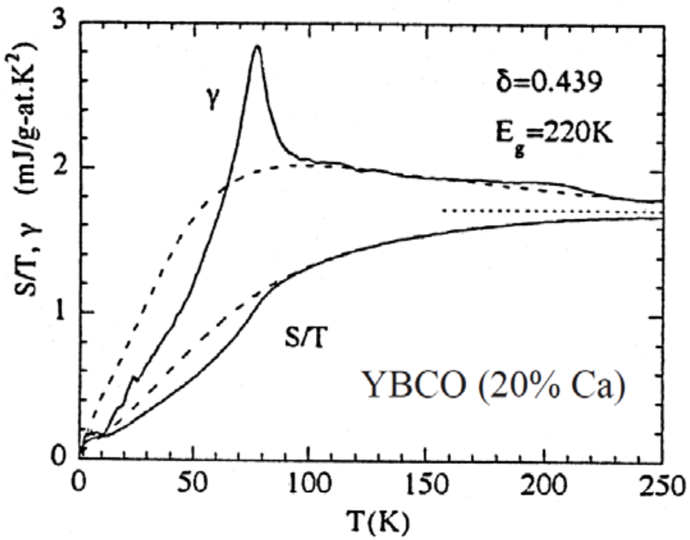




Renner *et al.*, PRL **80**, 3606 (1998).

Specific Heat





Loram *et al.*, J. Phys. Chem. Solids **59**, 2091 (1998).

$\Longrightarrow$ Pseudogap exists below T_c.

Part II

Theoretical Formalism

Introduction to BCS Theory

Interacting Hamiltonian

$$H^{BCS} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$

Pairing at low T

$$\epsilon_{\mathbf{k}} = \frac{k^2}{2m} - \mu$$

$$\Delta_{\mathbf{k}} \equiv -\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \langle c_{-\mathbf{k}'} c_{\mathbf{k}'} \rangle = \Delta \varphi_{\mathbf{k}} \,, \qquad \Delta \equiv |g| \sum_{\mathbf{k}} \varphi_{\mathbf{k}} \langle c_{-\mathbf{k}} c_{\mathbf{k}} \rangle$$

BCS reduced Hamiltonian

$$H^{BCS} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta \varphi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} + \Delta^* \varphi_{\mathbf{k}} c_{-\mathbf{k}} c_{\mathbf{k}} \right) - \frac{\Delta^2}{g}$$

Assume $c_{-\mathbf{k}} c_{\mathbf{k}} - \langle c_{-\mathbf{k}} c_{\mathbf{k}} \rangle$ is very small

Separable potential $V(\mathbf{k},\mathbf{k}') = g \varphi_{\mathbf{k}} \varphi_{\mathbf{k}'}$

Grand canonical Hamiltonian – Beyond BCS

$$H - \sum_\sigma \mu_\sigma N_\sigma = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_\sigma) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}$$
$$+ \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} U_{eff}(\mathbf{k},\mathbf{k}') a^\dagger_{\mathbf{q}/2+\mathbf{k},\uparrow} a^\dagger_{\mathbf{q}/2-\mathbf{k},\downarrow} a_{\mathbf{q}/2-\mathbf{k}',\downarrow} a_{\mathbf{q}/2+\mathbf{k}',\uparrow}$$

BCS keeps only q=0 terms

Population imbalance: $\mu_\uparrow \neq \mu_\downarrow$

Fermi gases: Take contact potential $U_{eff}(\mathbf{k},\mathbf{k}') = U$

Cuprates: Separable potential: $U_{eff}(\mathbf{k},\mathbf{k}') = U \varphi_{\mathbf{k}} \varphi_{\mathbf{k}'}$

$$\varphi_{\mathbf{k}} = \cos k_x - \cos k_y$$

Can be derived from a

- Two-channel Hamiltonian

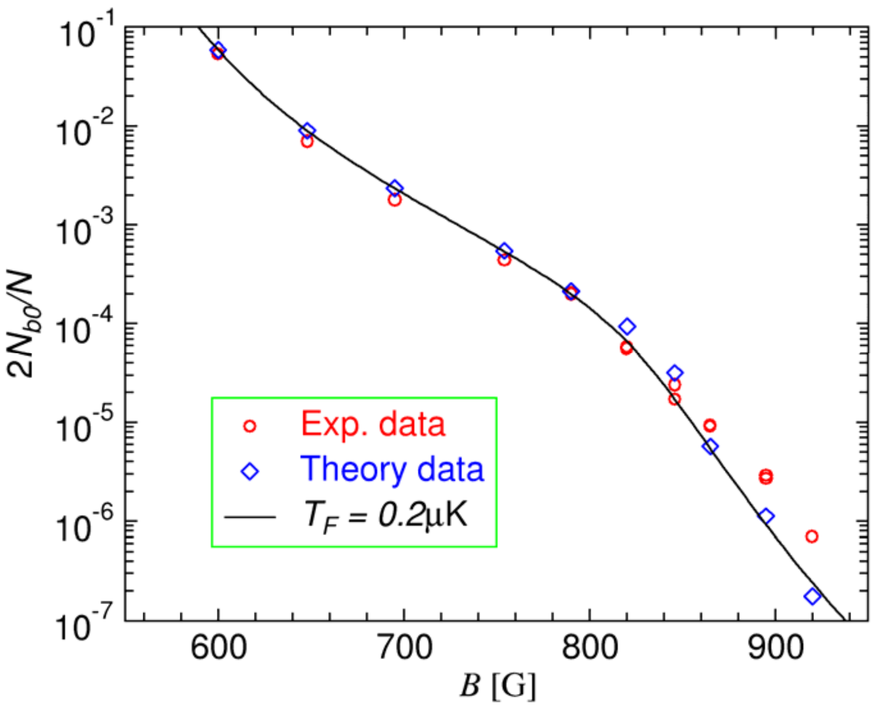
$$
\begin{aligned}
H &- \mu N = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \sum_{\mathbf{q}} (\epsilon^m_{\mathbf{q}} + \nu - 2\mu) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} \\
&+ \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} U(\mathbf{k},\mathbf{k}') a^\dagger_{\mathbf{q}/2+\mathbf{k},\uparrow} a^\dagger_{\mathbf{q}/2-\mathbf{k},\downarrow} a_{\mathbf{q}/2-\mathbf{k}',\downarrow} a_{\mathbf{q}/2+\mathbf{k}',\uparrow} \\
&+ \sum_{\mathbf{q},\mathbf{k}} \left(g(\mathbf{k}) b^\dagger_{\mathbf{q}} a_{\mathbf{q}/2-\mathbf{k},\downarrow} a_{\mathbf{q}/2+\mathbf{k},\uparrow} + h.c. \right)
\end{aligned}
$$

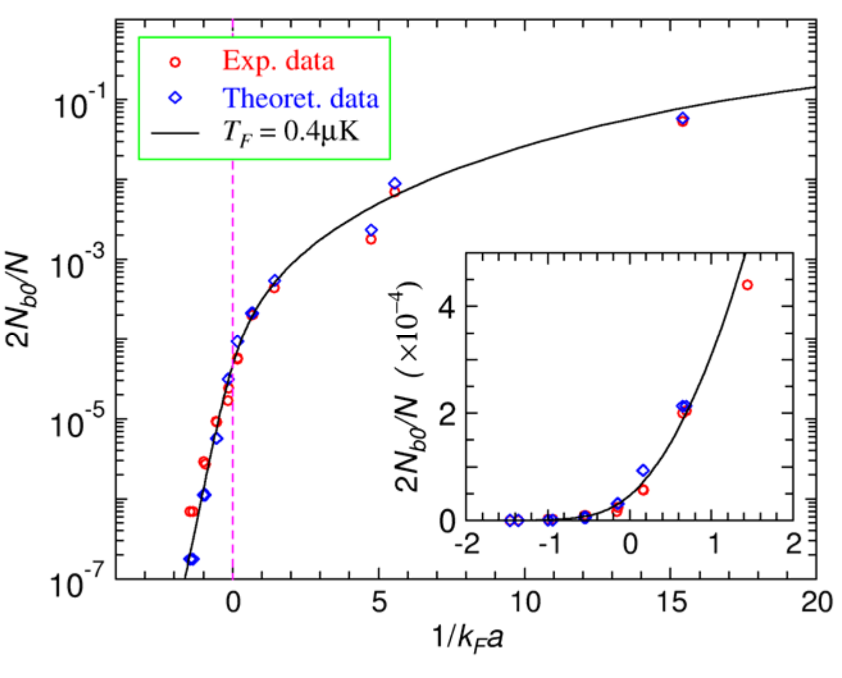
$$U_{eff}(Q) = U + g^2 D_0(Q)$$

Phys. Rep. 412, 1 (2005)

Closed channel fraction is small - Comparison with experiment

- $T = 0$; ^{6}Li, GB Partridge et al, PRL 95, 020404 (2005)





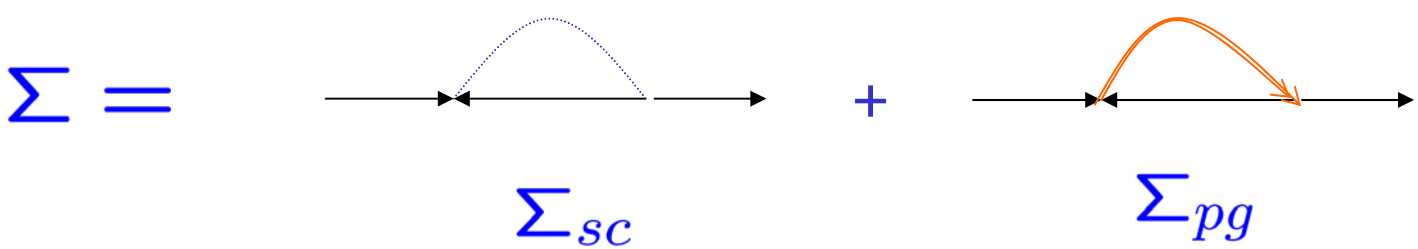
Ohio State workshop 2005

QC and K. Levin, PRL 95, 260406 (2005)

Pairing fluctuation theory -- Physical Picture

PRL 81, 4708 (1998)

- Pairs can be either condensed or fluctuating.



Equations of motion

$$i\dot{G} = [H, G] \sim G,\; G_2$$

$$i\dot{G}_2 = [H, G_2] \sim G, G_2, G_3$$

- Factorize G_3
- G and G_2 on equal footing
- → T-matrix approximation
  (G_0G scheme)
- Condensed and noncondensed pairs do not mix.

$$
\begin{aligned}
H &= \sum_{\alpha} \int d^3\mathbf{x}\ \psi^\dagger_\alpha(\mathbf{x}) \hat{T}(\mathbf{x}) \psi_\alpha(\mathbf{x}) \\
&\quad + \frac{1}{2} \sum_{\alpha\beta} \int d^3\mathbf{x} d^3\mathbf{x}'\ \psi^\dagger_\alpha(\mathbf{x}) \psi^\dagger_\beta(\mathbf{x}') V(\mathbf{x}, \mathbf{x}')_{\alpha,\beta} \psi_\beta(\mathbf{x}') \psi_\alpha(\mathbf{x})
\end{aligned}
$$

$$
V(1-1') \equiv V(\mathbf{x}, \mathbf{x}') \delta(t - t')
$$

$$
\psi(1) = e^{iHt} \psi(\mathbf{x}) e^{-iHt} \ ,
$$

$$
G_0^{-1}(1) = i \frac{\partial}{\partial t_1} - \hat{T}(1) \ .
$$

$$
\begin{aligned}
G(1-1') &= G^\alpha(1;1') = (-i) \langle T_t \psi_\alpha(1) \psi^\dagger_\alpha(1') \rangle \ , \\
G_2^{\alpha\beta}(12;1'2') &= (-i)^2 \langle T_t \psi_\alpha(1) \psi_\beta(2) \psi^\dagger_\beta(2') \psi^\dagger_\alpha(1') \rangle \ , \\
G_3^{\alpha\beta\gamma}(123;1'2'3') &= (-i)^3 \langle T_t \psi_\alpha(1) \psi_\beta(2) \psi_\gamma(3) \psi^\dagger_\gamma(3') \psi^\dagger_\beta(2') \psi^\dagger_\alpha(1') \rangle
\end{aligned}
$$

$$i\frac{\partial}{\partial t_1}\psi_\alpha(1) = \hat{T}(1)\psi_\alpha(1) + \sum_\beta V(1-\bar{1})\psi^\dagger_\beta(\bar{1})\psi_\beta(\bar{1})\psi_\alpha(1)$$

$$G_0^{-1}(1)G(1-1') = \delta(1-1') - iV(1-\bar{1})G_2^{+-}(1\bar{1};1'\bar{1}^+)$$

$$G_2^{\alpha\beta}(12;1'2') = G(1-1')G(2-2') + L^{\alpha\beta}(12;1'2')$$

$$\tilde{G}_0^{-1}(1) = G_0^{-1}(1) + iV(1-\bar{1})G(\bar{1}-\bar{1}^+)$$

$$\tilde{G}_0^{-1}(1)G(1-1') = \delta(1-1') - iV(1-\bar{1})L^{+-}(1\bar{1};1'\bar{1}^+)$$

$$G_0^{-1}(1)G_2^{\alpha\beta}(12;1'2') \quad = \quad G(2-2')\delta(1-1') - G(2-1')\delta(1-2')\delta_{\alpha\beta}$$

$$\begin{aligned}
G_3^{\alpha\beta,-\alpha}(12\bar{1};1'2'\bar{1}^+) \quad = \quad & G(2-2')G_2^{+-}(1\bar{1});1'\bar{1}^+) - \delta_{\alpha\beta}G(2-1')G_2^{+-}(1\bar{1};2'\bar{1}^+) \\
& - \delta_{\alpha,-\beta}G(2-\bar{1}^+)G_2^{+-}(1\bar{1};1'2') + G(\bar{1}-\bar{1}^+)L^{\alpha\beta}(12;1'2') \\
& + \delta_{\alpha\beta}G(\bar{1}-\bar{1}^+)G(1-2')G(2-1') - \delta_{\alpha\beta}G(1-2')L^{+-}(2\bar{1};1'\bar{1}^+) \\
& - \delta_{\alpha,-\beta}G(\bar{1}-2')L^{+-}(12;1'\bar{1}^+) + G(1-1')L^{-\alpha\beta}(\bar{1}2;\bar{1}^+2') \\
& + \delta_{\alpha,-\beta}G(1-1')G(2-\bar{1}^+)G(\bar{1}-2') + L_3^{\alpha\beta,-\alpha}(12\bar{1};1'2'\bar{1}^+)\ . \quad (2.8)
\end{aligned}$$

$$
\begin{aligned}
G_3^{+--}(12\bar{1};1'2'\bar{1}^+) \approx\ & G(2-2')L^{+-}(1\bar{1};1'\bar{1}^+) + G(1-1')G(2-2')G(\bar{1}-\bar{1}^+) \\
& - G(2-\bar{1}^+)G_2^{+-}(1\bar{1};1'2') + G(\bar{1}-\bar{1}^+)L^{+-}(12;1'2') \\
& - G(\bar{1}-2')L^{+-}(12;1'\bar{1}^+)\,. \qquad (2
\end{aligned}
$$

$$
\begin{aligned}
G_0^{-1}(1)G_2^{+-}(12;1'2') =\ & G(2-2')\delta(1-1') - iV(1-\bar{1})G_3^{+--}(12\bar{1};1'2'\bar{1}^+) \\
=\ & G(2-2')\delta(1-1') - iV(1-\bar{1})\left[G(2-2')L^{+-}(1\bar{1};1'\bar{1}^+)\right. \\
& + G(1-1')G(2-2')G(\bar{1}-\bar{1}^+) - G(2-\bar{1}^+)G_2^{+-}(1\bar{1};1'2') \\
& \left. + G(\bar{1}-\bar{1}^+)L^{+-}(12;1'2') - G(\bar{1}-2')L^{+-}(12;1'\bar{1}^+)\right].
\end{aligned}
$$

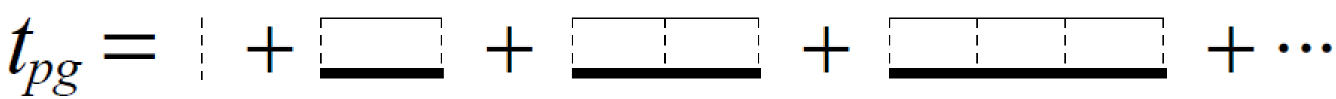
$$
\tilde{G}_0^{-1}(1)L^{+-}(12;1'2') = iV(1-\bar{1})\left[G(2-\bar{1}^+)G_2^{+-}(1\bar{1};1'2') + G(\bar{1}-2')L^{+-}(12;1'\bar{1}^+)\right]
$$

Details available at http://zimp.zju.edu.cn/~qchen/PhDThesis/Thesis.pdf

T-matrix Formalism

Q, K -- 4-momentum

- T-matrix



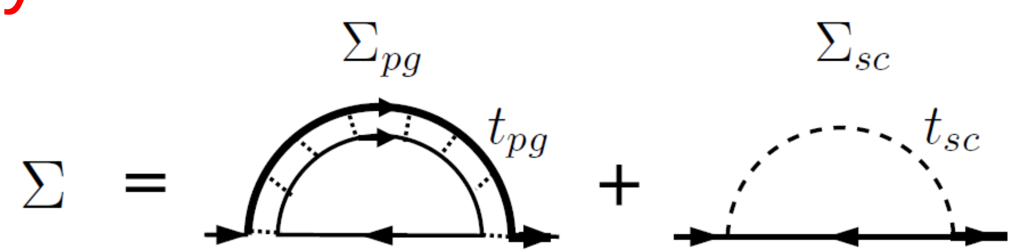
$$t_{pg}(Q) = \frac{U}{1+U\chi(Q)} \approx \frac{Z}{i\Omega - \Omega_q + \mu_{pair}}$$

- Fermion self-energy:

$$\chi(Q) = \sum_K G_0(Q-K)G(K)$$

$$1 + U\chi(0) = 0 = \mu_{pair}$$

$$(T \leq T_c)$$

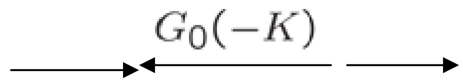


$$\Delta^2 = \Delta_{pg}^2 + \Delta_{sc}^2$$

Self Energy

$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

$$\Sigma_{sc}(K) = $$



$$= \frac{\Delta^2_{sc,\mathbf{k}}}{i\omega_l + \xi_{\mathbf{k}}}$$

$$\Sigma_{pg}(K) = \sum_Q t_{pg}(Q) G_0(Q-K)$$

$$\begin{aligned} \Sigma_{pg}(K) &= \sum_Q \frac{t_{pg}(Q)}{i\Omega_n - i\omega_l - \xi_{\mathbf{q}-\mathbf{k}}} \\ &= -\sum_Q \frac{t_{pg}(Q)}{i\omega_l + \xi_{\mathbf{k}}} + \delta\Sigma \\ &= \frac{\Delta^2_{pg}}{i\omega_l + \xi_{\mathbf{k}}} + \delta\Sigma \end{aligned}$$

$$\Delta^2_{pg} = -\sum_Q t_{pg}(Q)$$

$$\Delta^2 = \Delta^2_{pg} + \Delta^2_{sc}$$

$$\Sigma = -\Delta^2 G_0$$

$$Q \equiv (i\Omega_n, \mathbf{q})$$

Self-consistent Equations

- Gap equation: $\Longleftrightarrow$ BEC condition

$$1 + U\chi(0) = 0 \quad \Longleftrightarrow \quad \mu_{pair} = 0$$

$$1 + U\sum \frac{1-2f(E_k)}{2E_k} = 0 \qquad E_k = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$

- Pseudogap equation: Pair density (Boson number)

$$\Delta_{pg}^2 = -\sum_{Q\neq 0} t(Q) \qquad \Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2$$

- Number equation(s)

Trap effect: $\mu \to \mu - V_{trap}(r)$

$$n = 2\sum_K G(K) \qquad N = \int d^3r\, n(r)$$

$$\Delta_{sc}^2 \sim n_{cond} \qquad \Delta_{pg}^2 \sim n_{nc,pair}$$

Result to anticipate

- Weak coupling: $\Delta_{pg} = 0, \quad \Longrightarrow \quad$ BCS.
- Setting $\Delta_{sc} = 0 \quad \Longrightarrow \quad T_c$, $\mu(T_c)$ and $\Delta_{pg}(T_c)$.
- At $T < T_c \quad \Longrightarrow \quad \mu(T)$, $\Delta_{sc}(T)$, and $\Delta_{pg}(T)$.
- At $T = 0 \quad \Longrightarrow \quad \Delta_{pg} = 0, \; + \;$ Leggett ground state.
- $\Delta_{pg}^2 \sim T^{3/2}$ at low T.

Behavior of gaps vs T

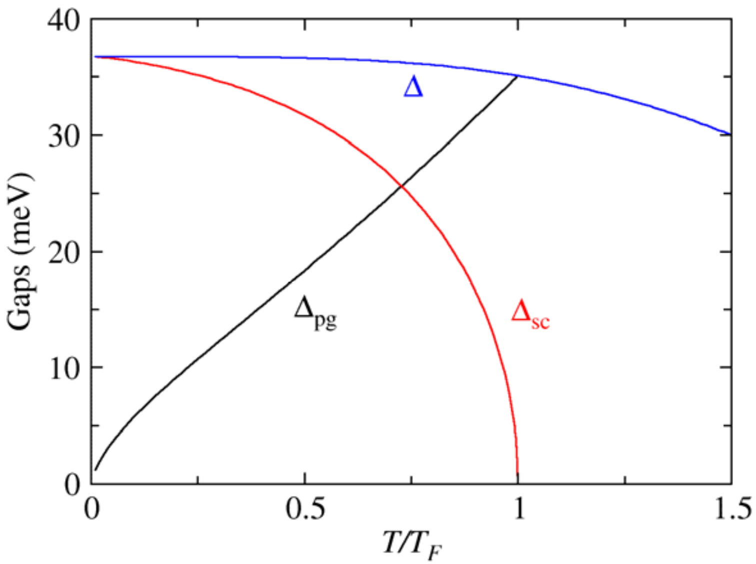
$$1 + U \sum_{\mathbf{k}} \frac{1 - 2f(E_k)}{2E_k} \varphi_{\mathbf{k}}^2 = 0$$

$$n = 2 \sum_K G(K)$$

$$\Delta_{pg}^2 = -\sum_Q t_{pg}(Q)$$

$$\Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2$$

How to determine Tc?

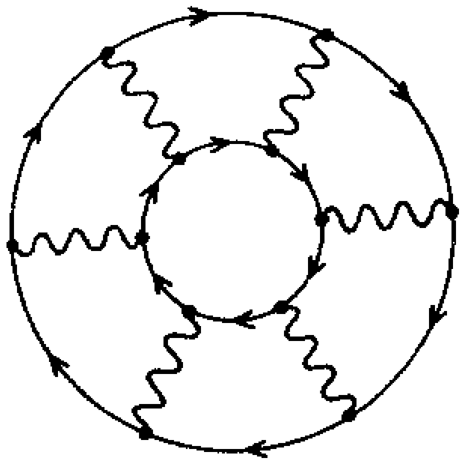


$$\frac{\Delta_{sc}^2}{\Delta^2}$$ --- Fraction of condensed pairs.

