Dissipatively Driven Many-Body Pairing State for Cold Fermions in an Optical Lattice

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Outline

- General idea of reservoir engieering
- Some examples
- Dissipatively driven pairing states of Fermions
  - Simple case: Anti-ferromagnetic Néel state
  - d-wave pairing state
  - General strategy
  - p-wave pairing state. (topological order?)
- Mean field theory
- Physical implementation
- Application: effective cooling scheme via adiabatic connection
- Summary

Conventional ground state preparation

Start from a given many body Hamiltonian H, cool the system into the ground state

$$\rho \xrightarrow{T \to 0} |\psi_g\rangle\langle\psi_g|$$

Open system with drive and dissipation

$$\frac{d\rho}{dt} = -i\,[H, \rho] + \mathcal{L}\rho$$

$$\rho(t) \xrightarrow{t \to \infty} \rho_{\mathrm{steady}}$$

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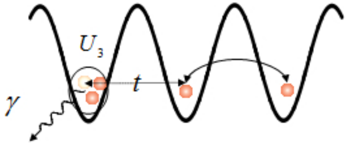
Engineer system-reservoir coupling, so that ρ_{steady} is localized in the Hilbert space.

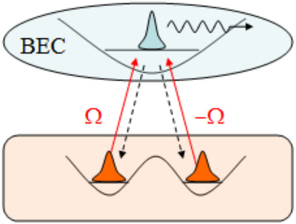
Example I: Stabilization of p-wave superfluid



- System cannot thermalize due to large three-body losses
- In an optical lattice, three-body losses suppressed by quantum Zeno effect, i.e. loss blockade

Y.-J, Han, Y.-H. Chan, W. Yi, A.J. Daley, S. Diehl, P. Zoller, L.-M. Duan, Phys. Rev. Lett. 103, 070404 (2009)

Example II: Driven dissipative BEC



- Local dissipation locks the relative phase between adjacent sites
- Long range order established by sequence of local dissipations
- The coherence of BEC eventually comes from the laser
- Final steady state not dependent upon initial density matrix

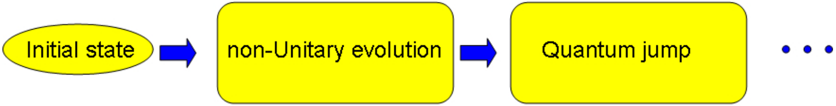
S. Diehl, A. Micheli, A. Kantian, B. Kraus, H.P. Büchler, P. Zoller, Nature Physics 4, 878 (2008)

Master equation

$$i\frac{\partial\rho}{\partial t} = -iH_{\text{eff}}\rho + i\rho H_{\text{eff}}^{\dagger} + \kappa\sum_{l} J_l \rho J_l^{\dagger}$$

$$H_{\text{eff}} = H - \frac{i}{2}\kappa\sum_{l} J_l^{\dagger} J_l$$

Quantum trajectory picture



$|\psi\rangle$ $\quad$ $e^{-iH_{\text{eff}}\Delta t}|\psi\rangle$ $\quad$ $J_l|\psi(\Delta t)\rangle$ with rate $\kappa\| J_l|\psi\rangle\|^2$

State preparation based on dissipative processes

Look for appropriate set of quantum jump operators $\{J_l\}$, so that they have a unique dark state

$$J_l|\varphi\rangle = 0 \qquad \forall l$$

Requirement

- Non-Hermitian
- Particle number conserving
- Possible to connect any state to the dark state
- Quasi-local
- Single particle operator

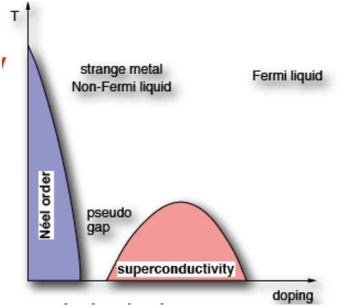
More examples:

- η-condensate of fermion pairs

  S. Diehl *et al.*, Nature Physics 4, 878 (2008)
- Reservoir engineering for general quantum simulation of spin-models

  H. Weimer *et al.*, Nature Phys. 6, 382-388 (2010)
- Fermion pairing states with various symmetries, e.g. d-wave symmetry

High Tc superconductors



- Repulsive interaction
- Experimental evidence for fermion pairing with d-wave symmetry
- Simplified model: 2-d Fermi-Hubbard Model
- Difficult to solve

Quantum simulation of Fermi-Hubbard Model

- Fermi-Hubbard Hamiltonian can be implemented with an optical lattice potential
- How to cool the system to its ground state?

$$T_c/T_F \sim 0.02 \qquad T_{\exp}/T_F \sim 0.2 \quad \text{(ETH)}$$

A possible solution

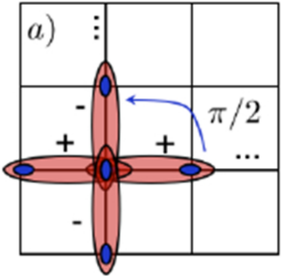
- Prepare pure state with appropriate symmetry
- Connect to the ground state of the Fermi-Hubbard model via adiabatic passage

BCS-type d-wave pairing state

$$|d\rangle = \left(d^\dagger\right)^N |vac\rangle$$
$$d^\dagger = \sum_i \left[\left(a^\dagger_{i+e_x,\uparrow}a^\dagger_{i,\downarrow} - a^\dagger_{i+e_x,\downarrow}a^\dagger_{i,\uparrow}\right) - \left(a^\dagger_{i+e_y,\uparrow}a^\dagger_{i,\downarrow} - a^\dagger_{i+e_y,\downarrow}a^\dagger_{i,\uparrow}\right)\right]$$

Under translational symmetry

$$d^\dagger = \Big[\left(a^\dagger_{i+e_x,\uparrow} + a^\dagger_{i-e_x,\uparrow}\right)a^\dagger_{i,\downarrow} - \left(a^\dagger_{i+e_y,\uparrow} + a^\dagger_{i-e_y,\uparrow}\right)a^\dagger_{i,\downarrow}\Big]$$



Simplified version

$$d^\dagger = \sum_{i,\lambda} f(\lambda) S^{\pm}_{i+\lambda}$$

$$f(\lambda) = \begin{cases} 1 & \lambda = e_x \\ -1 & \lambda = e_y \end{cases}$$

With

$$S^{+}_{i+\lambda} = a^\dagger_{i+\lambda}\sigma^{+} a^\dagger_i = a^\dagger_{i+\lambda,\uparrow} a^\dagger_{i,\downarrow}$$

$$S^{-}_{i+\lambda} = a^\dagger_{i+\lambda}\sigma^{-} a^\dagger_i = a^\dagger_{i+\lambda,\downarrow} a^\dagger_{i,\uparrow}$$

Anti-ferromagnetic Néel state

$$|AF\pm\rangle = \prod_{i\in A,\lambda} a^\dagger_{i+\lambda}\sigma^{\pm} a^\dagger_i |vac\rangle = \prod_{i\in A,\lambda} S^{\pm}_{i+\lambda}|vac\rangle$$

How to prepare anti-ferromagnetic Néel state?

$$|AF+\rangle = \prod_{i\in A,\lambda} a^\dagger_{i+\lambda,\uparrow} a^\dagger_{i,\downarrow}|vac\rangle$$

Consider jump operators on the two-site unit cell

$$a^\dagger_{j,\uparrow} a_{i,\downarrow}|\uparrow,\downarrow\rangle = 0$$
$$a^\dagger_{j,\downarrow} a_{i,\uparrow}|\uparrow,\downarrow\rangle = 0$$
$$a^\dagger_{j,\uparrow} a_{i,\downarrow}|\downarrow,\uparrow\rangle = 0$$
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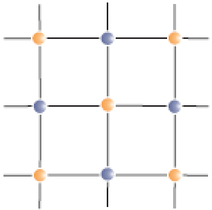

The set of jump operators $\left\{a^\dagger_{j,\uparrow} a_{i,\downarrow}, a^\dagger_{j,\downarrow} a_{i,\uparrow}\right\}$ dictates that any given site should have opposite spin with its neighbouring sites.