Underlying microscopic theory for this -- PRL 81, 4708 (1998)

Remarks about Nozieres - Schmitt-Rink theory

Pairing fluctuation contribution to thermodynamic potential: [All G_0's]



Gap equation is given by:

$$1 + U\chi_0(0) = 0,$$

$$\chi_0(Q) = \sum_Q G_0(K)G_0(Q-K)$$

Number equation:

$$n = -\frac{\partial\Omega}{\partial\mu}$$

$$G = G_0 + G_0\Sigma_0 G_0,$$

$$\Sigma_0(K) = \sum_Q t_0(Q)G_0(Q-K)$$

Lowest order term in the Dyson equation expansion

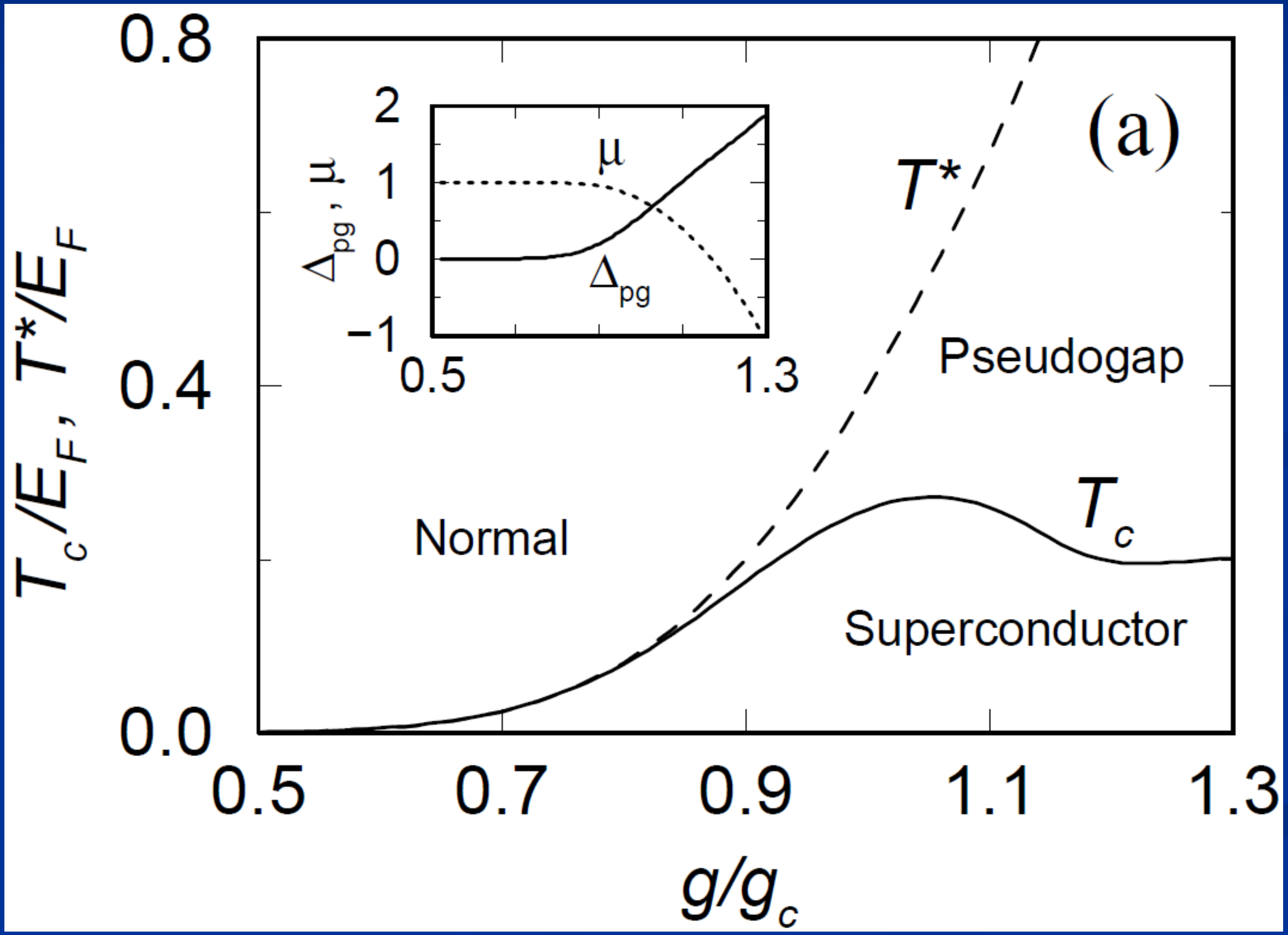
No pseudogap or self-energy feedback in gap equation.

Part III

Superfluidity From a Pseudogapped State

--- Comparing theory and experiment

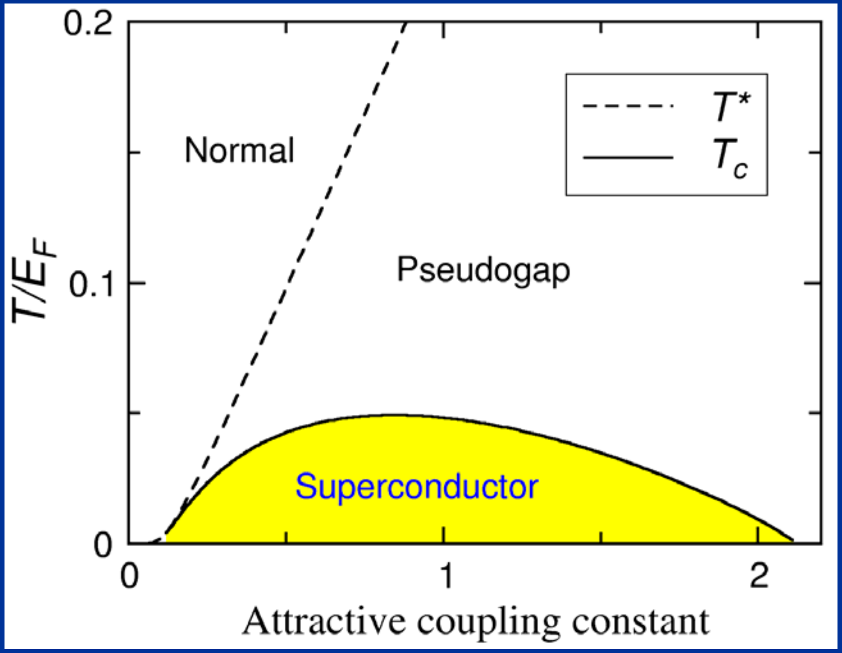
Phase diagram of a 3D Fermi gas

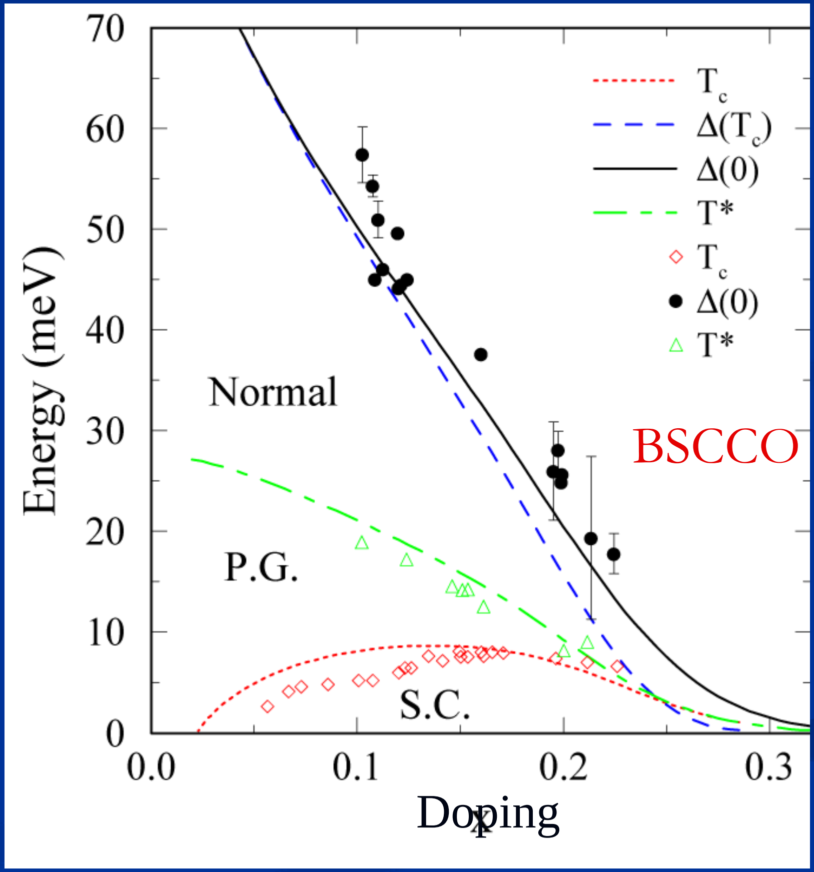


Cuprate phase diagram

QC et al, PRL 81, 4708 (1998).

Apply crossover theory to *d*-wave lattice case





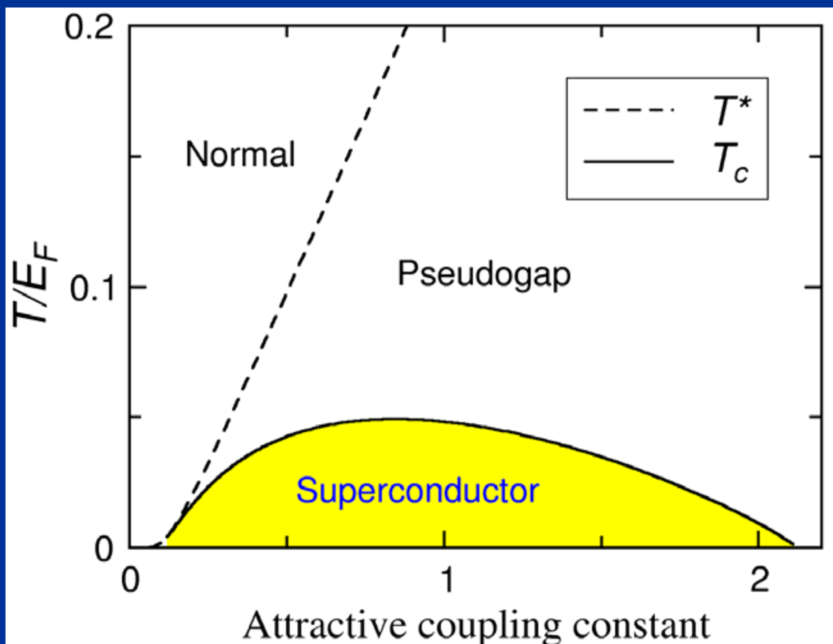
BCS : $2\Delta/T^* = 4.3$

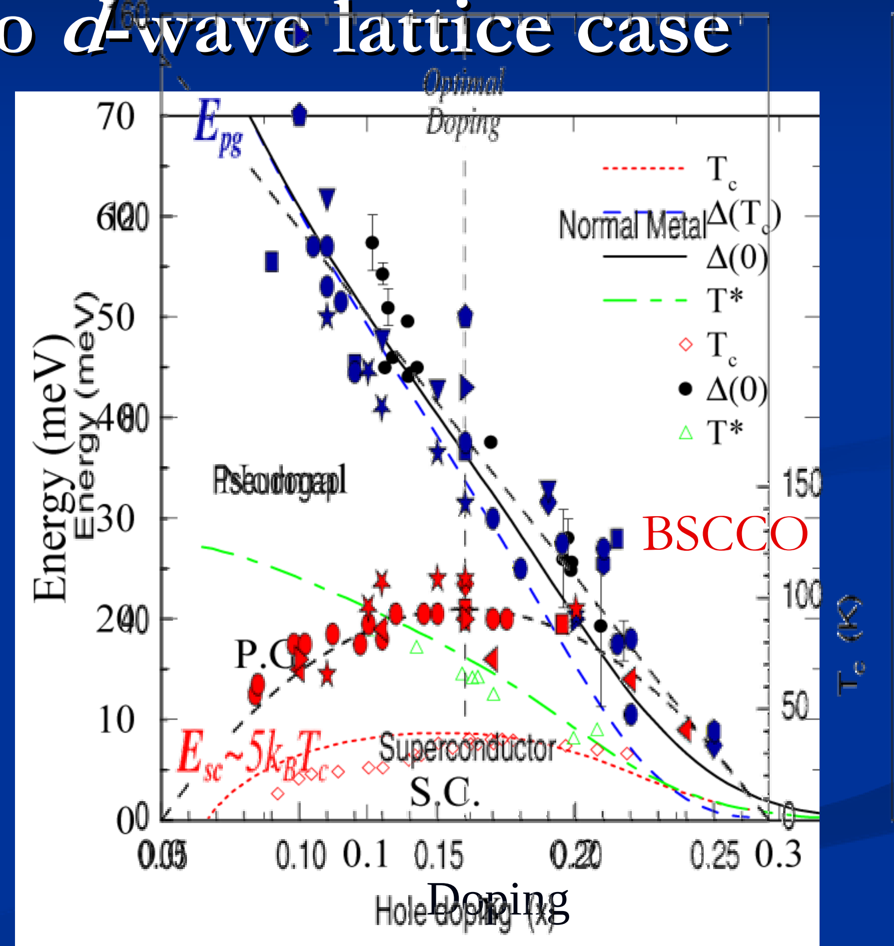
Pairs localize and Tc vanishes well before BEC.

Cuprate phase diagram

QC et al, PRL 81, 4708 (1998).

Apply crossover theory to *d*-wave lattice case





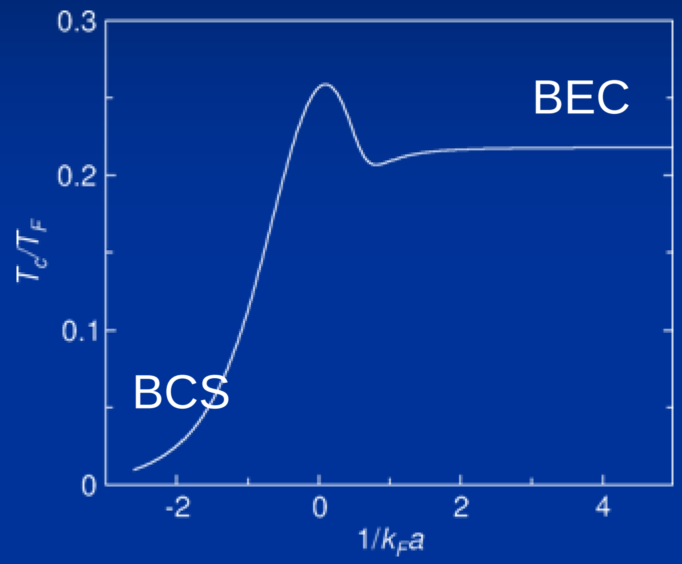
BCS : $2\Delta/T^* = 4.3$

Hufner et al, Rep. Prog. Phys. 71, 062501 (2008)

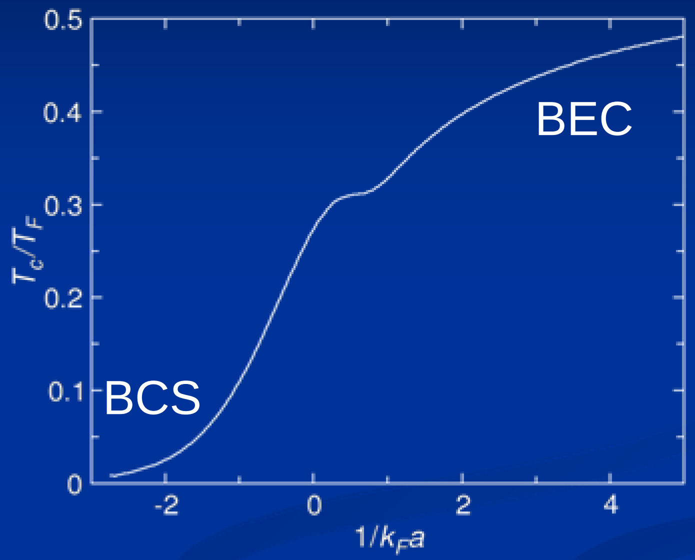
Atomic Fermi gases

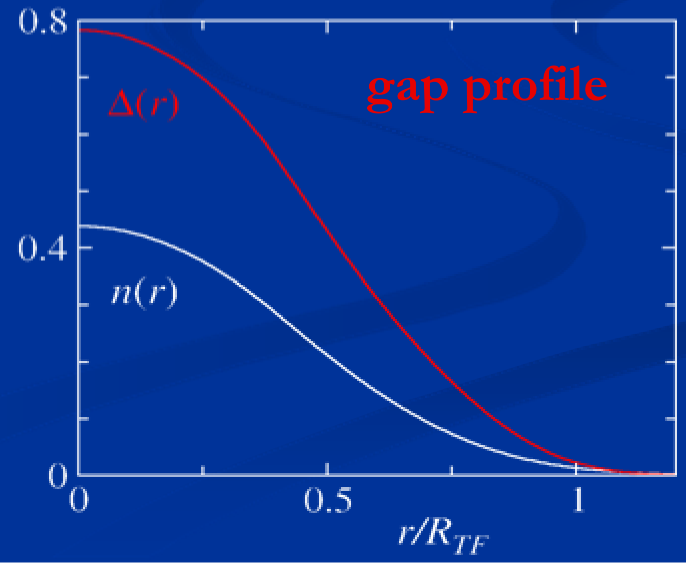
Critical Temperature/Trap Effects

Homogeneous case



In trap: use local density approx





- ***Spatial variation of gap, density***
- ***Will excite fermions at edge, bosons in middle***

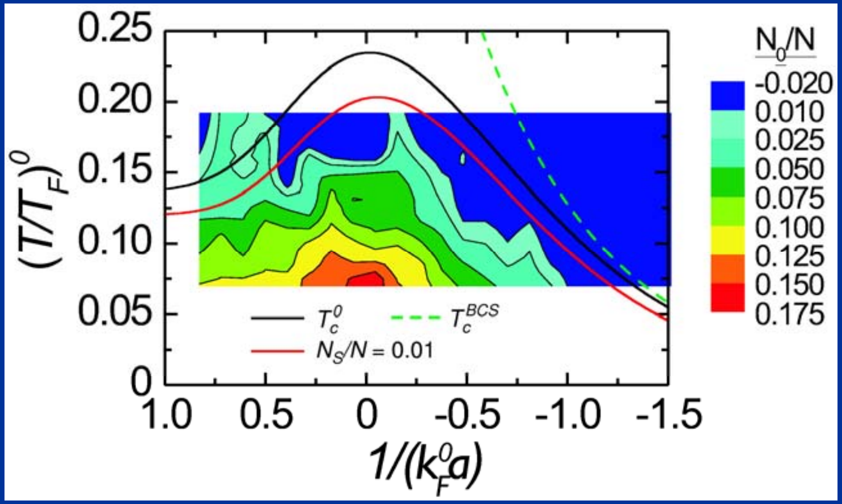
Phase Diagram of Ultracold Atoms

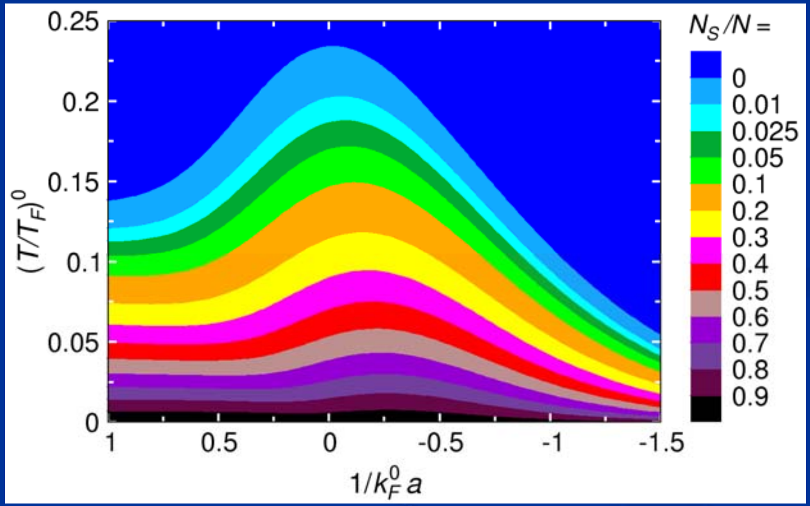
Present Theory and JILA Data

PRL 95, 260405 (2005)

Contour plot

Theory





PRA 73, 041601(R) (2006)

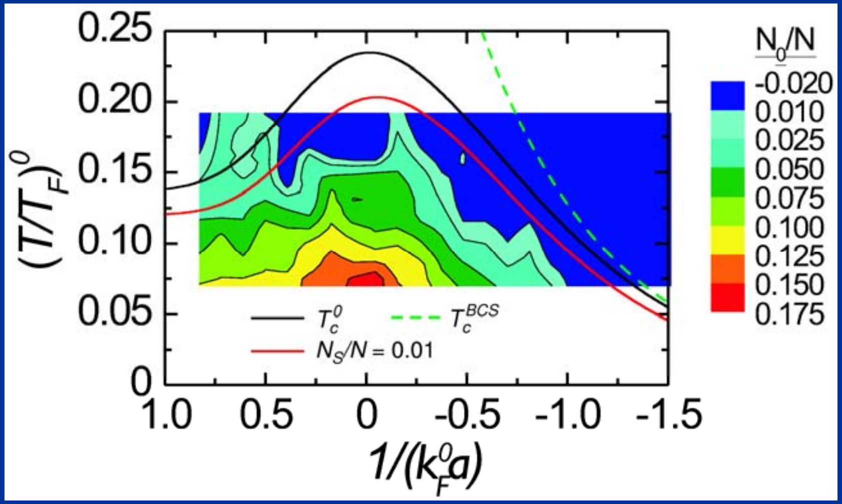
Equilibrium phase diagram

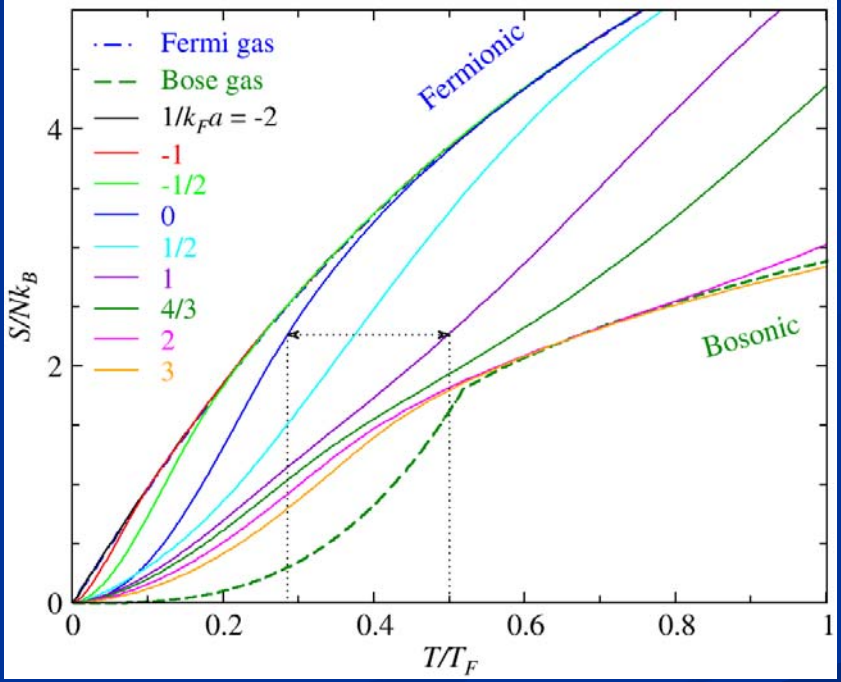
Use sweep projected temperature to plot effective Tc or at given superfluid density.