Additional jump operator to get rid of degeneracy of 'dark' state:

$$J_i = a_{i,\sigma}^\dagger a_{i,\sigma}$$

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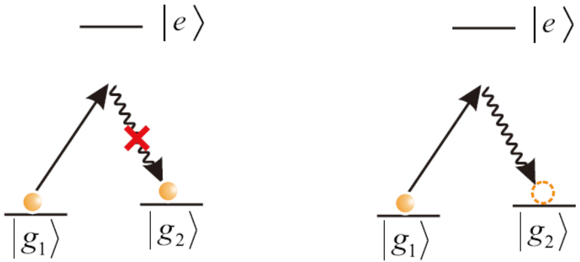
Physical origin: Pauli blocking



Building d-wave jump operators

- Local singlet pairing
- Spatial d-wave symmetry

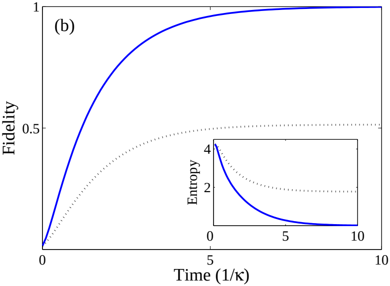
- Singlet pairing state in $1d$

$$|d\rangle_{1d} = \left[\sum_i (a^\dagger_{i+1,\uparrow} + a^\dagger_{i-1,\uparrow}) a^\dagger_{i,\downarrow}\right]^N |vac\rangle$$

- Jump operator

$$J_i^{\pm} = (a^\dagger_{i+1} + a^\dagger_{i-1})\sigma^{\pm} a_i$$
$$J_i^{\pm}|d\rangle_{1d} = 0$$

Uniqueness

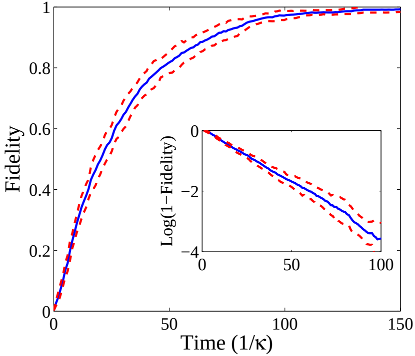


- Dark state has two-fold degeneracy with $\{J_i^{\pm}\}$
- The degeneracy can be removed by

$$J_i^z = (a_{i+1}^{\dagger} + a_{i-1}^{\dagger})\sigma^z a_i$$

- Both dark state and steady state unique under $\{J_i^{\pm}, J_i^z\}$

What about $2d$? (2x6 ladder with 4 atoms)



With jump operators

$$J_i^{\pm} = \left(a_{i+e_x}^{\dagger} + a_{i-e_x}^{\dagger}\right)\sigma^{\pm} a_i - \left(a_{i+e_y}^{\dagger} + a_{i-e_y}^{\dagger}\right)\sigma^{\pm} a_i$$

$$J_i^{z} = \left(a_{i+e_x}^{\dagger} + a_{i-e_x}^{\dagger}\right)\sigma^{z} a_i - \left(a_{i+e_y}^{\dagger} + a_{i-e_y}^{\dagger}\right)\sigma^{z} a_i$$

States that can be constructed from Néel state unit cell operators

$$\beta_i^\dagger = \sum_\nu \rho_\nu a^\dagger_{i+e_\nu,\sigma_1} a^\dagger_{i,\sigma_2}$$

Jump operator

$$J_i = \sum_\nu \rho_\nu a^\dagger_{i+e_\nu,\sigma_1} a_{i,\sigma_2}$$

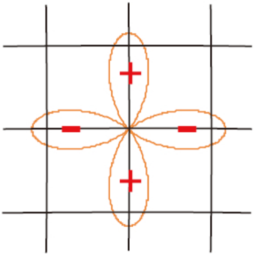
With

$$\sum_{\mu,\nu} \rho_\mu \rho_\nu a^\dagger_{j+\mathbf{e}_\nu,\sigma_1} a^\dagger_{j+\mathbf{e}_\mu,\sigma_1} = 0, \qquad \text{for } \sigma_1 \neq \sigma_2$$

$$\sum_{\mu,\nu} \rho_\mu \rho_\nu (a^\dagger_{j+\mathbf{e}_\nu} - a^\dagger_{j-\mathbf{e}_\nu}) a^\dagger_{j+\mathbf{e}_\mu} = 0, \qquad \text{for } \sigma_1 = \sigma_2$$