Phase Diagram of Ultracold Atoms
Present Theory and JILA Data

PRL 95, 260405 (2005)

Contour plot

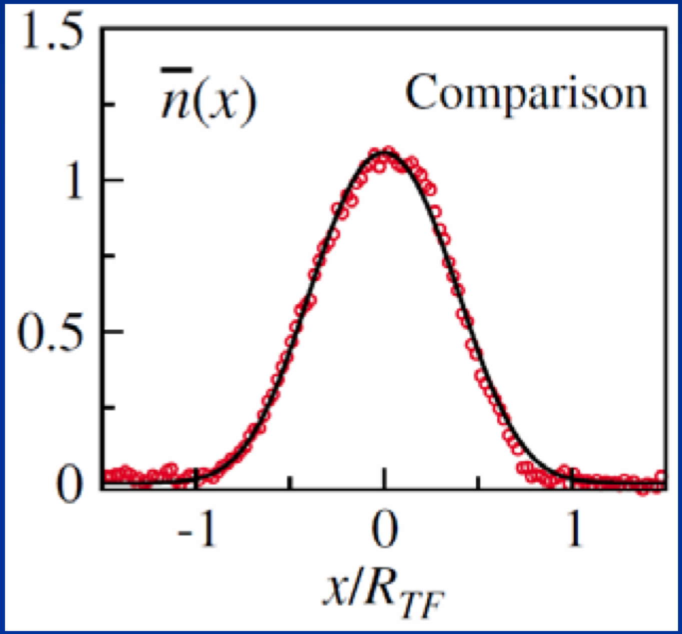




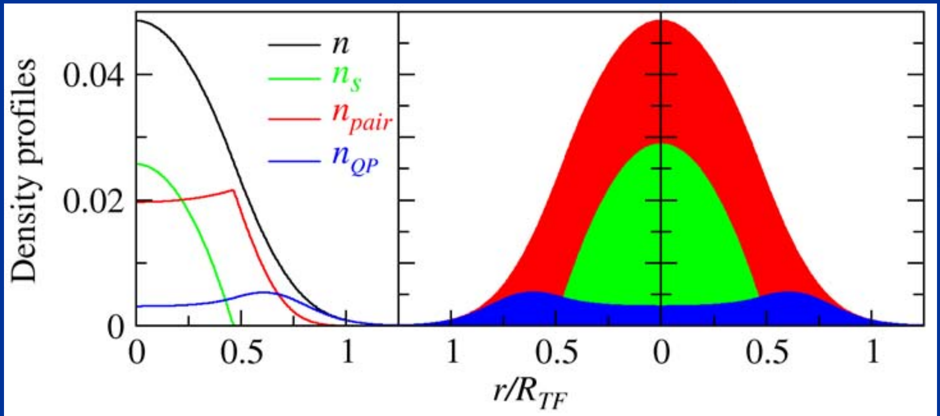
PRA 73, 041601(R) (2006)

Use sweep projected temperature to plot effective Tc or at given superfluid density.

Uncondensed Pairs Smooth out the Profiles



Phys. Rev. Lett. 94, 060401 (2005)

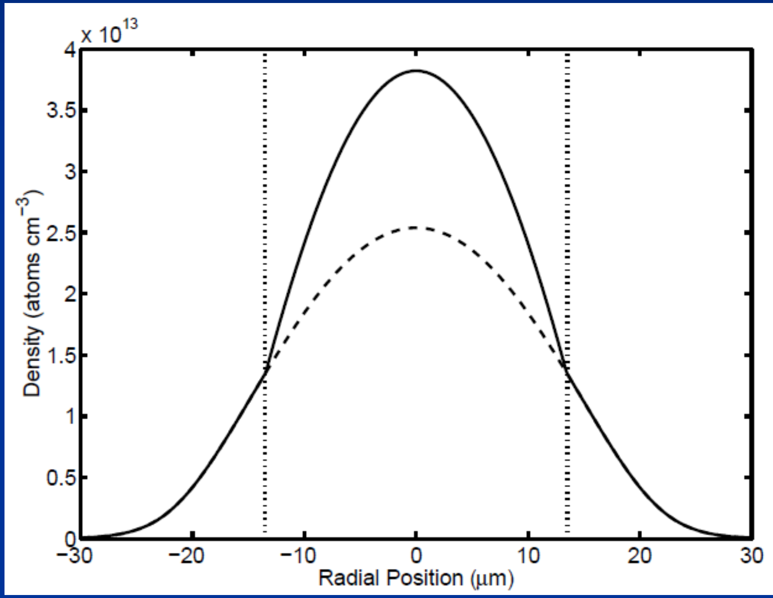


Unitary profile Below Tc

Condensate Noncondensed pairs Fermions

Data from Duke

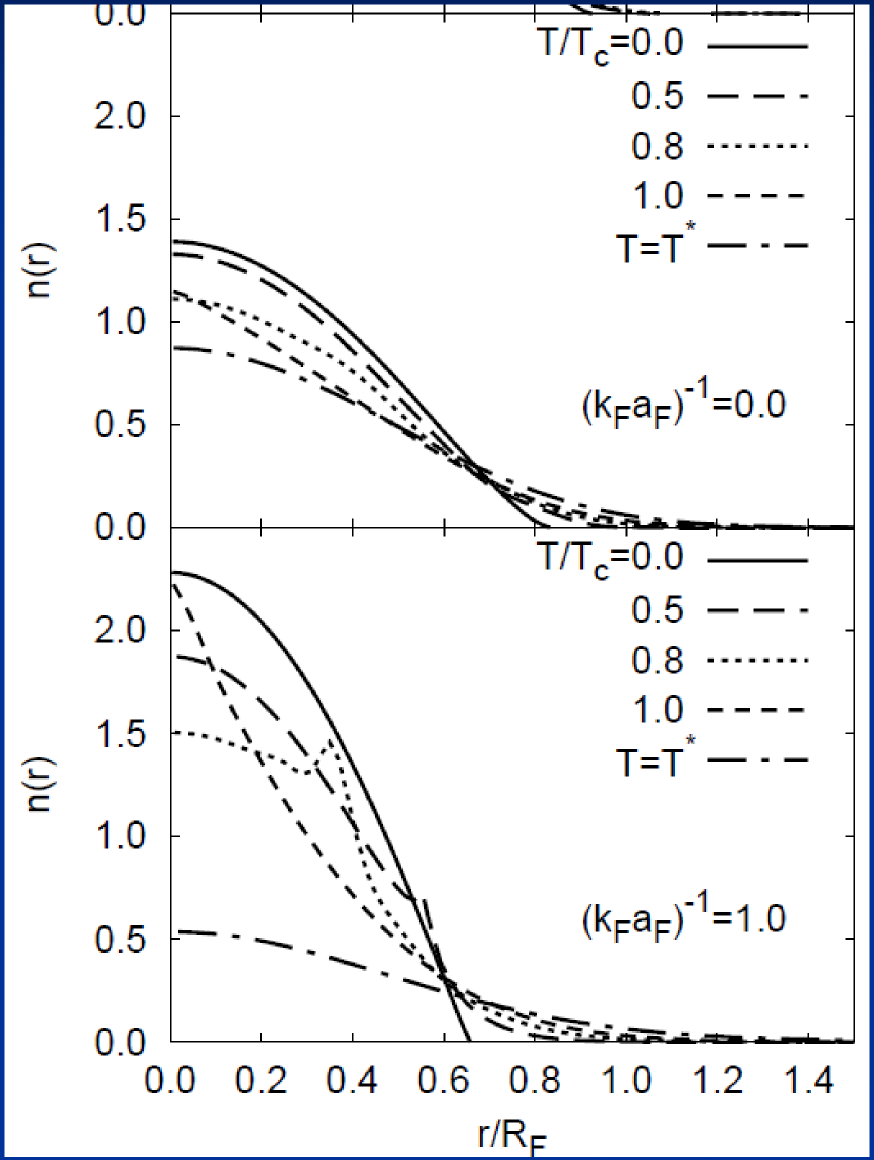
Other theoretical predictions



Mean-field calculation: Kink at superfluid/normal boundary

Chiofalo, Kokkelmans, Milstein, Holland, PRL 88, 090402 (2002)

Other theoretical predictions (cont'd)



Perali, Pieri, Pisani, Strinati, PRL 92, 220404 (2004)

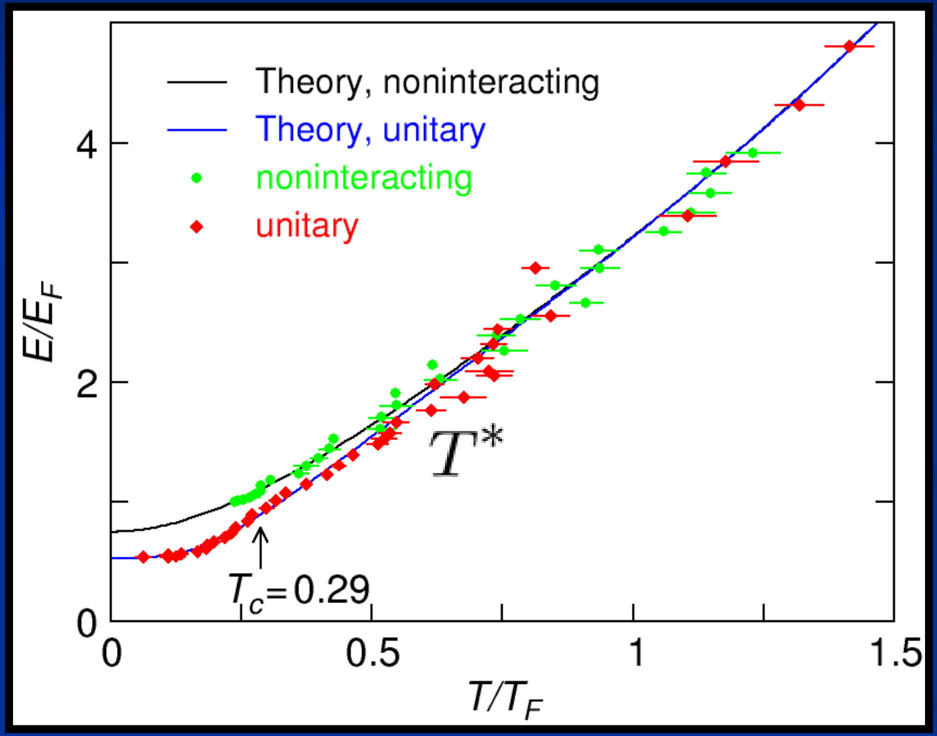
n(r) nonmonotonic as a function of *r* and *T* --- unphysical behavior.

Other theoretical predictions (cont'd)

- Hui Hu and Peter Drummond group:
- Add an extra (erroneous) $\partial\Delta/\partial\mu$ related term to number equation but keep gap equation at mean-field level.
- Consequences:
  - Different Tc when approached from above and below.
  - Homogeneous case: Chemical potential jump across Tc
  - In trap: Density jump at superfluid/normal state boundary
  - They never (dare to) show their density profiles!

Hu, Liu and Drummond, Europhys. Lett. 74, 574 (2006)

Thermodynamics and pseudogap effects



Duke data

No fitting parameter!

T* appears as temperature where 2 curves meet

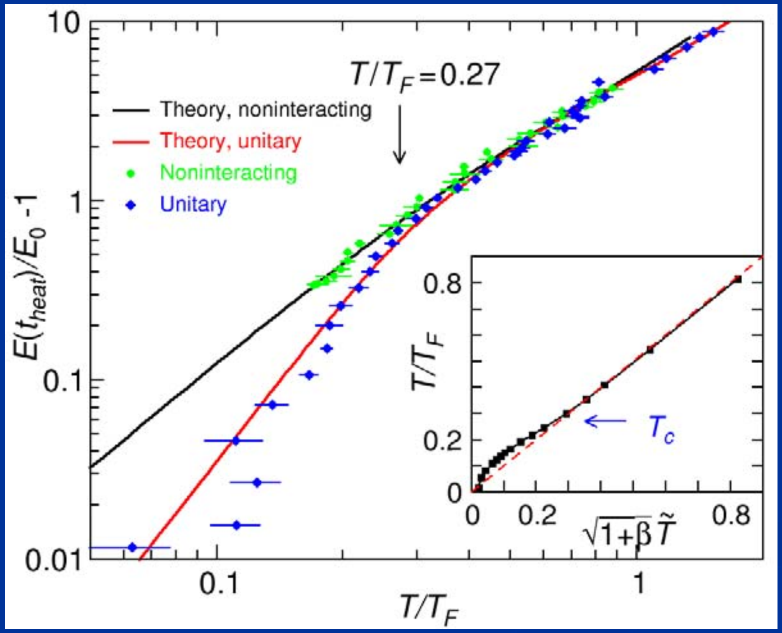
Science 307, 1296 (2005)

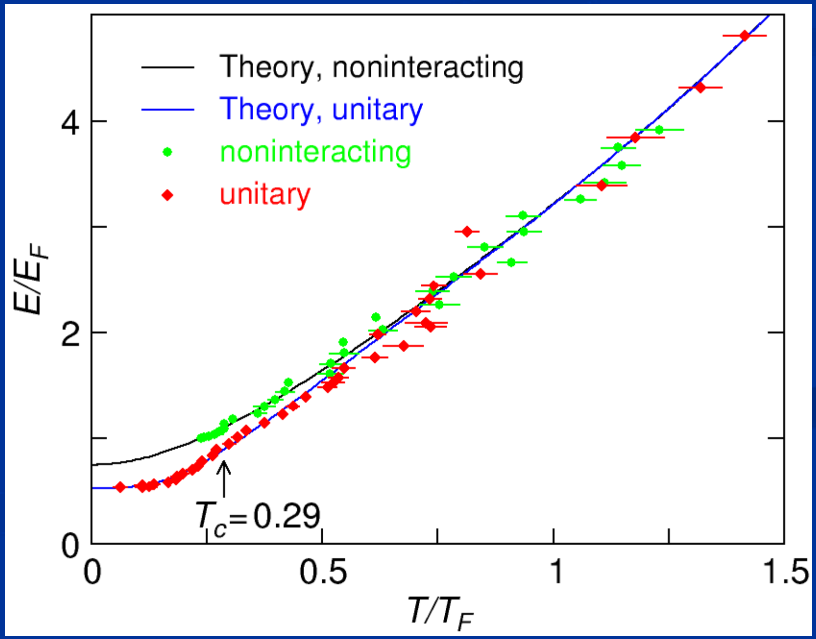
Thermodynamics and pseudogap effects

- First evidence (with experiment) for a superfluid phase transition

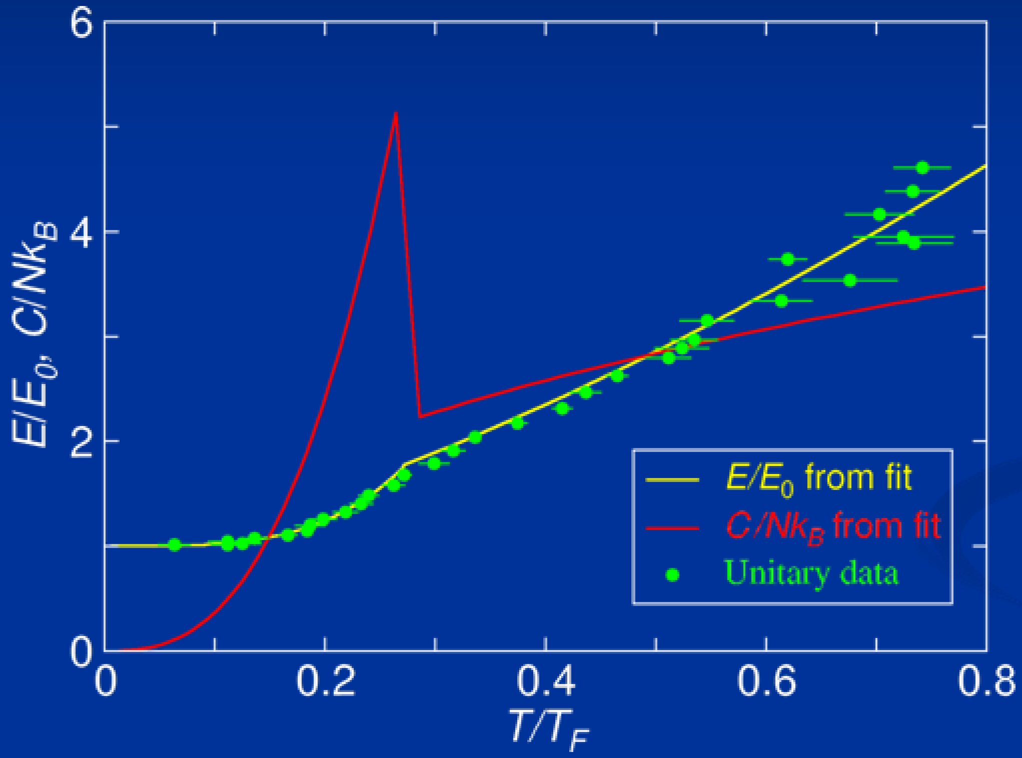
Science 307, 1296 (2005)

➢Thermodynamic properties of strongly interacting trapped gases





Specific heat



BCS like jump

Ground breaking work -- Ketterle

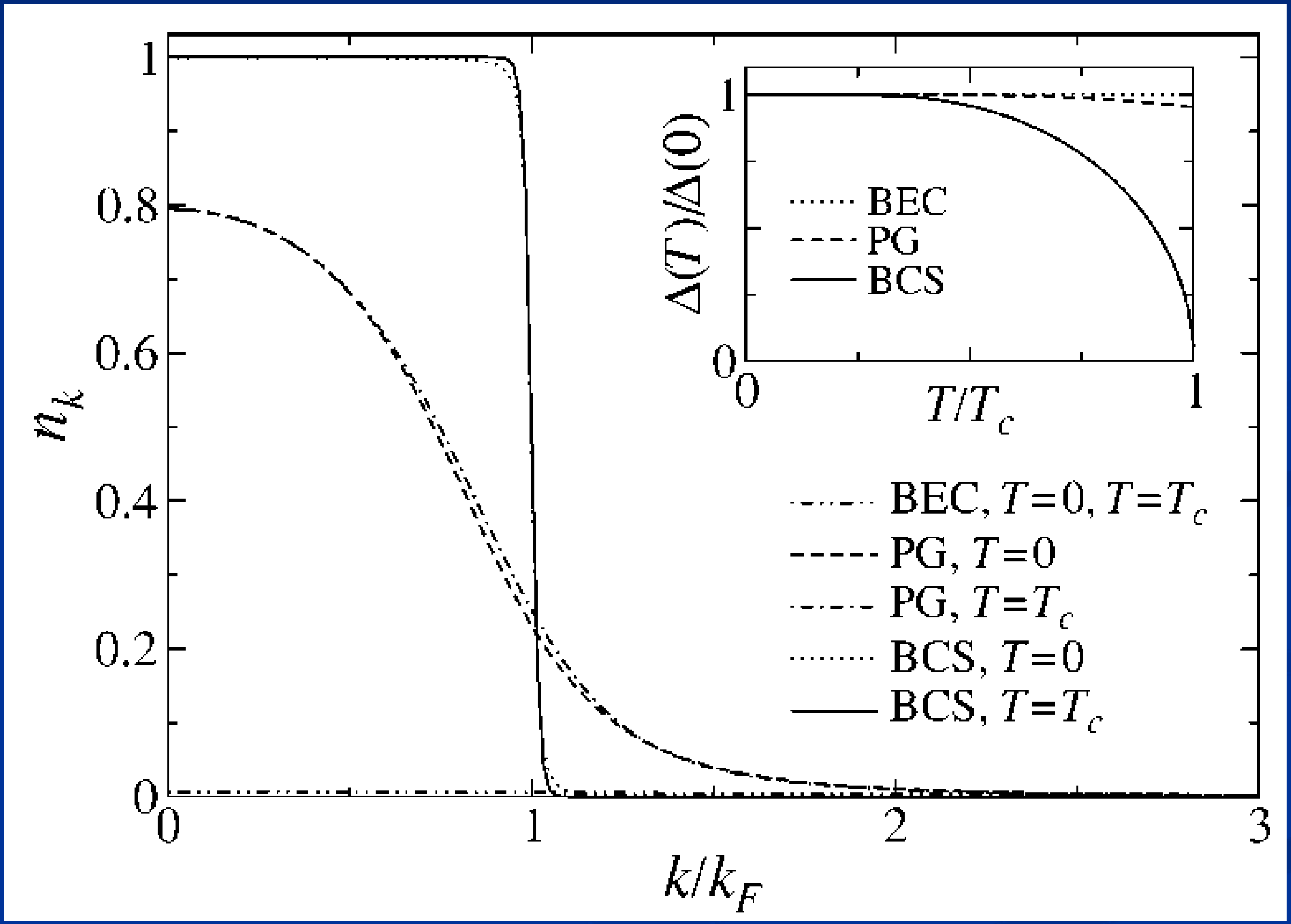
Many other successful comparison btw our theory and experiment.

How to measure excitation gap?

Use RF spectroscopy --

- Difficult to measure excitation gap in atomic Fermi gases.
  - Tiny system
  - Charge neutral
- RF spectroscopy is one of the most direct probes.
- Interpretation subject to complications from trap inhomogeneity, etc., --- Needs theoretical support.