Examples

$$\rho_{\pm x} = -\rho_{\pm y} = 1$$



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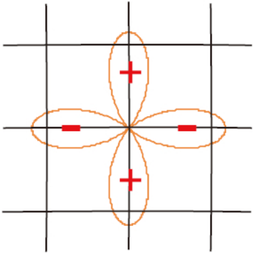
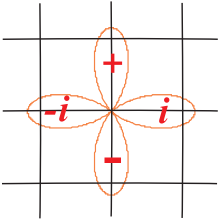
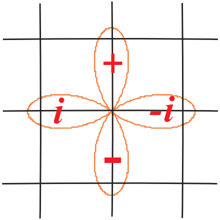


$$\rho_x = -\rho_{-x} = \pm i$$
$$\rho_y = -\rho_{-y} = 1$$

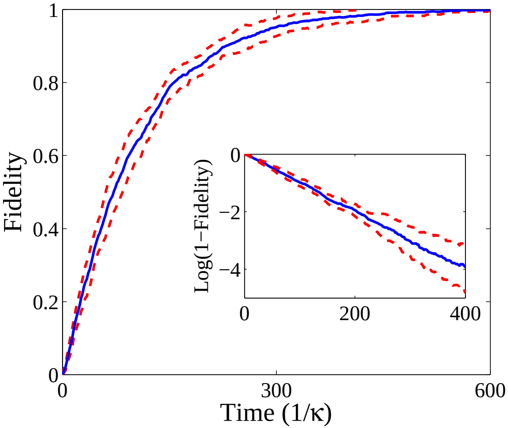


$p_x + ip_y$



$p_x - ip_y$

p-wave pairing state (4x4 plaquette with 4 atoms)



- Final steady state is approached exponentially fast, as in the d-wave case

Discussion on uniqueness condition

Consider the ‘parent’ Hamiltonian

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

- H_p semi-positive definite
- Dark state is unique if the ground state of H_p is non-degenerate
- Check if symmetry operations of H_p leave the dark state unaltered
- For example, the reduced ‘parent’ Hamiltonian $H_p^r = \sum_{i,\sigma=\pm} \left(J_i^\sigma\right)^\dagger J_i^\sigma$ has a discrete symmetry

$$\begin{aligned} T_d: \quad & a_{i,\uparrow} \to -a_{i,\uparrow}; \quad a_{i,\downarrow} \to a_{i,\downarrow} \text{ for } i \in A, \\ & a_{i,\uparrow} \to a_{i,\uparrow}; \quad a_{i,\downarrow} \to a_{i,\downarrow} \text{ for } i \in B \end{aligned}$$

Mean field description

- The BCS pairing state in coherent state form

$$|D(\theta)\rangle = \prod_{\mathbf{q}\in BZ}\left[\frac{1}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}} + \frac{e^{i\theta}\varphi_{\mathbf{q}}}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}} a^\dagger_{\mathbf{q},\uparrow} a^\dagger_{-\mathbf{q},\downarrow}\right]|\mathsf{vac}\rangle,$$

- Gutzwiller ansatz in momentum space

$$\rho = \prod_{\mathbf{q}} \rho_{\mathbf{q}}, \quad \rho_{\mathbf{q}} = \mathsf{tr}_{\neq\mathbf{q}}\rho, \quad \mathsf{tr}\rho_{\mathbf{q}} = 1\,\forall\ \mathbf{q}$$

- Trace over the master equation

$$\mathsf{tr}_{\neq\mathbf{q}}\left\{\frac{d\rho}{dt}\right\} = \mathsf{tr}_{\neq\mathbf{q}}\left\{-i\left[H,\rho\right] + \mathcal{L}\rho\right\}$$

- The Louisvillian part

$$\mathrm{tr}_{\neq\mathbf{q}}\mathcal{L}\rho = 2\kappa_{\mathbf{q}} \times \left\{\gamma_{\mathbf{q},\uparrow}\rho_{\mathbf{q}}\gamma^{\dagger}_{\mathbf{q},\uparrow} + \gamma_{\mathbf{q},\downarrow}\rho_{\mathbf{q}}\gamma^{\dagger}_{\mathbf{q},\downarrow} - \frac{1}{2}\left[\gamma^{\dagger}_{\mathbf{q},\uparrow}\gamma_{\mathbf{q},\uparrow} + \gamma^{\dagger}_{\mathbf{q},\downarrow}\gamma_{\mathbf{q},\downarrow}, \rho_{\mathbf{q}}\right]\right\}$$

- The new jump operators

$$\gamma_{\mathbf{q},\uparrow} = \frac{1}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}}\,(c_{\mathbf{q},\uparrow} - \varphi_{\mathbf{q}}c^{\dagger}_{-\mathbf{q},\downarrow}),$$
$$\gamma_{\mathbf{q},\downarrow} = \frac{1}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}}\,(c_{-\mathbf{q},\downarrow} + \varphi_{\mathbf{q}}c^{\dagger}_{\mathbf{q},\uparrow})$$

- The damping spectrum is gapped

$$\kappa_{\mathbf{q}} = 2A\kappa(1+|\varphi_{\mathbf{q}}|^2), \qquad A \equiv \frac{1}{(2\pi)^d}\int d\mathbf{q}\frac{|\varphi_{\mathbf{q}}|^2}{1+|2\varphi_{\mathbf{q}}|^2}$$

- Approach to the steady state exponentially fast at late times

Examples

- 1d singlet pairing $\varphi_q = \cos q, A = (1 - 1/\sqrt{2})$
- 2d d-wave pairing $\varphi_{\mathbf{q}} = \cos q_x - \cos q_y, A \sim 0.36$
- Generalize to spinless fermions: p-wave pairing

$$|P\rangle = \mathcal{N} \prod_{\mathbf{q}} \left(\frac{1}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} + \frac{2\varphi_{\mathbf{q}}}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} c^\dagger_{\mathbf{q}} c^\dagger_{-\mathbf{q}} \right) |\text{vac}\rangle,$$
$$\varphi_{\mathbf{q}} = 2i(\sin q_x \pm i \sin q_y), A \sim 0.12$$

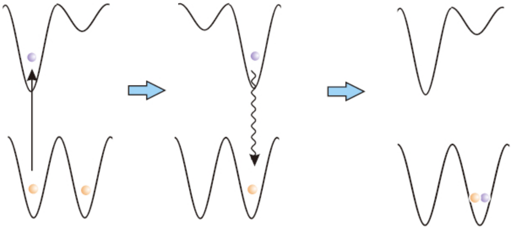
- Jump operator for p-wave pairing

$$\gamma_{\mathbf{q}} = \frac{1}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} (c_{\mathbf{q}} - 2\varphi_{\mathbf{q}} c^\dagger_{-\mathbf{q}})$$

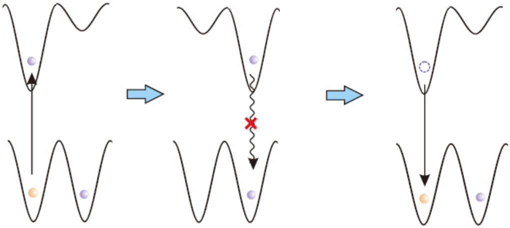
- However, this p-wave state is in the strong pairing phase and topologically trivial

Jump operators for AF Néel state

$$a^\dagger_{j,\downarrow} a_{i,\uparrow} |\uparrow, \uparrow\rangle \Rightarrow |0, \uparrow\downarrow\rangle$$

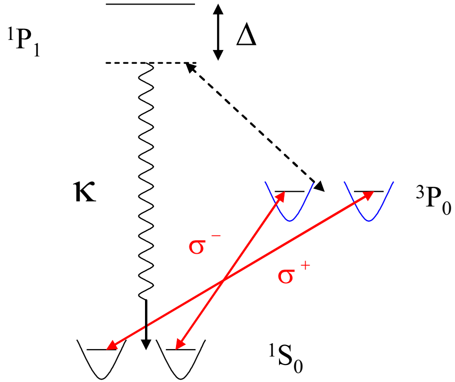


$$a^\dagger_{j,\downarrow} a_{i,\uparrow} |\uparrow, \downarrow\rangle \Rightarrow |\uparrow, \downarrow\rangle$$



Alkaline earth like atoms

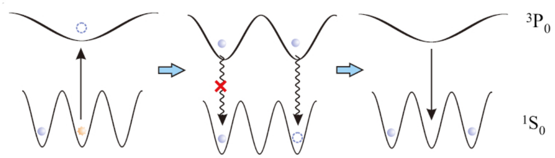
^{171}Yb



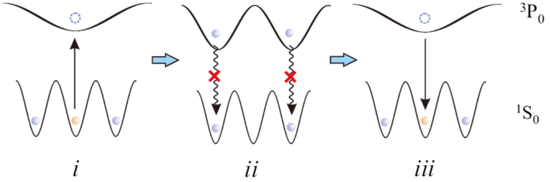
- Long lived metastable state $^3P_0$, $\Gamma \sim 2\pi \times 10\text{mHz}$
- Electronic spin and nuclear spin decoupled in the ground state