Behavior of n_k in different regimes



Stajic *et al*, PRA 69, 063610 (2004)

Behavior of DOS in different regimes

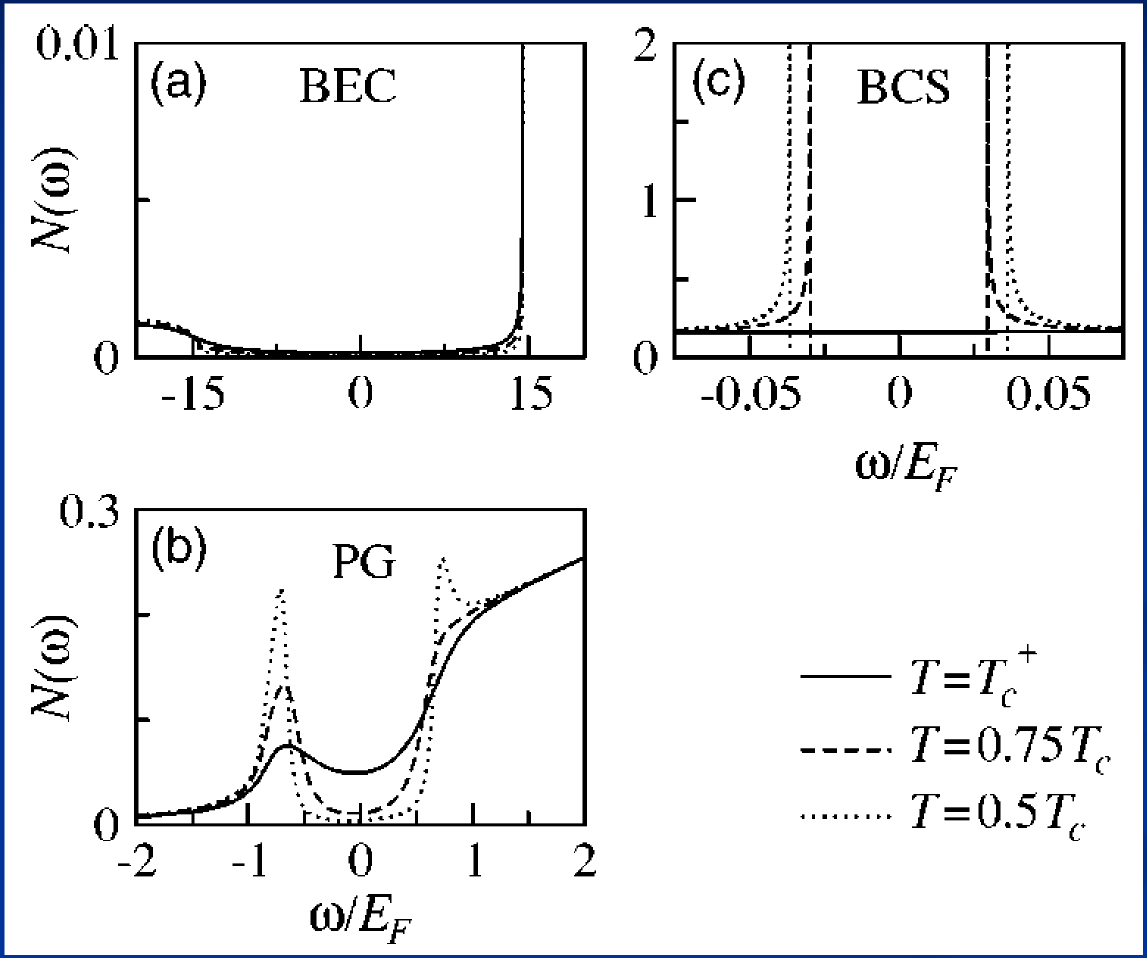
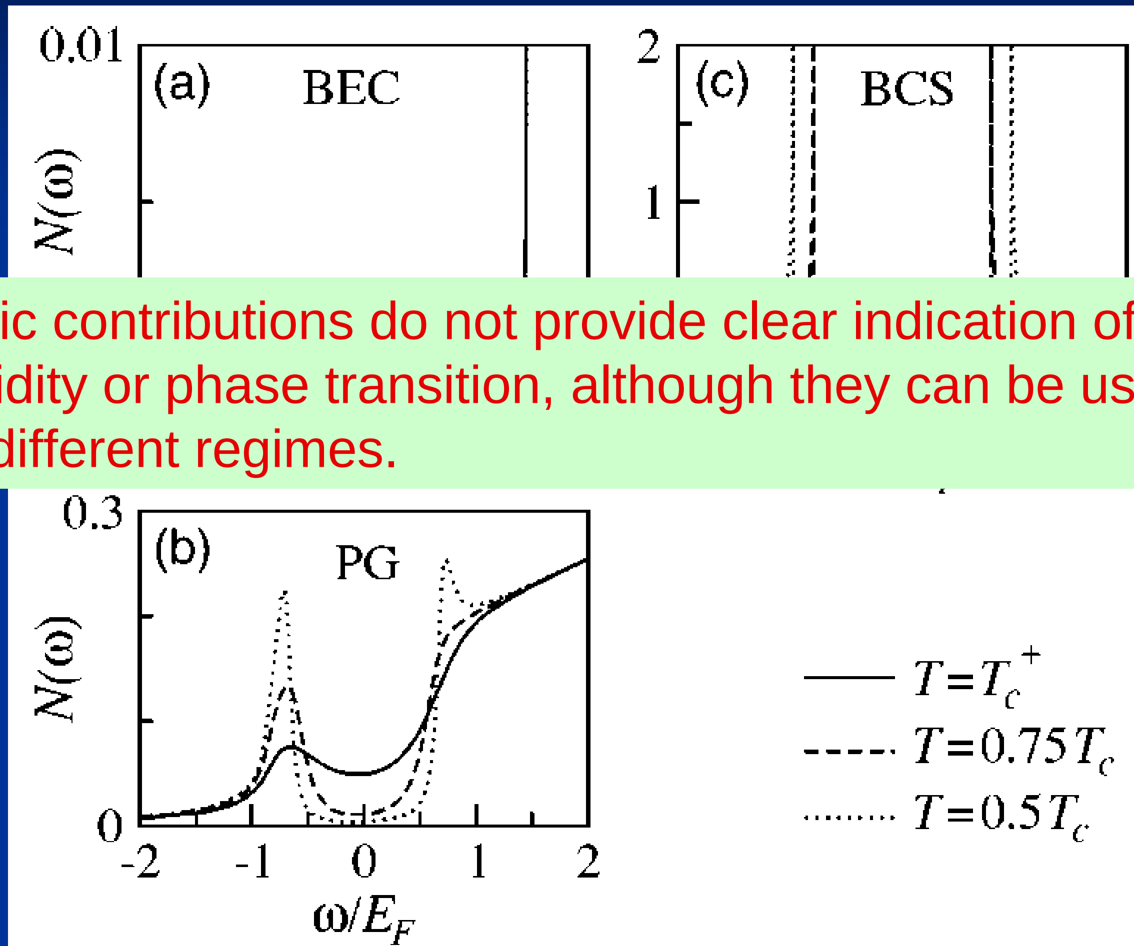


FIG. 3. Fermionic density of states vs energy for the three regimes at three indicated temperatures. Note the difference in the scales.

Behavior of DOS in different regimes



Fermionic contributions do not provide clear indication of superfluidity or phase transition, although they can be used to identify different regimes.

FIG. 3. Fermionic density of states vs energy for the three regimes at three indicated temperatures. Note the difference in the scales.

What is RF spectroscopy

- Atoms in hyperfine level 1 and 2 are paired.
- Atoms between levels 1 and 3 do not pair.
- Using RF field to excite an atom in hyperfine level 2 into level 3.
- Extra energy (detuning) needed if atoms in level 2 are paired than if level 2 is free.

Linear response theory for RF

State 3 empty

1-2 superfluid with 2-> 3 transition

$$H_{rf} = e^{-i\Omega t} \int d^3x \, \psi_3^\dagger \psi_2 + h.c.$$

$$D(i\Omega_n) = T \sum_K G^{(2)}(K) G^{(3)}(K+Q)$$

$$A_3(\mathbf{k}, \omega) = 2\pi\delta(\omega - \xi_\mathbf{k} + \mu_3 - \mu)$$

$$I(\nu) = -\frac{1}{\pi} \mathrm{Im}\, D^R(\nu + \mu - \mu_3)$$

$$= -\frac{1}{2\pi} \sum_\mathbf{k} A(\mathbf{k}, \omega) f(\omega) \Big|_{\omega = \xi_\mathbf{k} - \nu}$$

In the absence of finate state effects

Spectral function -- Effects of phase coherence

$$A(\mathbf{k},\omega) = -2\,\mathrm{Im}\,G(\mathbf{k},\omega+i0)\ ,$$

$$\Sigma_{pg}(K) = \frac{\Delta_{pg}^2}{i\omega_l + \xi_{\mathbf{k}}} + \delta\Sigma$$

$$\longrightarrow \quad \Sigma_{pg}(\omega + i0^+, \mathbf{k}) \approx \frac{\Delta_{pg}^2}{\omega + \xi_{\mathbf{k}} + i\gamma} + i\Sigma_0$$

Above Tc:

$$A(\mathbf{k},\omega) = \frac{2\Delta_{\mathbf{k}}^2\gamma}{(\omega^2 - E_{\mathbf{k}}^2)^2 + \gamma^2(\omega - \xi_{\mathbf{k}})^2}$$

Below Tc:

$$A(\mathbf{k},\omega) = \frac{2\Delta_{\mathbf{k},pg}^2\gamma(\omega+\xi_{\mathbf{k}})^2}{(\omega+\xi_{\mathbf{k}})^2(\omega^2 - E_{\mathbf{k}}^2)^2 + \gamma^2(\omega^2 - \xi_{\mathbf{k}}^2 - \Delta_{\mathbf{k},sc}^2)^2}$$

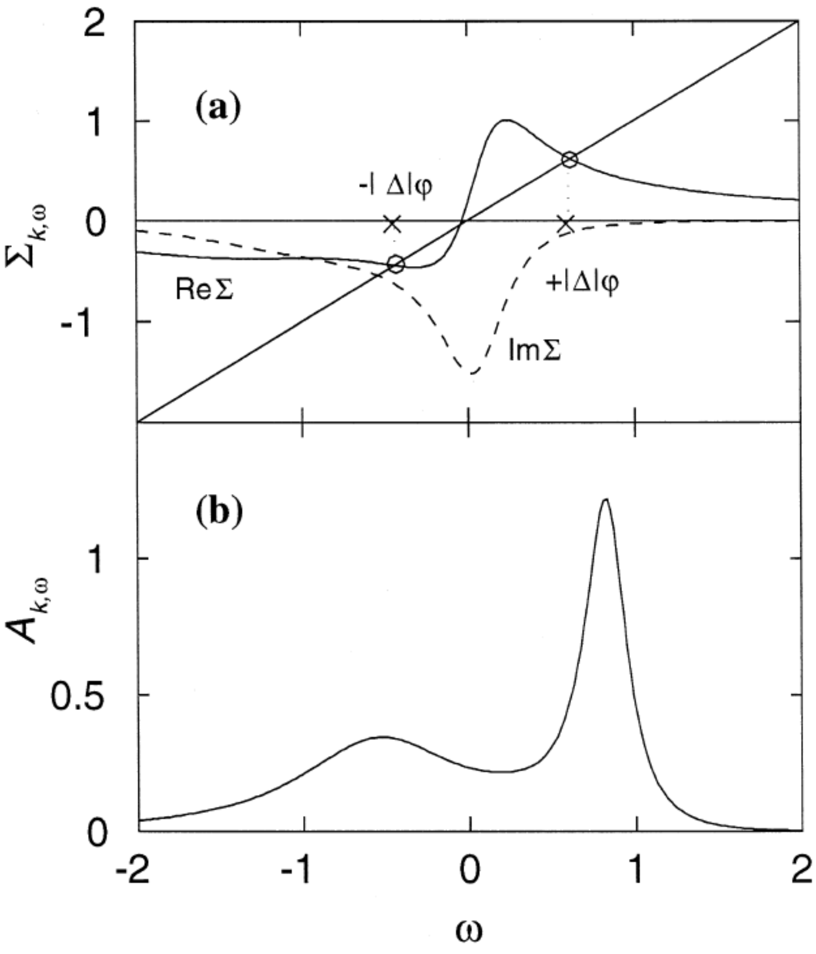
Exists an zero at $\omega = -\xi_{\mathbf{k}}$

Numerical basis

Extensive numerical analysis were done to arrive at the expression of Σ_{pg}

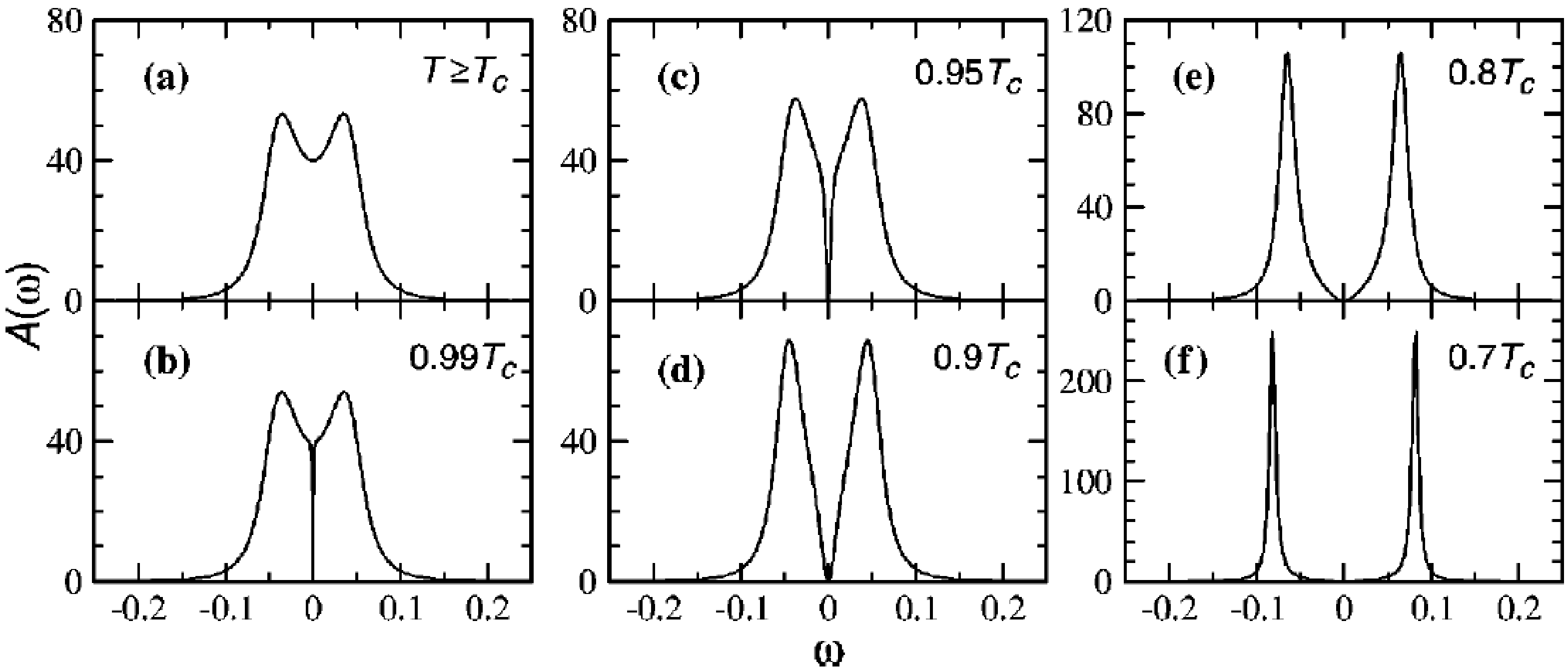
Janko, Maly, and Levin, PRB 56, R11407 (1997).

Maly, Janko, and Levin, PRB 59, 1354 (1999); Physica C 321, 113 (1999)



What is a pseudogap?

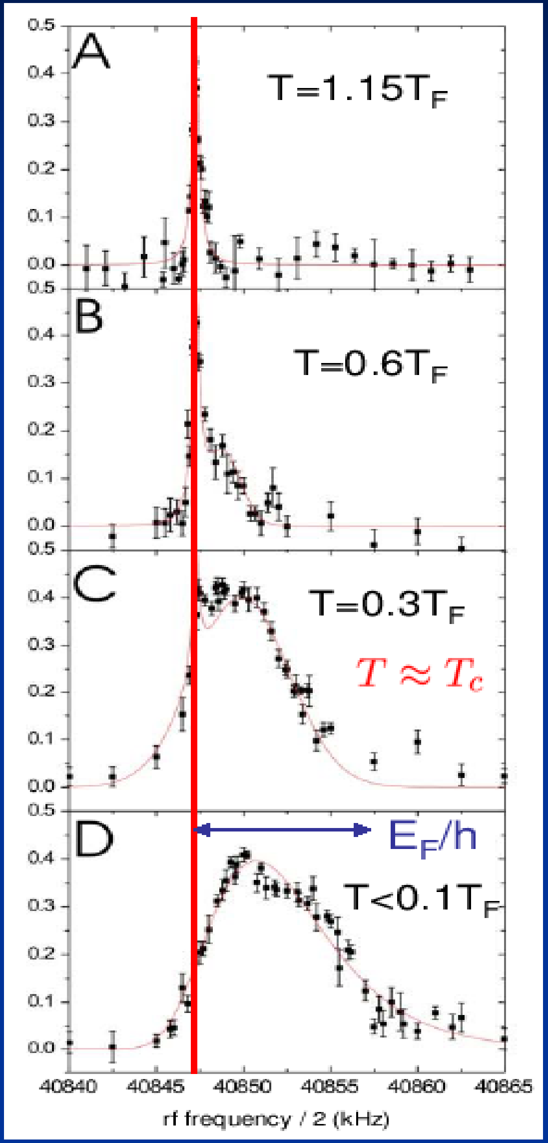
- Pseudogap becomes “real” upon phase coherence



Chen, Kosztin, and Levin, PRB 63, 184519 (2001)

RF Spectroscopy and Pseudogap Effects

Signal strength



C. Chin et al, Science 305, 1128 (2004).

New data at unitarity from Grimm

Red line – Free atom peak

-- Trap integrated!

Used as evidence of superfluid state, we pointed out that it may be just pseudogap state.

Pseudogap is evident as $T > T_c$ shoulder!

Questions

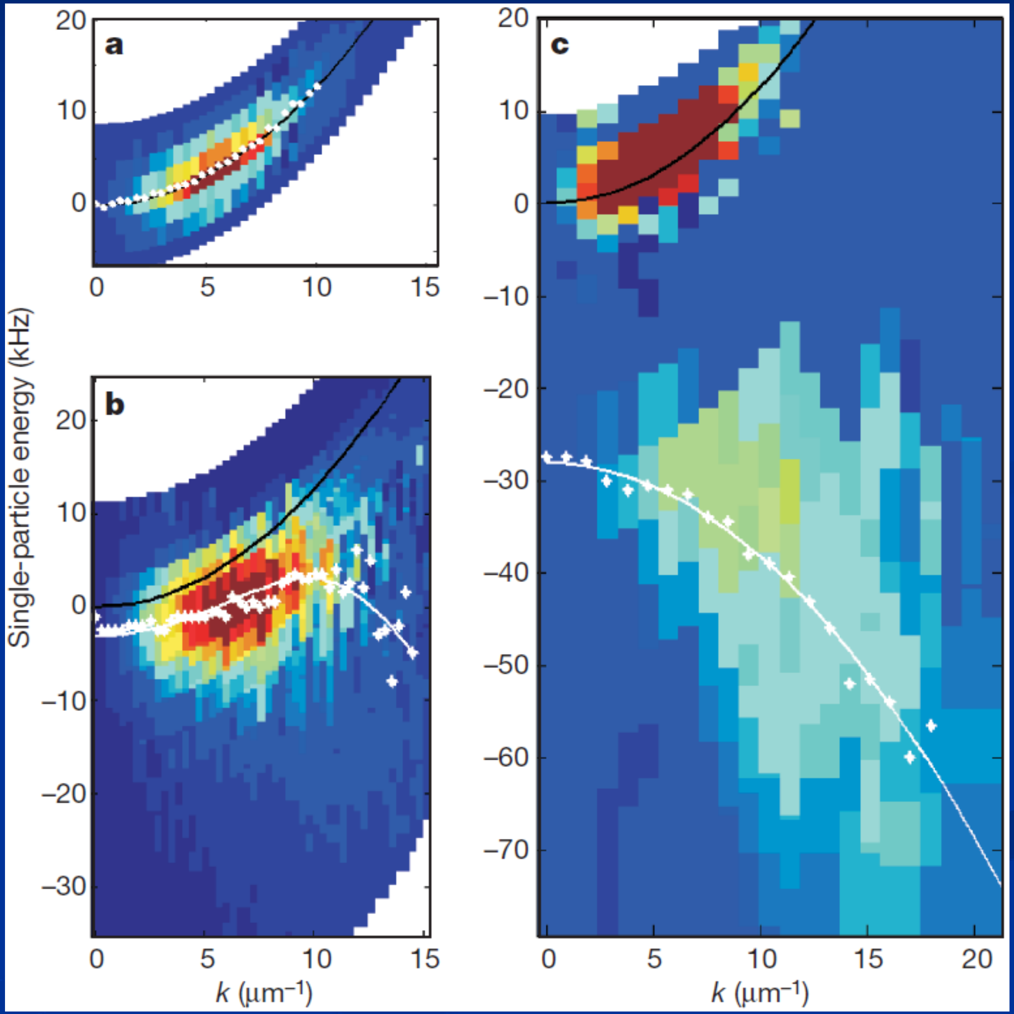
- Sounds great, BUT
  - Detuning NOT proportional to excitation gap
  - Trap inhomogeneity
  - Lack of momentum resolution

  – detuning depends on k for same Δ

$$\begin{aligned}\delta\nu &= \sqrt{(\xi_{\mathbf{k}}-\mu)^2+\Delta^2}+\xi_{\mathbf{k}}-\mu \\ &\geq \sqrt{\mu^2+\Delta^2}-\mu \approx \Delta^2/2\mu\end{aligned}$$

Momentum resolved RF probe

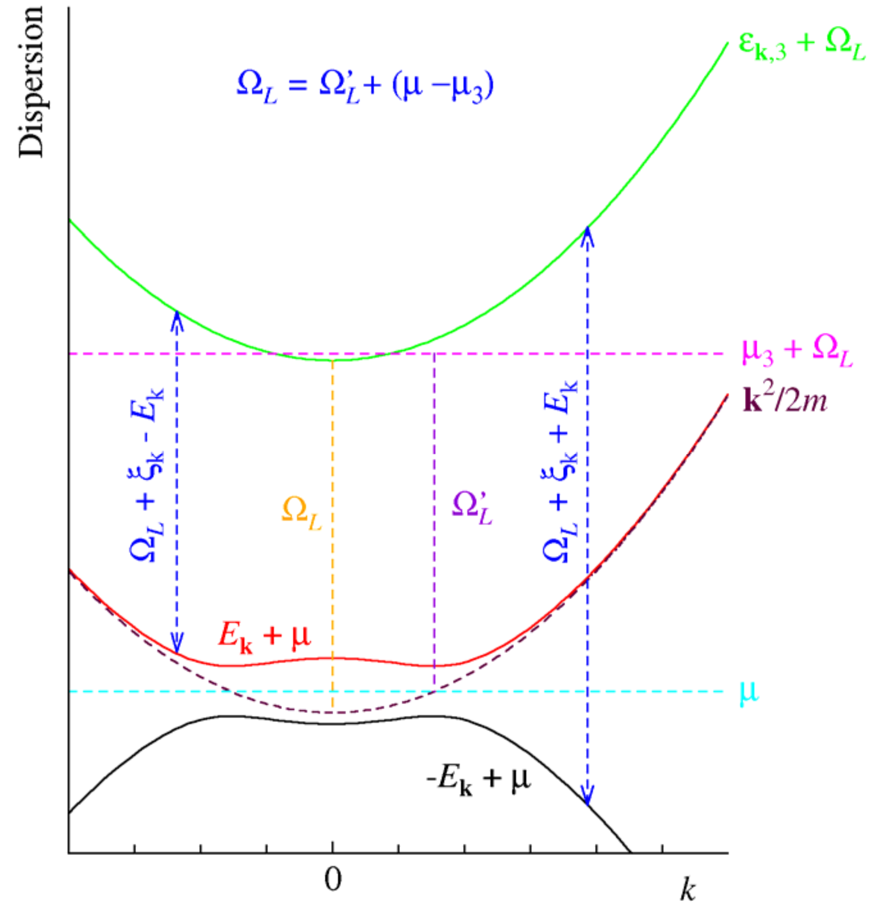
- Recent exciting experiment from JILA provides momentum resolution
- ^{40}K – free of final state interactions
- Great advance
- BUT
- Still plagued by inhomogeneity issue
- Can the RF probe give reasonable info about A(k,ω) in the trap?



JT Stewart, JP Gaebler, DS Jin, Nature 454, 744 (2008)

Momentum resolved RF -- ARPES

- In the absence of interaction, photon energy is fixed at Ω_L, indep. of k.
- No real emission
- 3D bulk probe



$$I(\mathbf{k}, \delta\nu)$$

$$= \frac{|T_k|^2}{2\pi} A(\mathbf{k}, \omega) f(\omega) \Big|_{\omega = \xi_{\mathbf{k}} - \delta\nu}$$

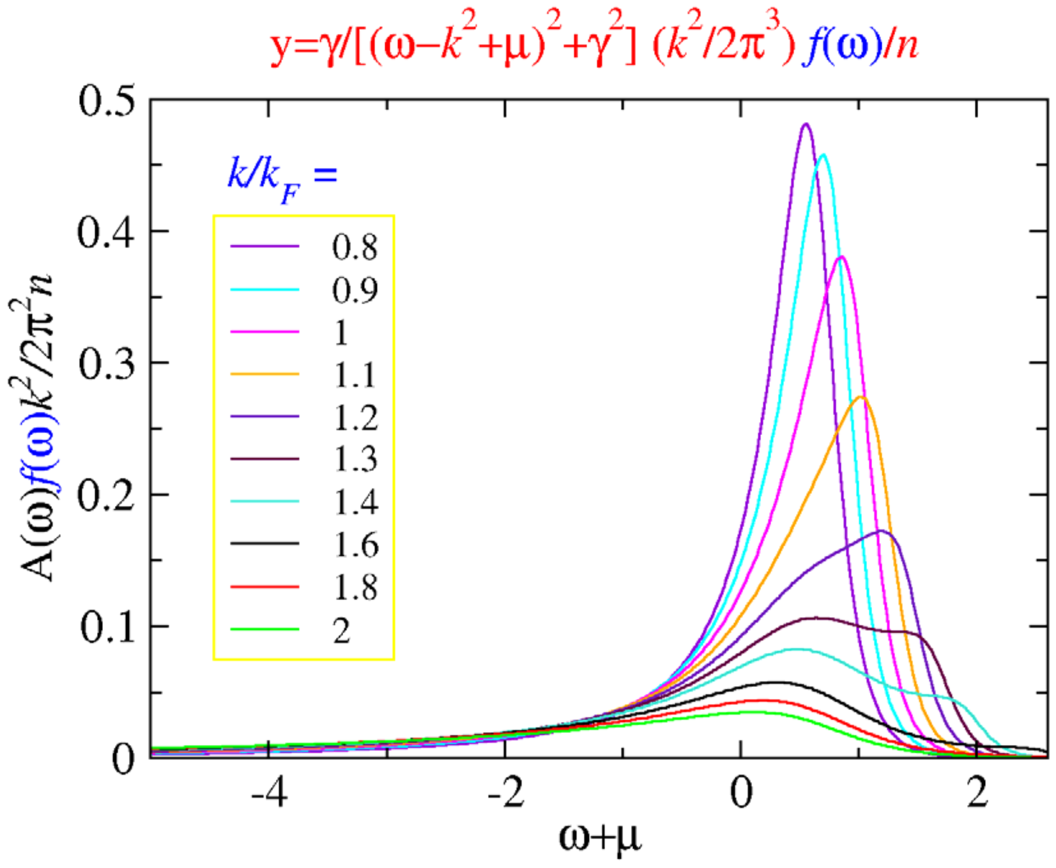
$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

Energy distribution curves

Homogeneous "free" Fermi gas

Broad peak emerges for high k as a result of Fermi function suppression

Energy determined by curve fitting at high k inaccurate.

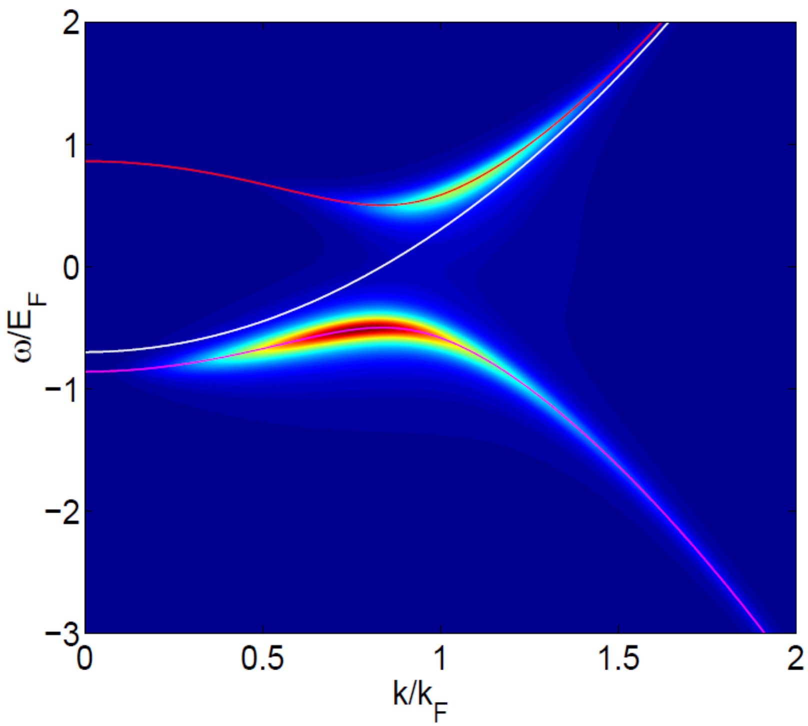


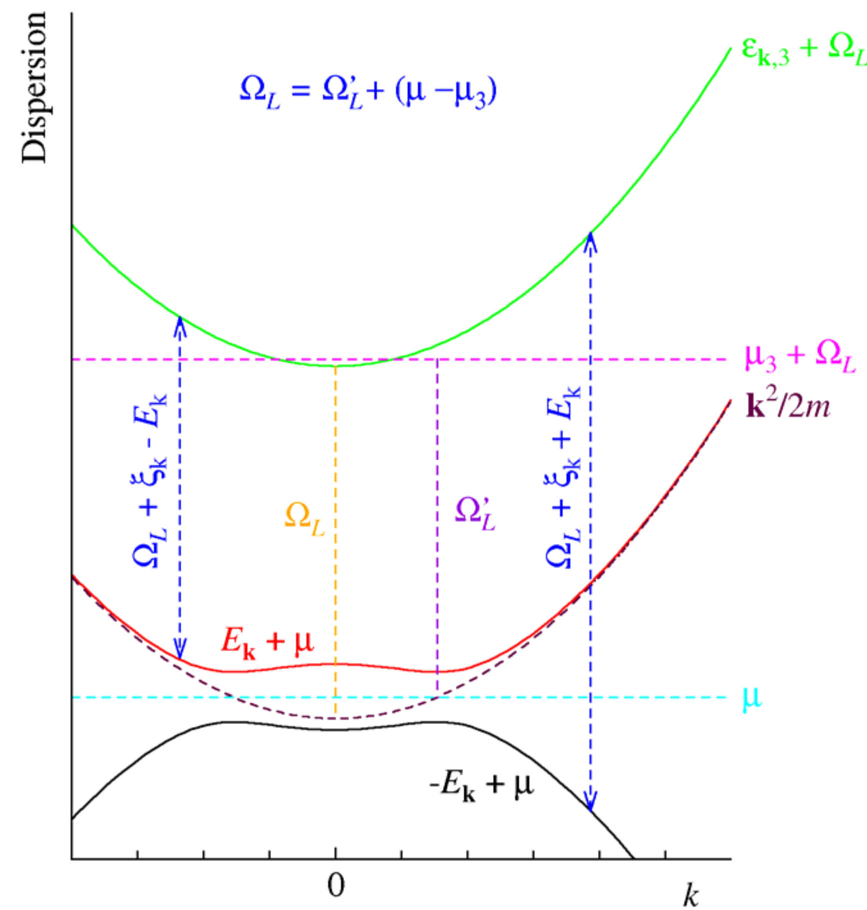
QC and K. Levin, PRL 102, 190402 (2009).

$\mu = 0.62$, $\gamma = 0.33$, $T/T_F = 0.3$

Two branches – Homogeneous case

Population of two branches self-consistently determined – not by hand.

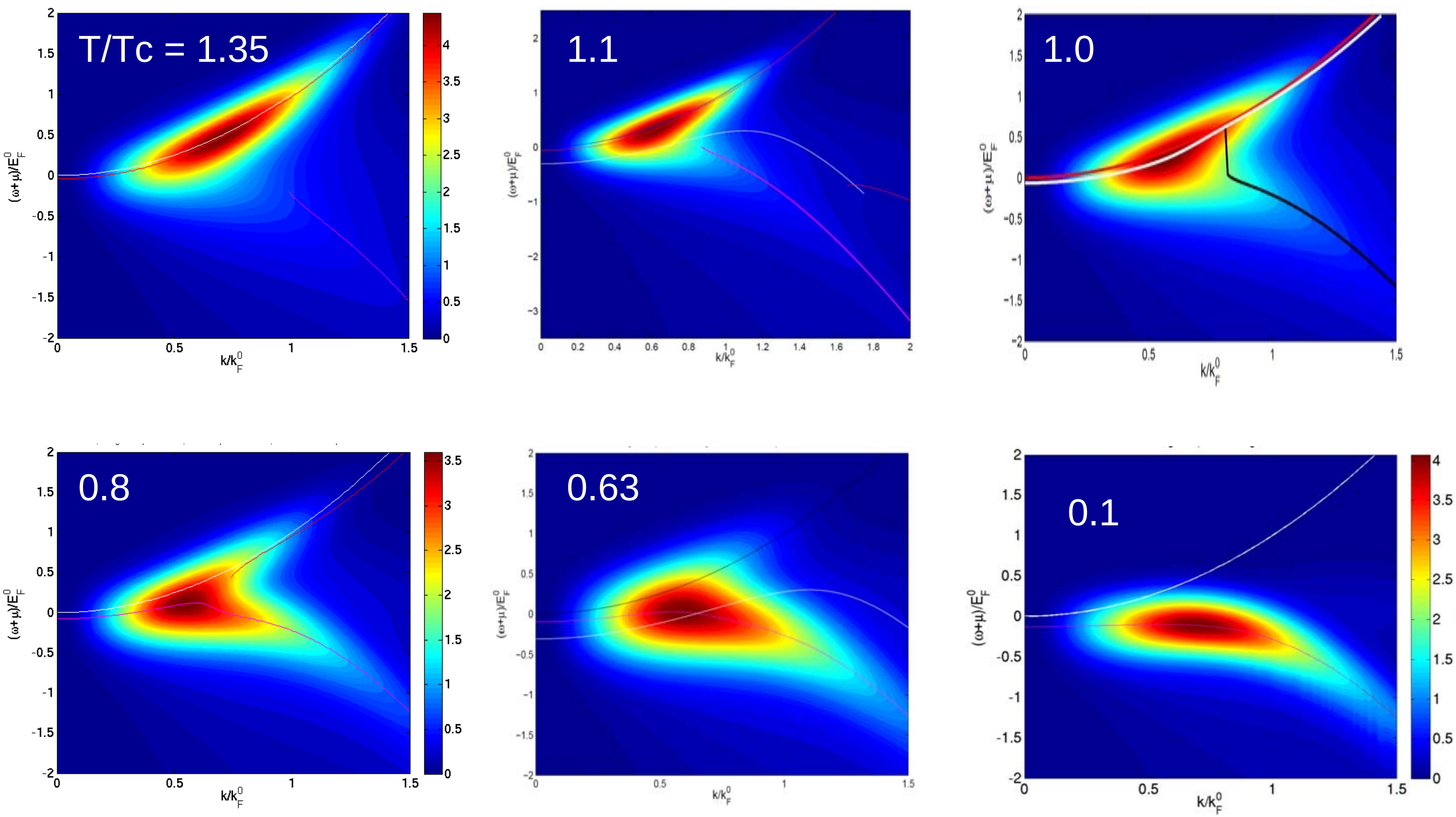




- Unitary, μ = 0.7, Δ_{pg}=0.5, γ = 0.1, T/T_F=0.4

Big pseudogap and relatively high T needed.

Momentum resolved RF -- In traps



$\Sigma_0 = 0.25 E_F^0$ and $\gamma = 0.25(T/T_c)E_F^0$ $\qquad \sigma = 0.22 E_F^0$

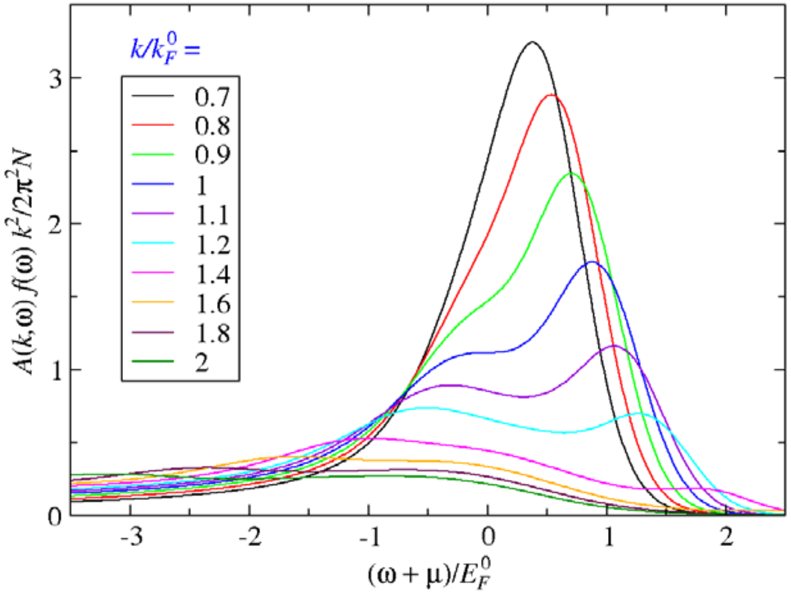
Comparison -- Energy distribution curves

Fermi gas at unitarity in a trap

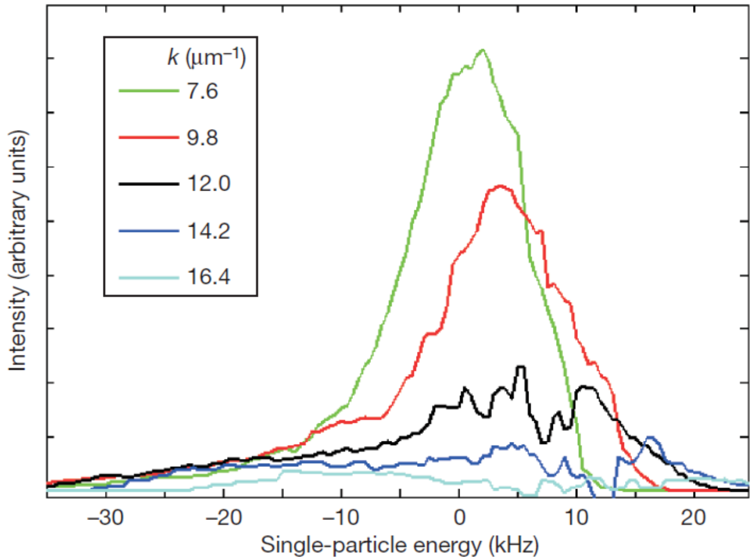
k_F^0 is $8.6 \pm 0.3\ \mu m^{-1}$

QC and K. Levin, PRL 102, 190402 (2009).

Theory, T/Tc = 1.1



Experiment



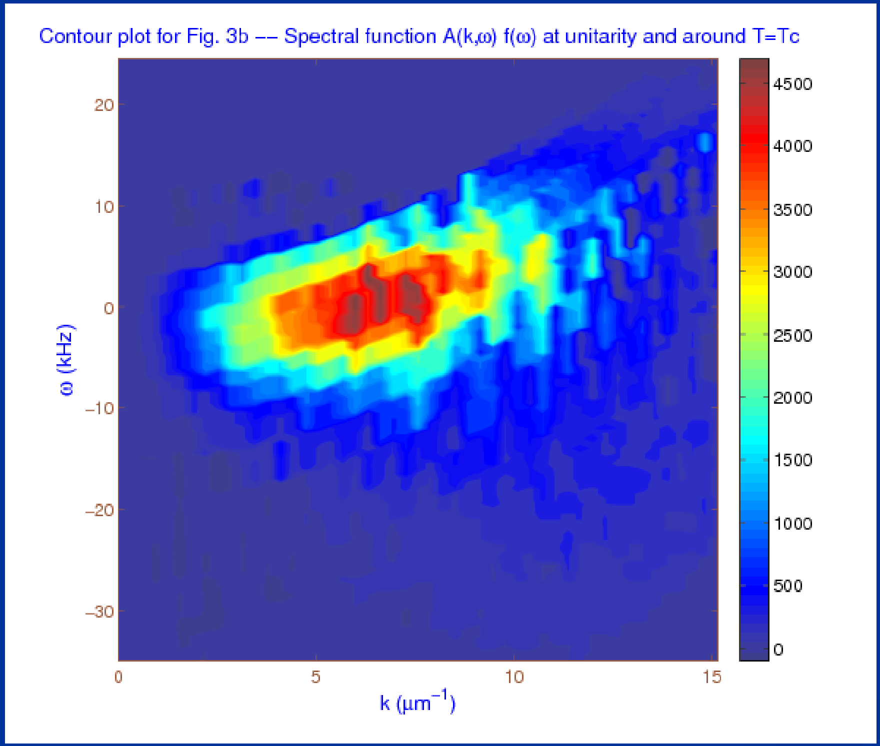
JT Stewart, JP Gaebler, DS Jin, Nature 454, 744 (2008)

Σ_0, γ determined by experiment

$\mu = h \times (12.6 \pm 0.7\,\text{kHz})$ and $\Delta = h \times (9.5 \pm 0.6\,\text{kHz})$

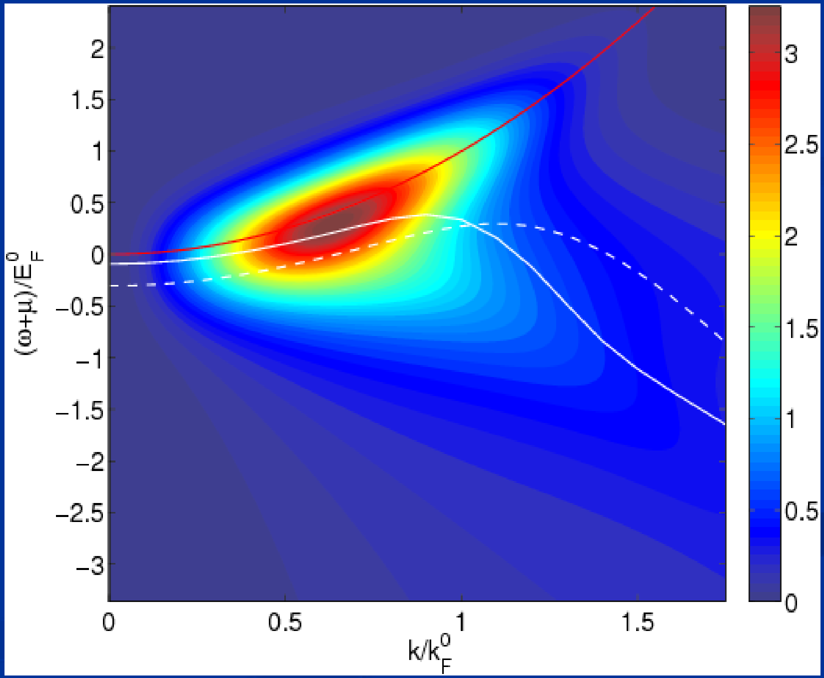
Intensity map of momentum resolved RF spectra for trapped Fermi gas at unitarity

JILA data



Stewart et al, Nature 454, 744 (2008)

Our theory

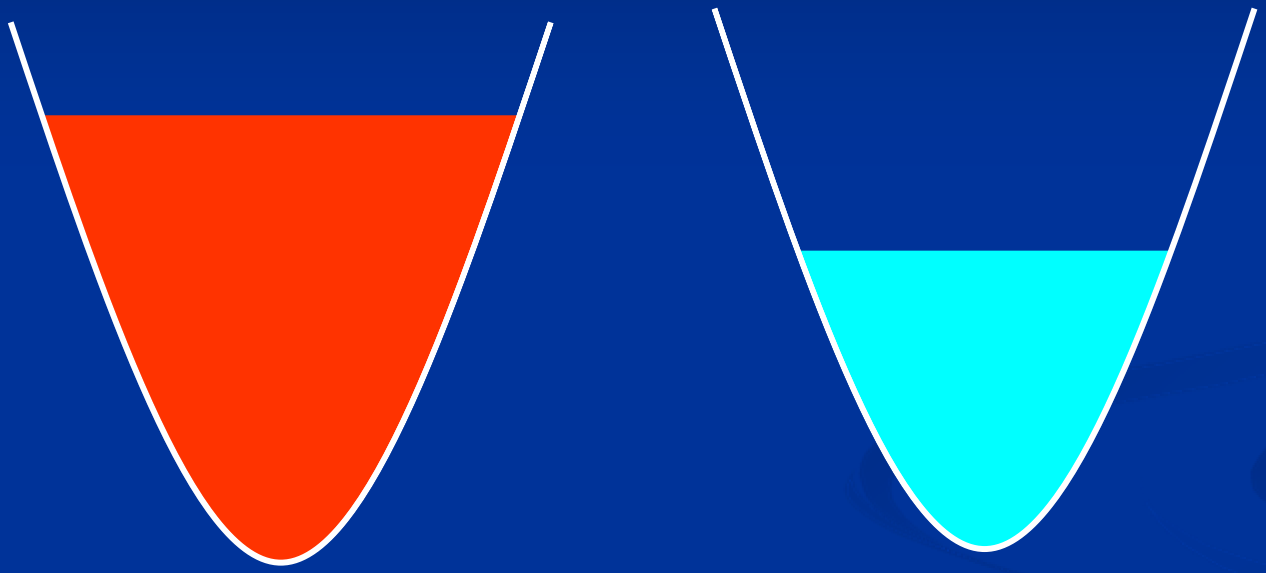


QC and K. Levin, PRL 102, 190402 (2009).

Comparison between theory and experiment

-- Population imbalanced Fermi gases

Population imbalanced superfluidity

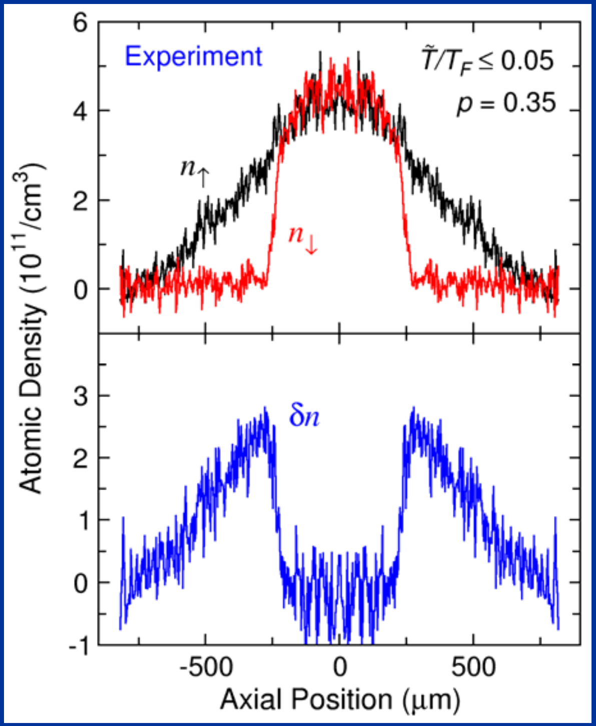


Naturally arises in nuclear matter; superconductors in magnetic field
Search for the elusive Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase.

Two groups: Rice and MIT

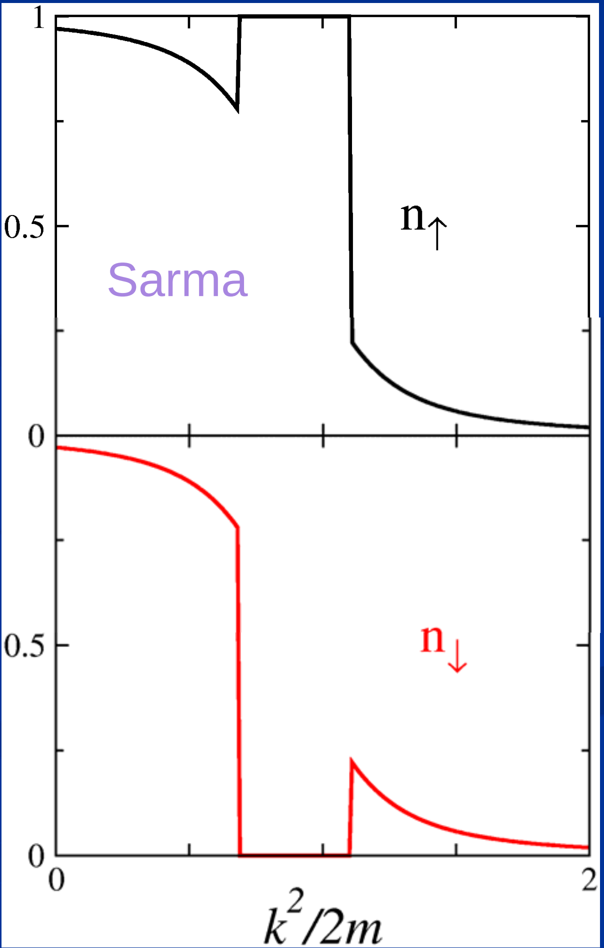
Three Ways to accommodate polarization

- Breached Pair (Sarma) State
- Phase Separation
- FFLO



Phase Sep.

Rice data



Equation set with population Imbalance

$$t^{-1}(P) = U^{-1} + \chi(P)$$

$$\chi(P) = \frac{1}{2}[\chi_{\uparrow\downarrow}(P) + \chi_{\downarrow\uparrow}(P)]$$

$$\chi_{\uparrow\downarrow}(P) = \sum_K G_{0\uparrow}(P-K)G_{\downarrow}(K)$$

$$\chi_{\downarrow\uparrow}(P) = \sum_K G_{0\downarrow}(P-K)G_{\uparrow}(K)$$

Gap equation

$$t^{-1}(0) = 0 = U^{-1} + \chi(0)$$

$$0 = 1 + U\chi(0) = 1 + U\sum_{\mathbf{k}} \frac{1-2\bar{f}(E_{\mathbf{k}})}{2E_{\mathbf{k}}}\varphi_{\mathbf{k}}^2$$

$$n = 2\sum_{\mathbf{k}} \left(v_{\mathbf{k}}^2 + \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\bar{f}(E_{\mathbf{k}}) \right),$$

$$pn = \sum_{\mathbf{k}} [f(E_{\mathbf{k}} - h) - f(E_{\mathbf{k}} + h)],$$

$$\Delta_{pg}^2 \equiv -\sum_{Q\neq 0} t(Q) = \frac{1}{Z}\sum_{\mathbf{q}} b(\Omega_{\mathbf{q}}).$$

$$\bar{f}(x) \equiv [f(x+h) + f(x-h)]/2$$

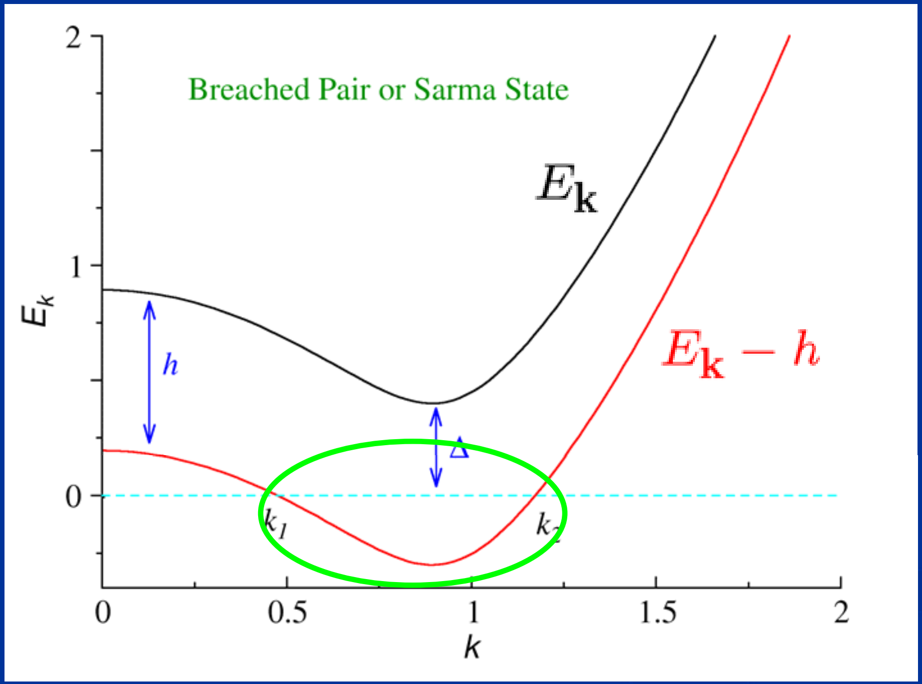
$$p = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}, \qquad n = n_\uparrow + n_\downarrow$$

$$E_k = \sqrt{(\xi_k - \mu)^2 + \Delta^2}, \qquad \mu = \frac{1}{2}(\mu_\uparrow + \mu_\downarrow),\ h = \frac{1}{2}(\mu_\uparrow - \mu_\downarrow)$$

Population imbalanced superfluidity

$$G_{\uparrow,\downarrow}(K) = \frac{u_{\mathbf{k}}^2}{i\omega_n \pm h - E_{\mathbf{k}}} + \frac{v_{\mathbf{k}}^2}{i\omega_n \mp h + E_{\mathbf{k}}}$$

$$E_k = \sqrt{(\xi_k - \mu)^2 + \Delta^2}, \quad \mu = \frac{1}{2}(\mu_\uparrow + \mu_\downarrow),\ h = \frac{1}{2}(\mu_\uparrow - \mu_\downarrow)$$



Gapless excitation spectrum !

Breached pair or Sarma state

Unstable when

Interaction weak

Imbalance high

Population imbalanced superfluidity

$$G_{\uparrow,\downarrow}(K) = \frac{u_{\mathbf{k}}^2}{i\omega_n \pm h - E_{\mathbf{k}}} + \frac{v_{\mathbf{k}}^2}{i\omega_n \mp h + E_{\mathbf{k}}}$$

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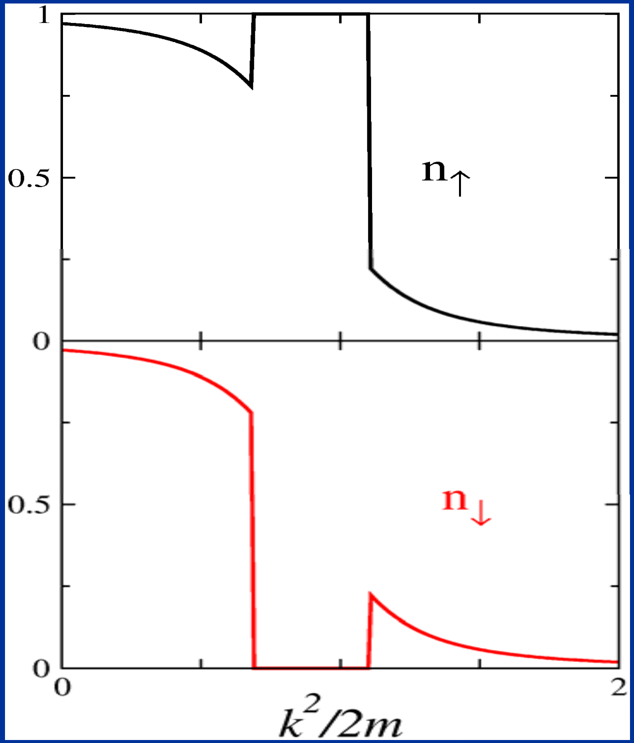


Gapless excitation spectrum !

Breached pair or Sarma state

Unstable when

Interaction weak

Imbalance high

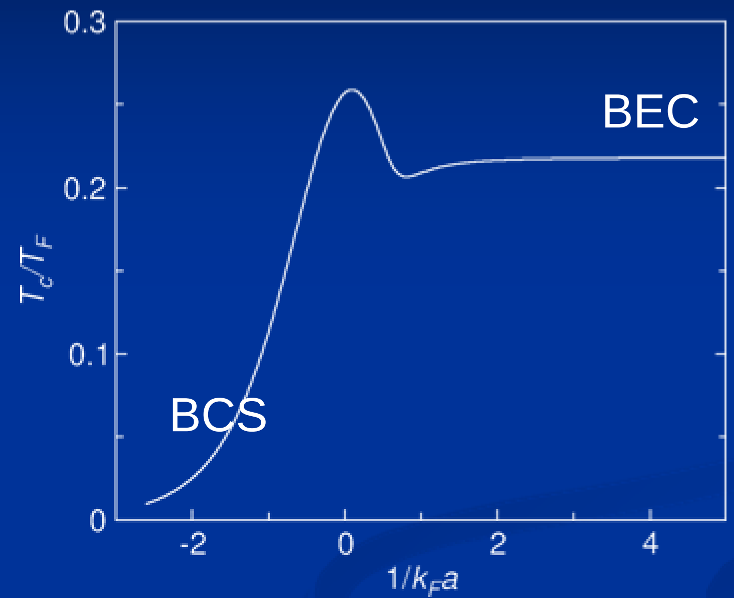
Temperature is essential to include!

- Experimental temperature is never strictly zero.
- Sarma phase only stable at finite T at unitarity and in BCS.
- Experimental profiles of polarized gases change at Tc.

Contrast with unpolarized case.

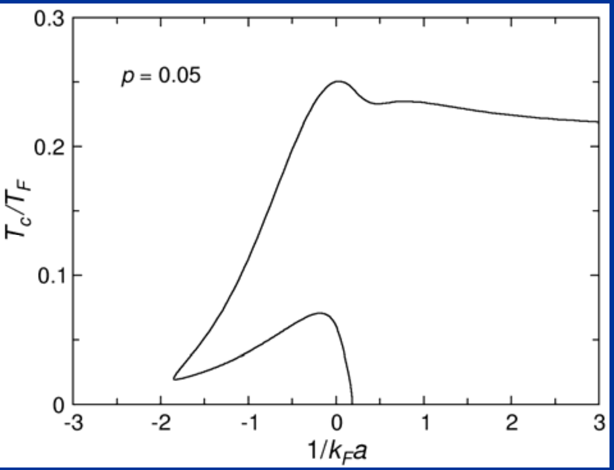
Homogeneous case – Behavior of Tc

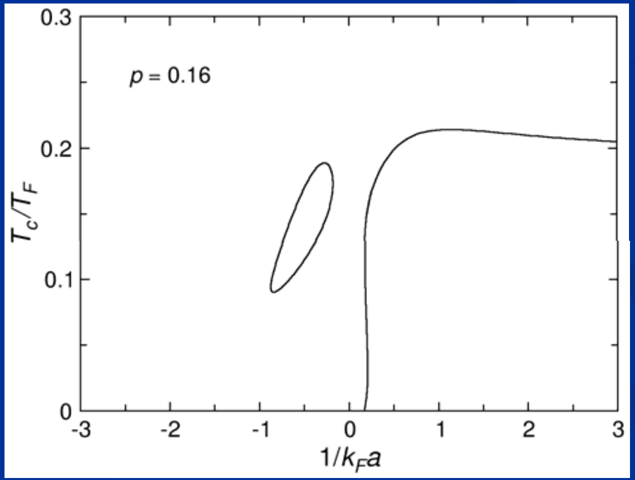
Unpolarized case

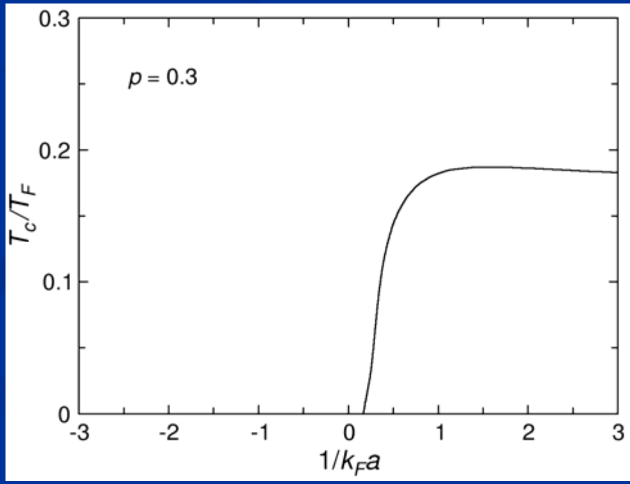


Phys. Rev. Lett. 97, 090402 (2006)

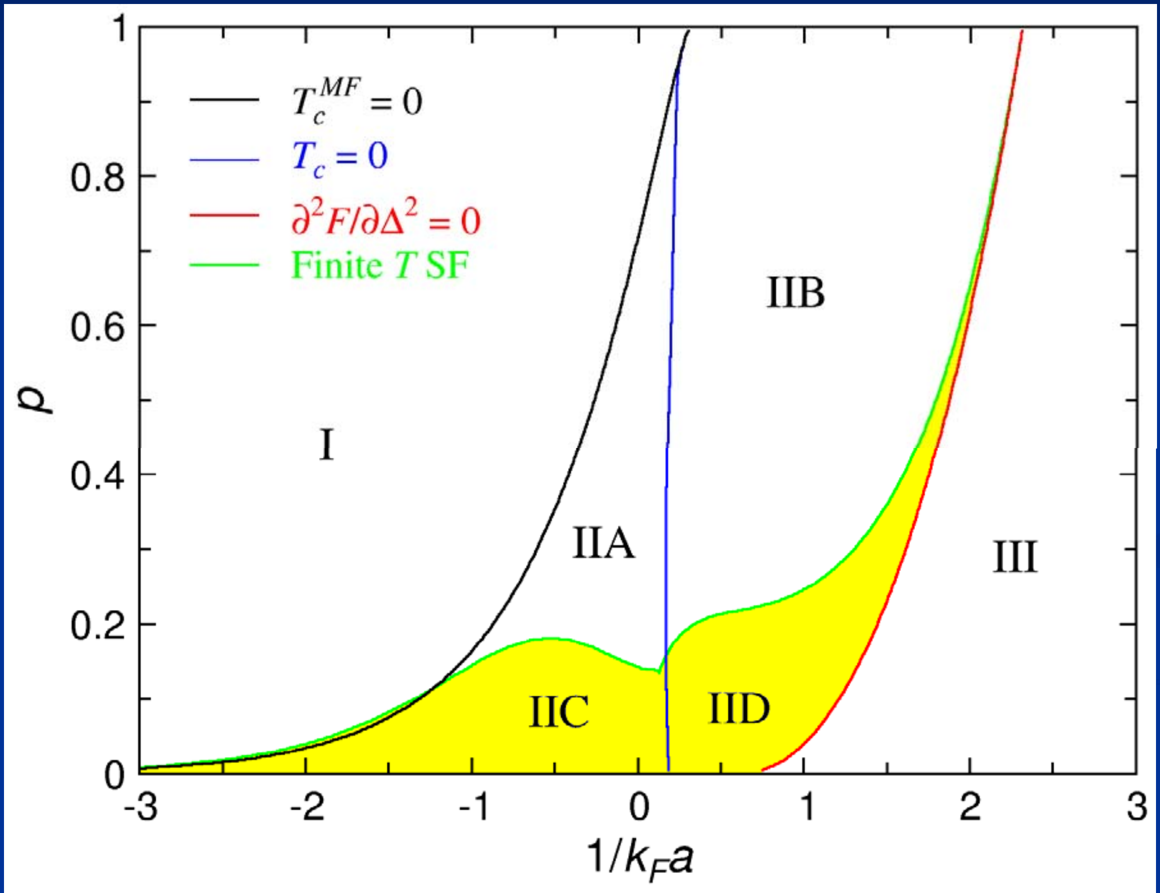
$$p = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$$







Homogeneous phase diagram -- Intermediate temperature superfluid



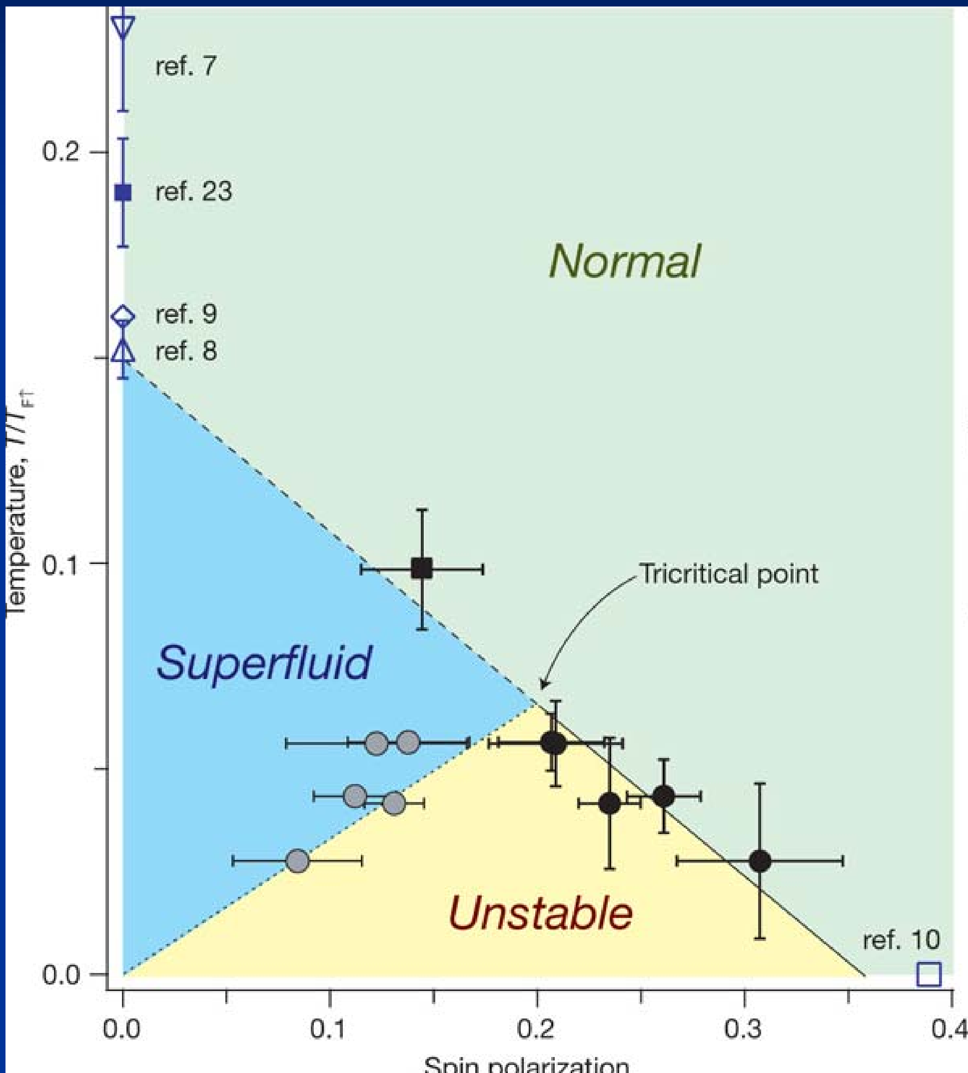
PRL 97, 090402 (2006)

Phase diagram (at T=0)

II: Phase separation

I: Fermi gas; III: Sarma SF (BEC regime)

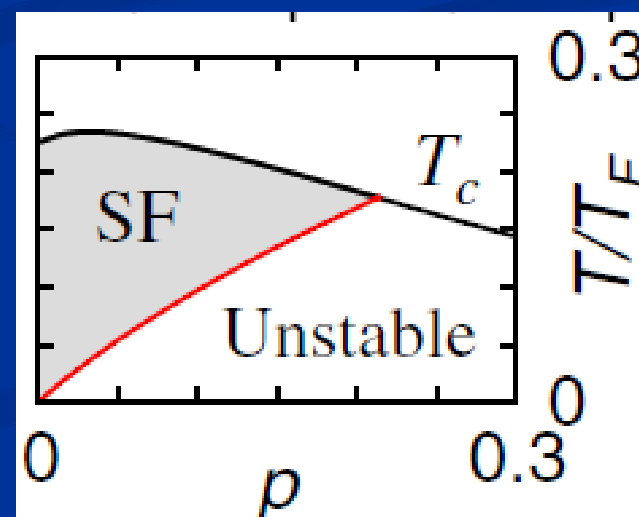
IIA-B: PS+FFLO; IIC-D: Intermediate *T* SF



- Yong-il Shin, Christian H. Schunck, André Schirotzek & Wolfgang Ketterle, Nature 451, 689-693 (7 February 2008)

Experiment evidence for intermediate temperature superfluid

Theory:



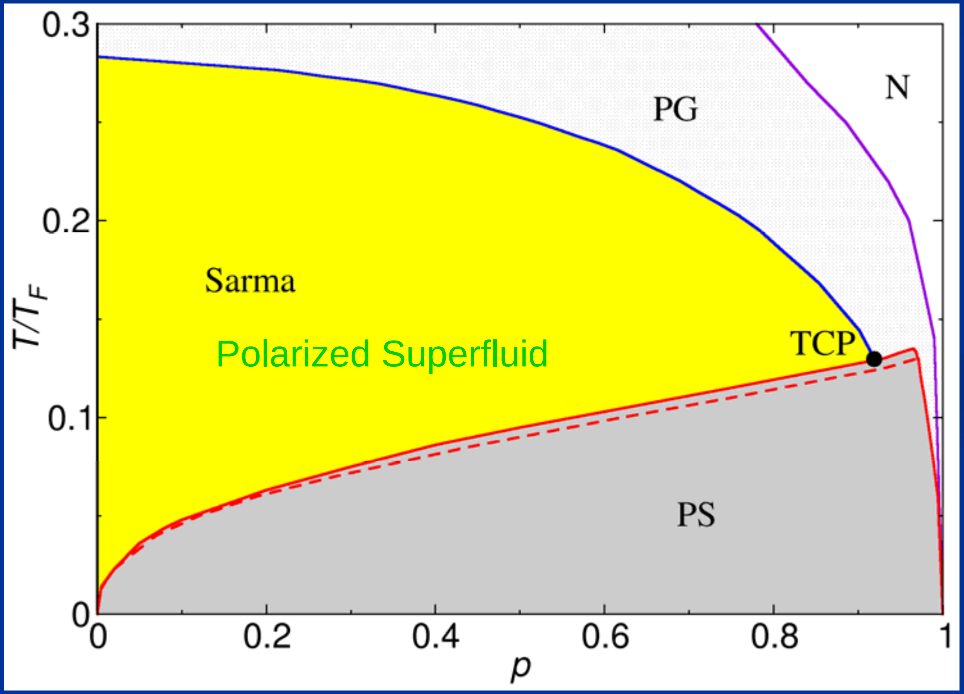
Comparison between theory and experiment

-- population imbalanced Fermi gases in traps

Population imbalance phase diagrams

PRL 98, 110404 (2007)

Unitary: $1/k_F a = 0$



From profiles, MIT reports“highly correlated *T>Tc* normal state”

In RF expts., MIT Reports “Normal State Pairing Gap”.

N = Normal

PG = pseudogap

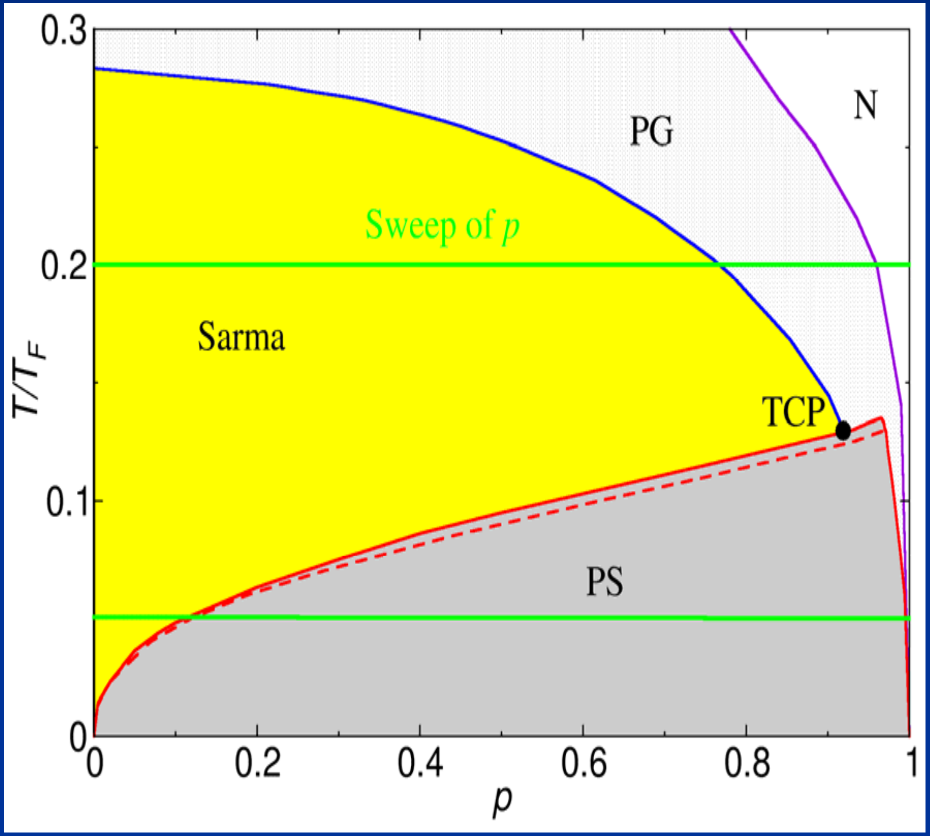
PS = Phase Separation

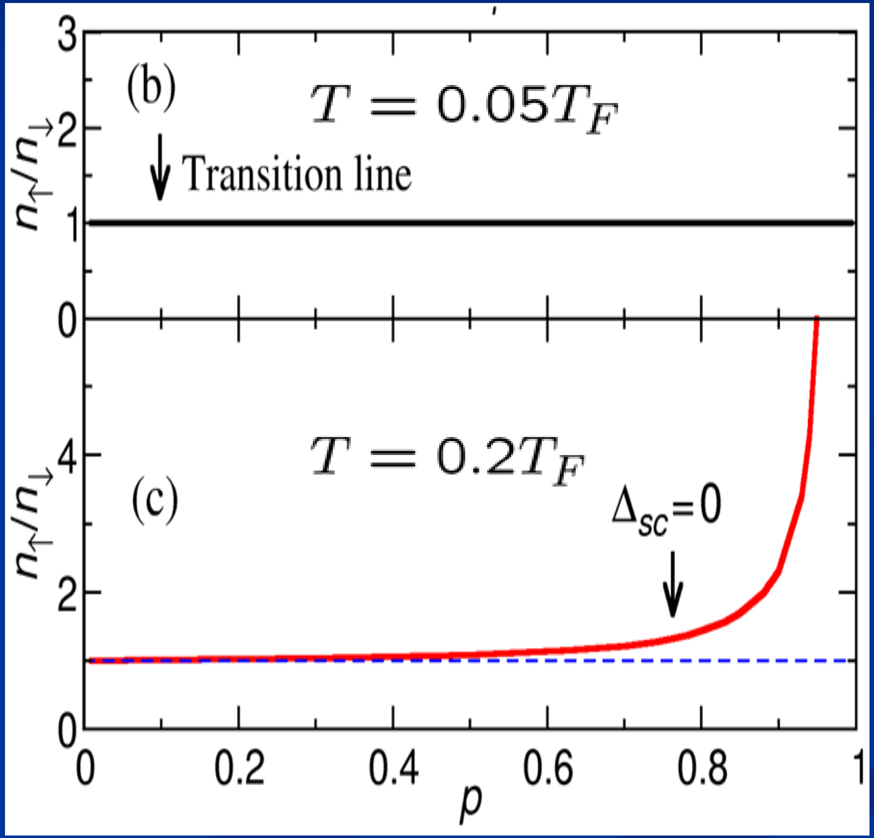
Solid lines separate different phases.

Comparison with Rice data

Unitary Phase Diagram

Theory

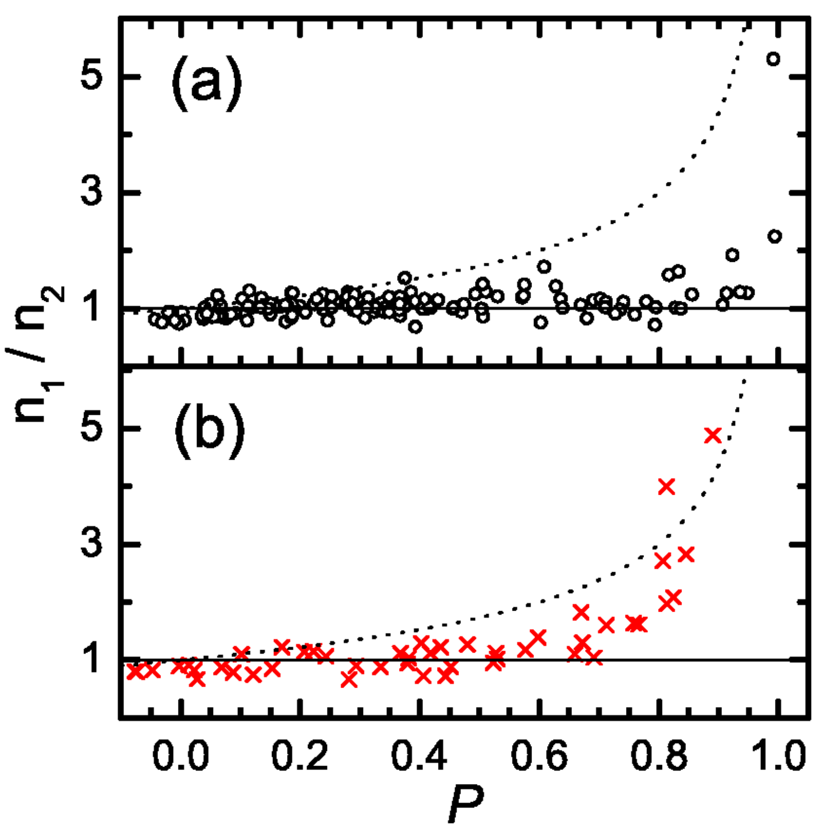




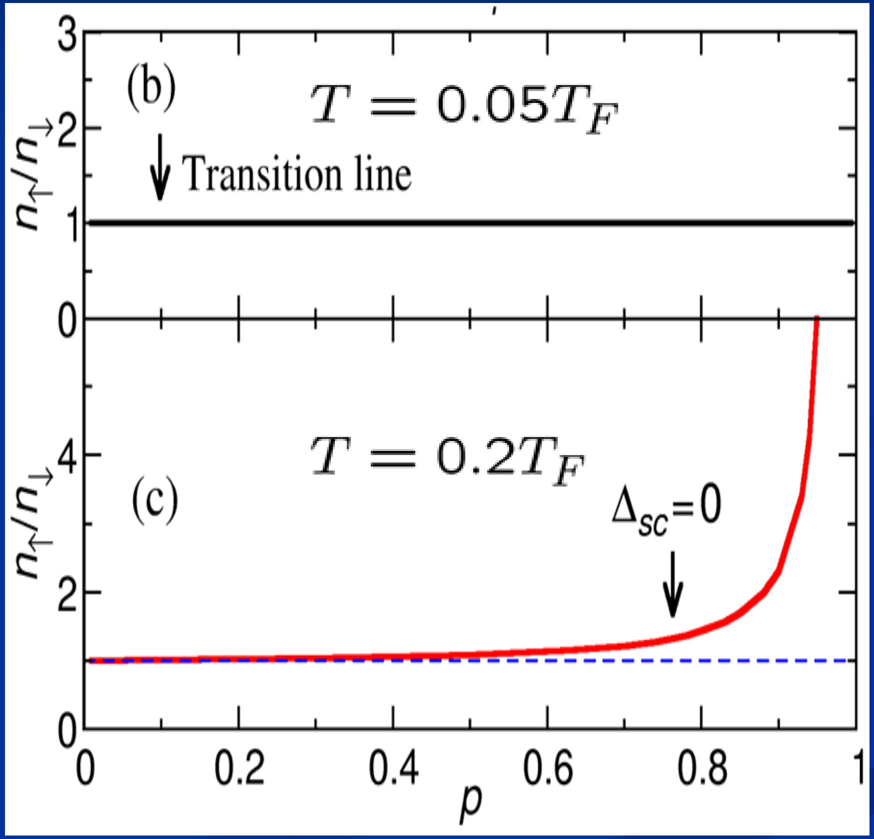
PRL 98, 110404 (2007)

Comparison with Rice data

Rice data



Theory



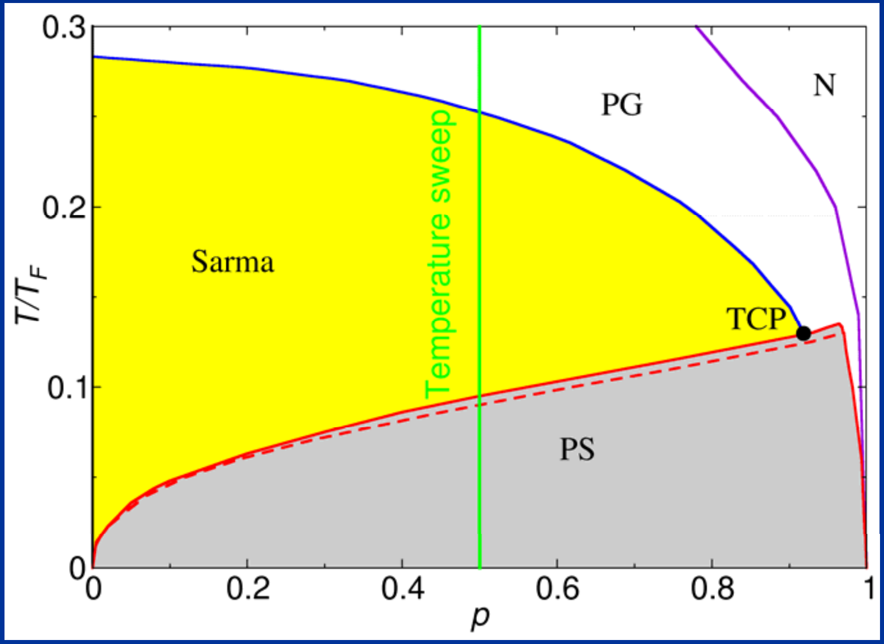
PRL 98, 110404 (2007)

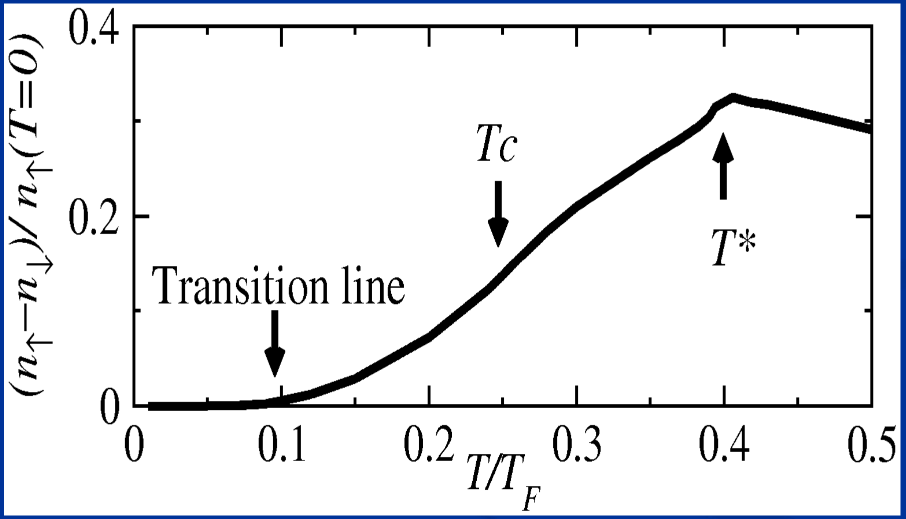
Comparison with MIT Data at Unitarity

PRL 98, 110404 (2007)

Unitary Phase Diagram

Theory



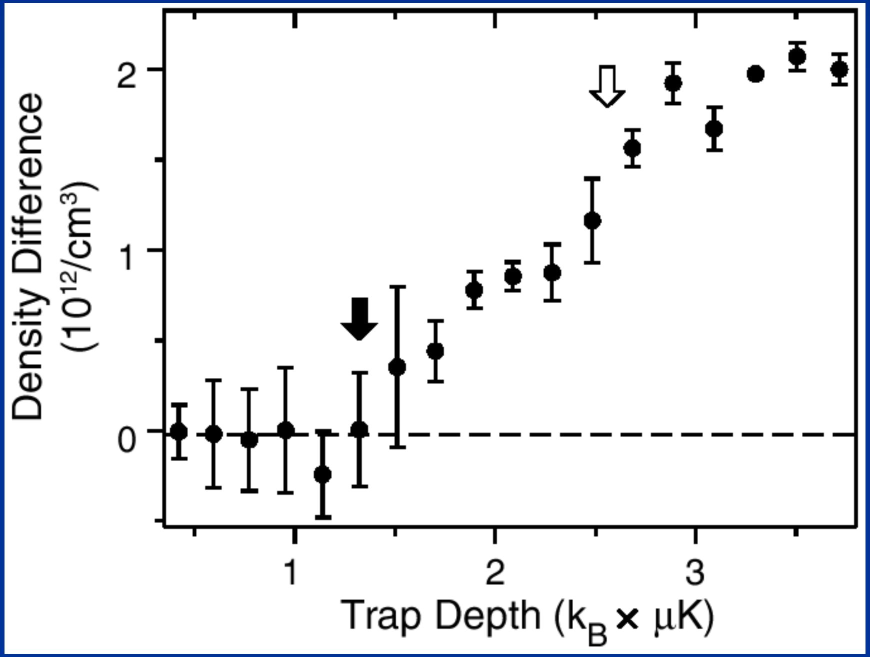


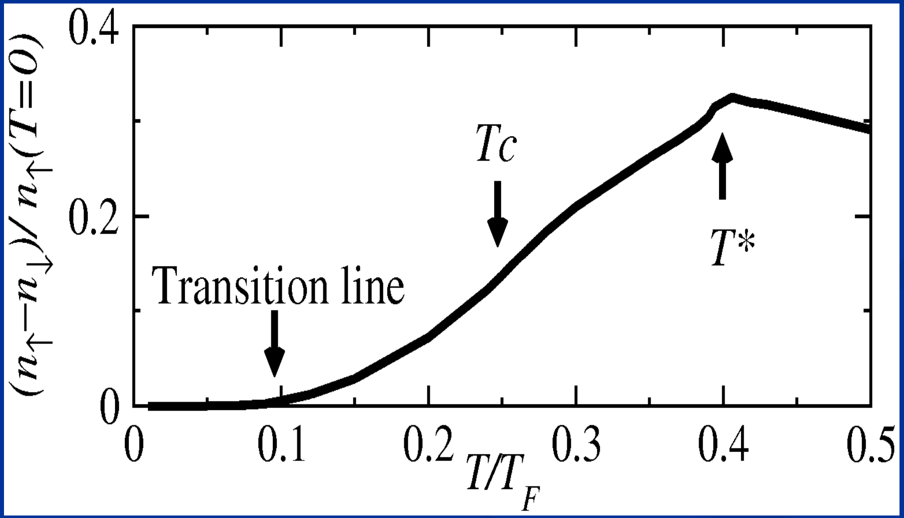
Comparison with MIT Data at Unitarity

PRL 98, 110404 (2007)

Experiment

Theory





Trap depth roughly proportional to *T*, *p* ~ 0.5 – 0.6.

Temperature Effects and MIT 2D Profiles in the near- BEC

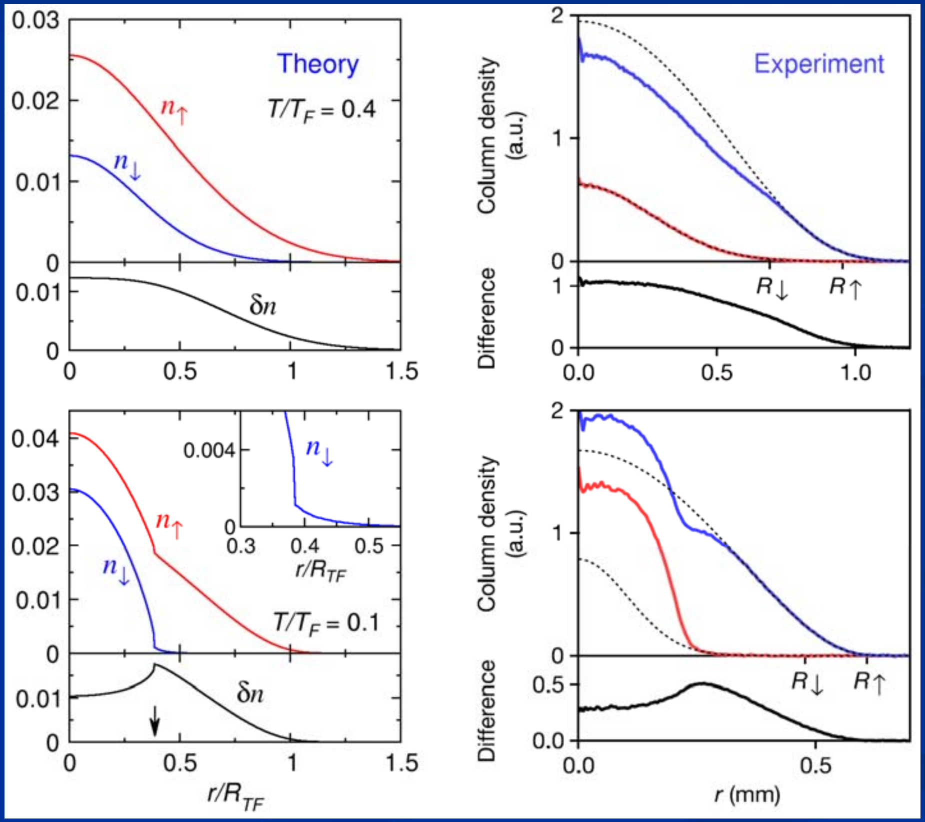
theory **MIT data**

Above Tc

Phys. Rev. A 74, 021602(R) (2006)

cond-mat/0610006

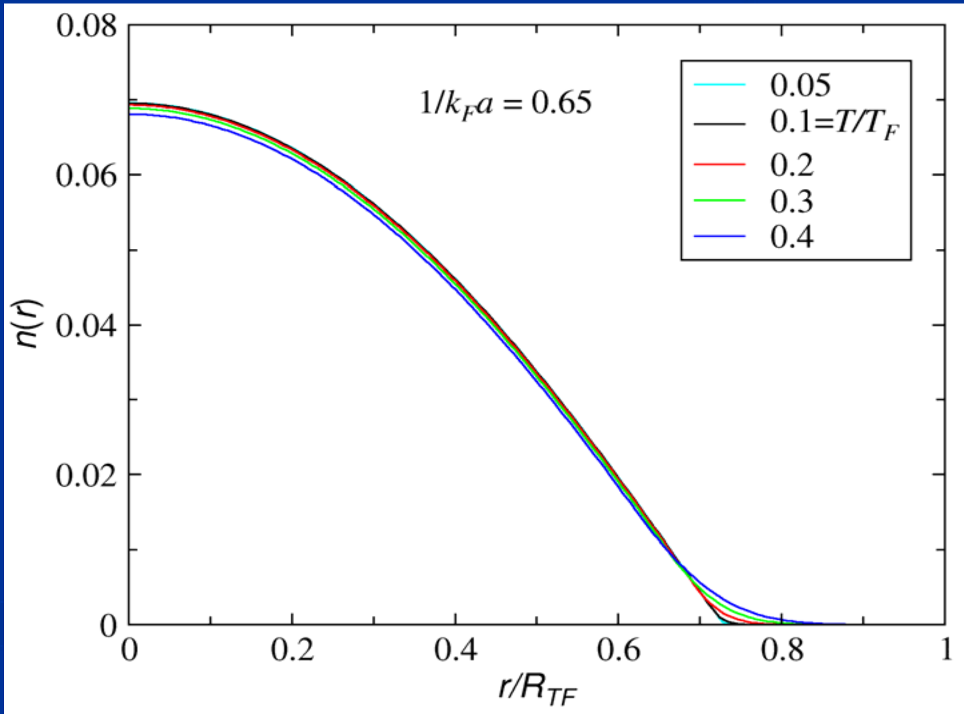
Below Tc



See clear signatures of superfluidity!

Profile at near-BEC for equal spin mixture

- No significant bi-modal distribution
- The kink in the **deep** BEC regime in the "bimodal" distribution moves in opposite direction with *T*.



Temperature Effects and MIT Profiles in the near- BEC

theory

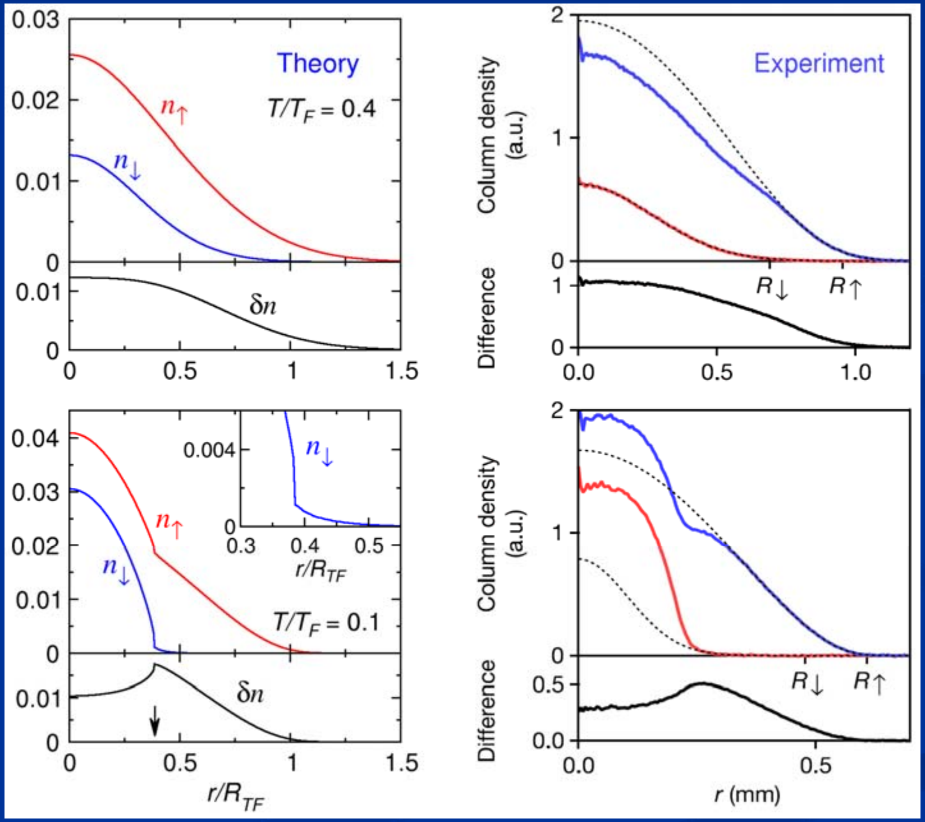
MIT data

Above Tc

Phys. Rev. A 74, 021602(R) (2006)

cond-mat/0610006

Below Tc



See clear signatures of superfluidity!

Probing the spectral function of a homogeneous gas

- Momentum resolved rf spectroscopy so far lacks spatial resolution. Trap averaging severely limits the resolution in the (k,ω) plane.

- Spectral function A(k,ω) is of central importance → Self energy, interaction, test theories

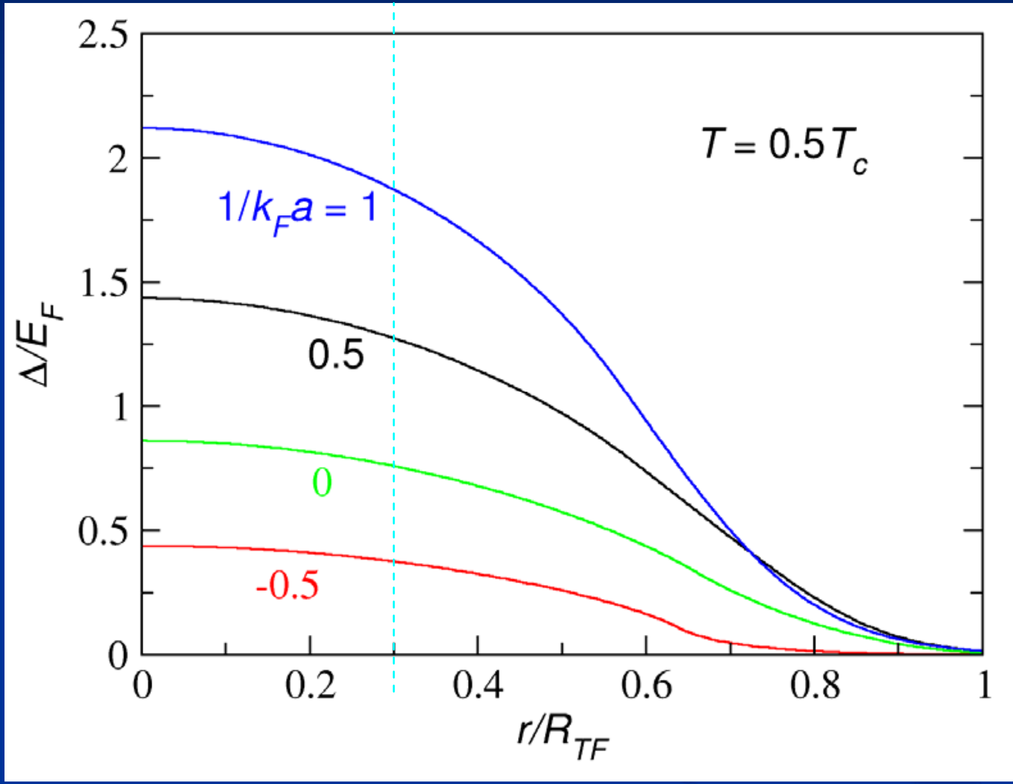
- Need spatial resolution -- Tomography
- Need momentum resolution – ARPES
- But cannot have both simultaneously
- or can we?

-- What we really really really really want !!!

No method exists or has been proposed so far !

Gap profiles from BCS to BEC

Equal spin mixture



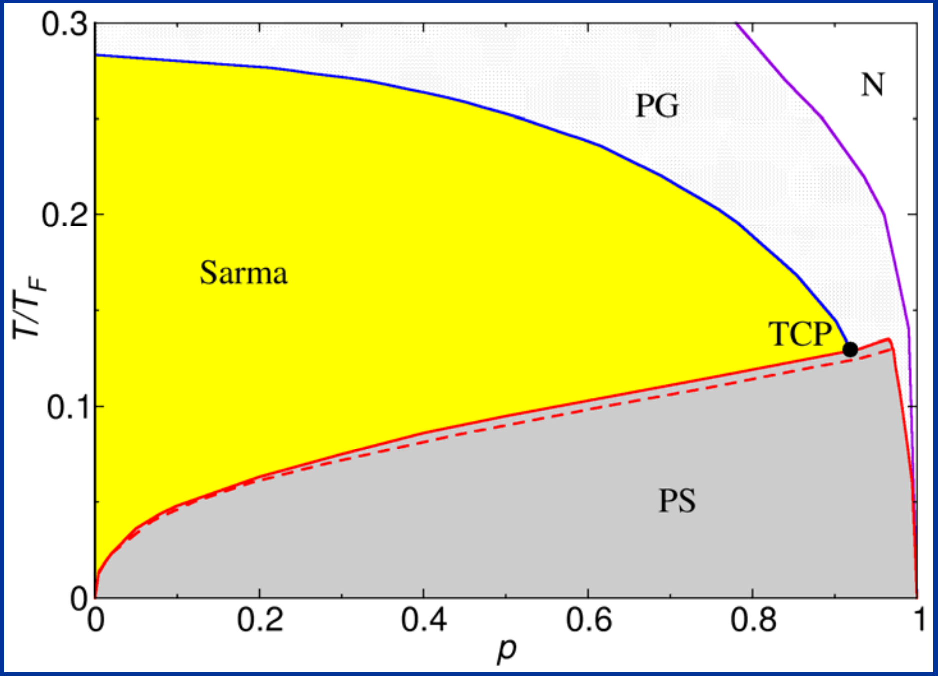
- Gap flat at trap center
- Very small minority cloud size requires extremely high imbalance – may not be possible
- No unlimited spatial resolution
- But will show it is ok

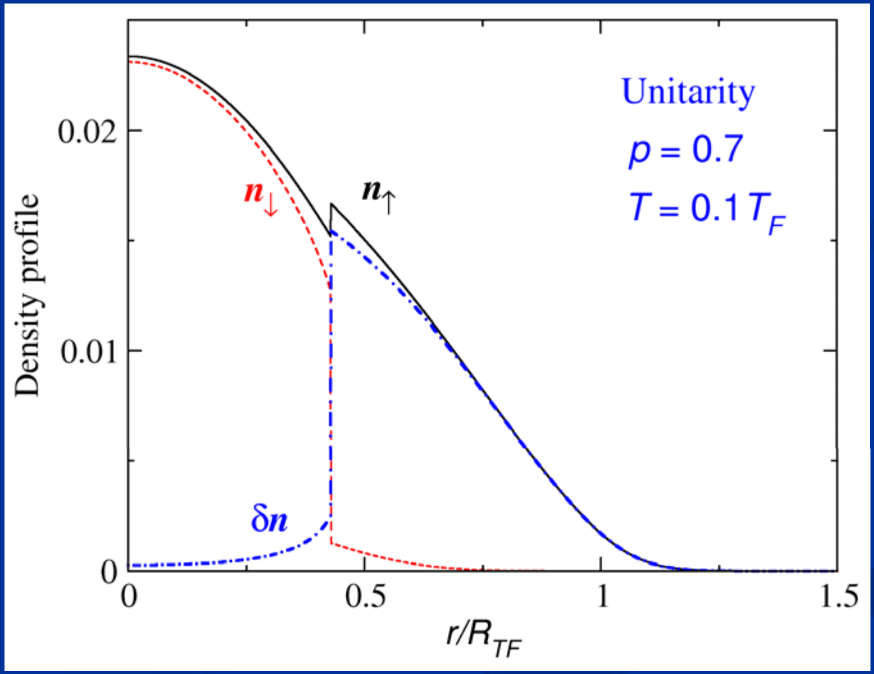
Phase diagram in the presence of population imbalance

Chien, Chen, He and Levin, PRL 98, 110404 (2007)

- Unitary

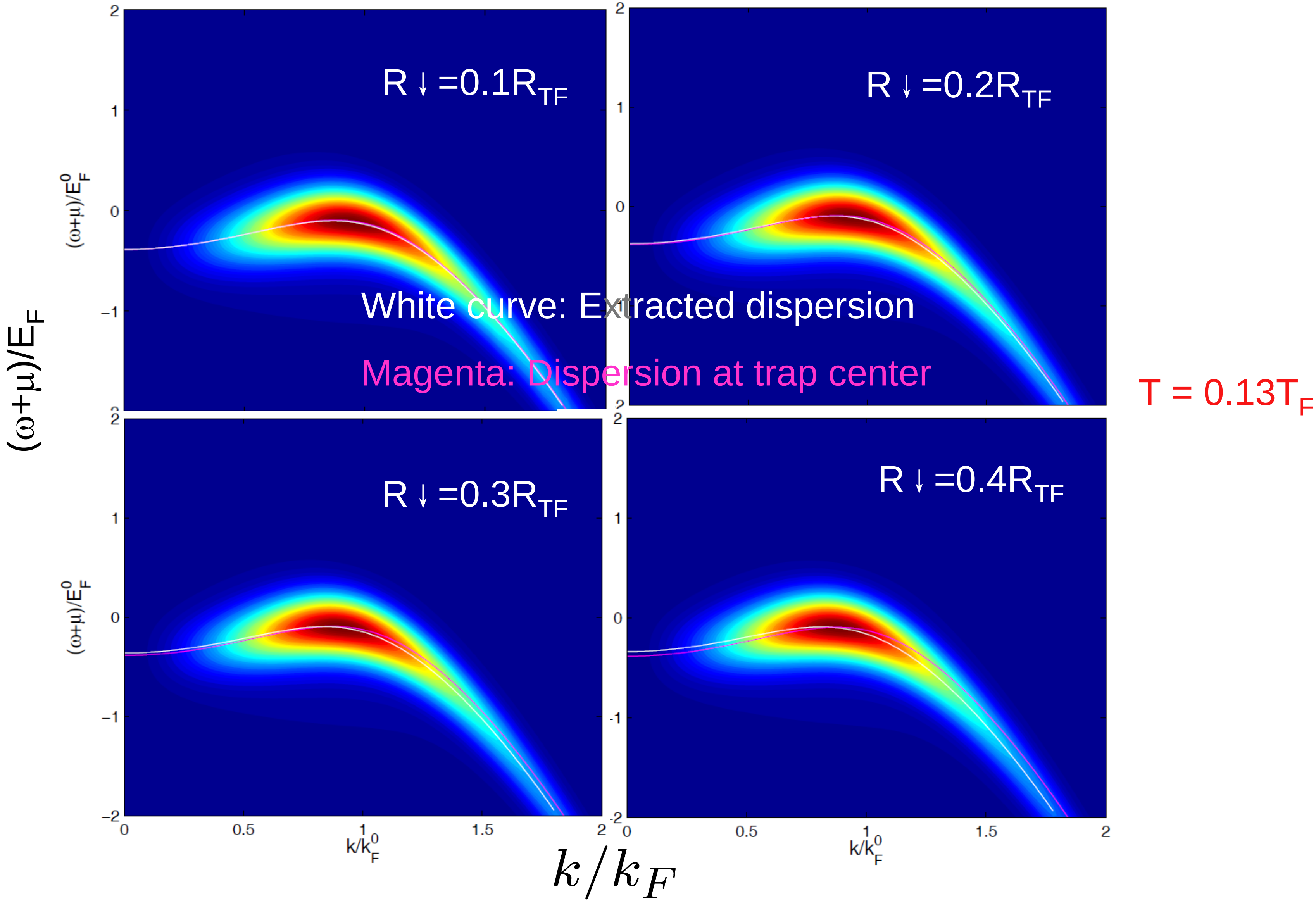
Density profile



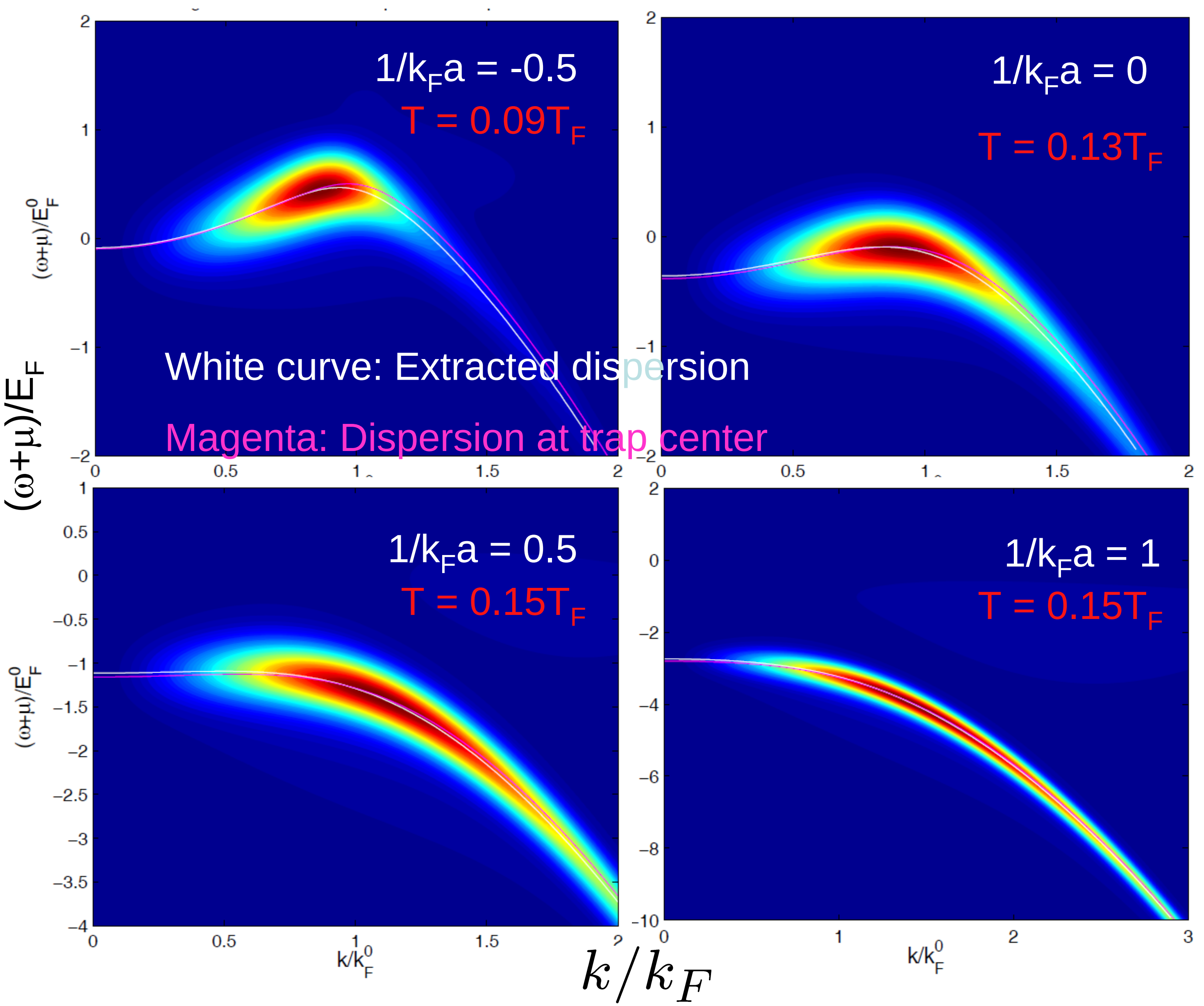


Phase separation at low T and high p.

Phase separated minority RF at unitarity for different imbalances



Phase separated minority RF from BCS to BEC

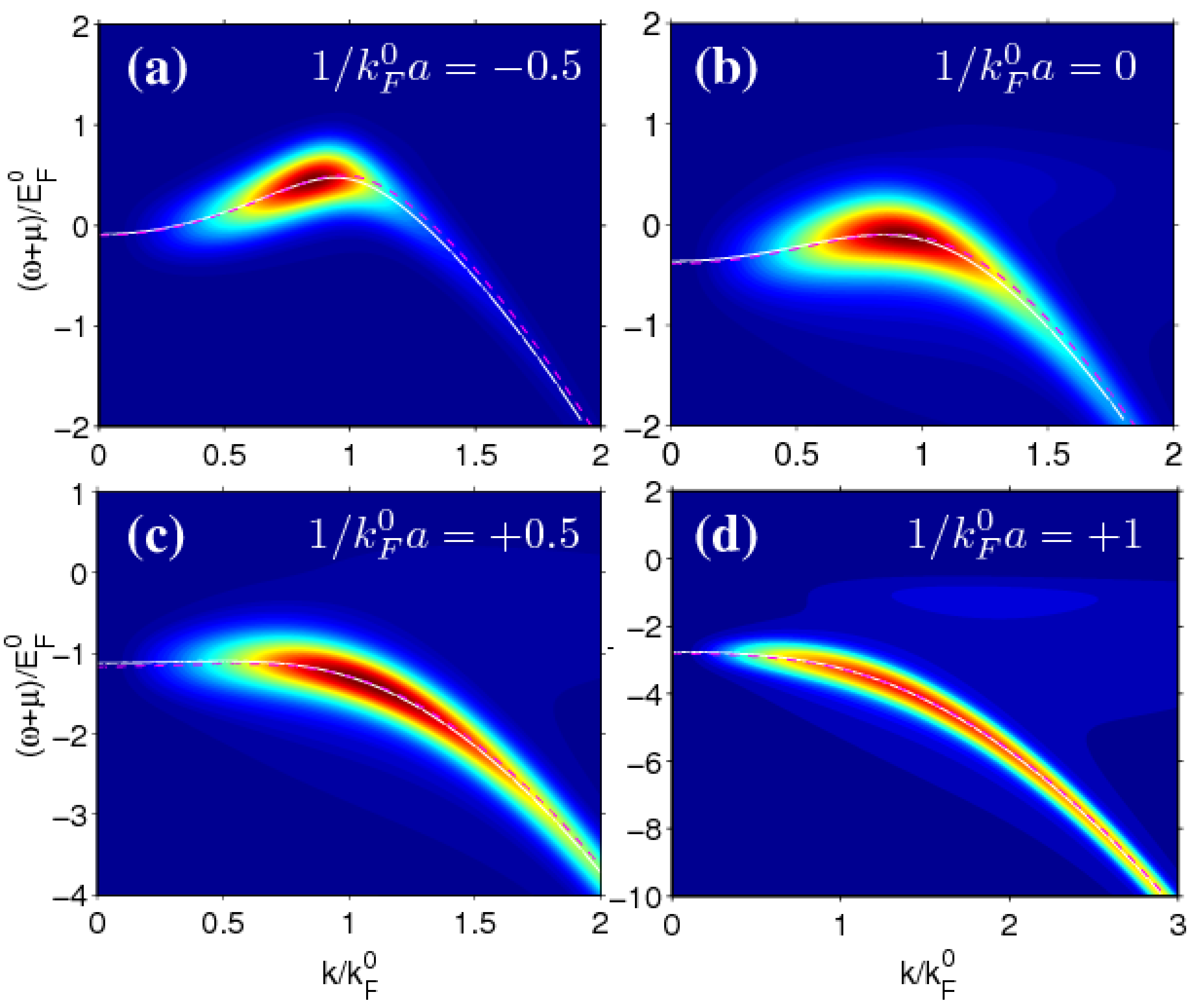


$R\downarrow = 0.3R_{TF}$

$p = 0.7 \sim 0.9$

Extracted dispersion very close to that at trap center.

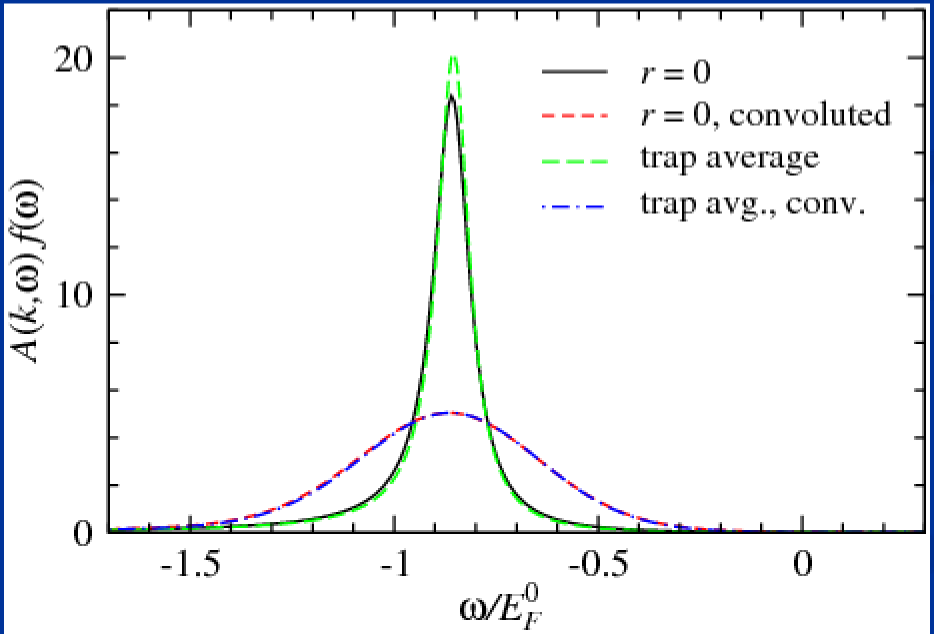
Model independent



Calculated using simple broadened mean-field BCS self energy.

Spectral function A(k,ω)

- At Fermi level



Summary

- New state of superfluidity: non-condensed pairs present -- pseudogap effects are evident.
- Pairing is not equal to superfluidity.
- Pseudogap persists into the superfluid phase.
- Successfully applied to multiple cold atom expts and cuprates
- Inclusion of effects of temperature and noncondensed pairs is crucial to arrive at a meaningful quantitative comparison with experiments.
- Lots of potential applications and interest from astrophysics, nuclear and even particle physics.
- BCS-BEC crossover theory is thought to be relevant to high Tc, where the pairs are small.
- Optical lattices – beyond one trap physics

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