Singlet pairing in $1d$

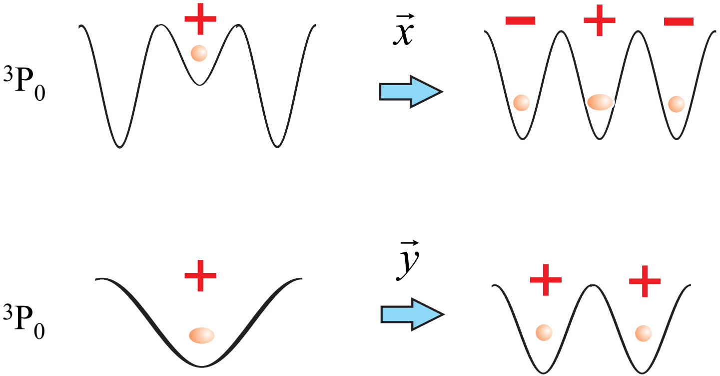
$$(a^\dagger_{i+1,\downarrow}+a^\dagger_{i-1,\downarrow})a_{i,\uparrow}|\downarrow,\uparrow,0\rangle$$



$$(a^\dagger_{i+1,\downarrow}+a^\dagger_{i-1,\downarrow})a_{i,\uparrow}|\downarrow,\uparrow,\downarrow\rangle$$



d-wave pairing



Effective cooling scheme

- Dissipatively prepare steady state with the desired symmetry properties
- Adiabatically connect to the ground state of the Hamiltonian with the same symmetry

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Competition between H, e.g. FHM, and dissipative process leads to mixed state

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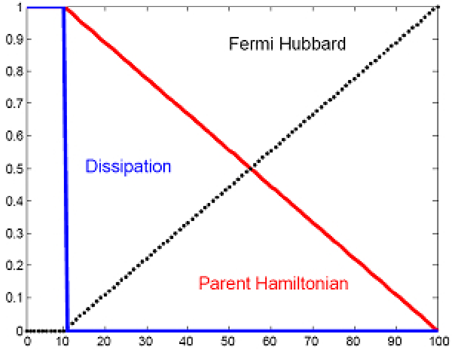
Solution

Recall the ‘parent’ Hamiltonian

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

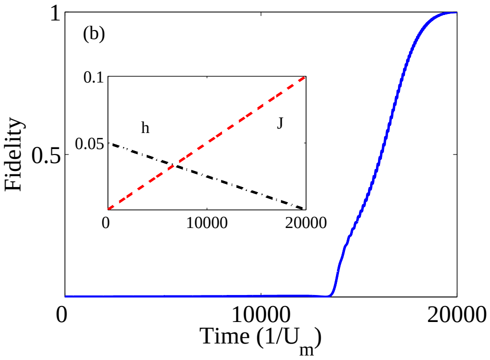
$$H_p|d\rangle = 0$$

Adiabatic passage



$$H(t) = h(t)H_p + U(t)\sum_i n_{i,\uparrow}n_{i,\downarrow} - J(t)\sum_{\langle i,j\rangle,\sigma} a^\dagger_{i,\sigma}a_{j,\sigma}$$

Adiabatic passage for small plaquette (4 atoms on 2x4 ladder)



- The dissipative gap becomes an excitation gap of H_p
- The adiabatic process will be gapped so long as all the symmetry properties of the Hamiltonian are captured by the steady state

Complete vs. reduced H_p

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

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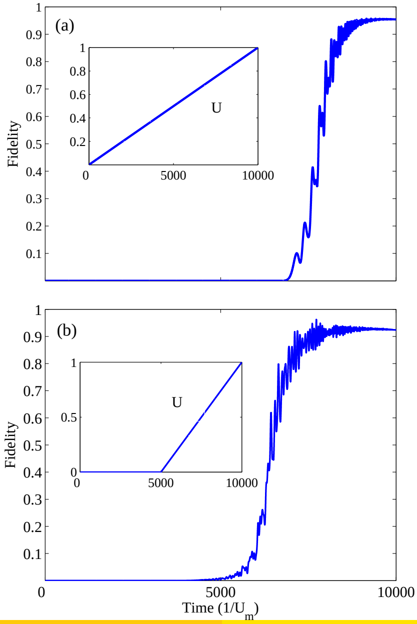
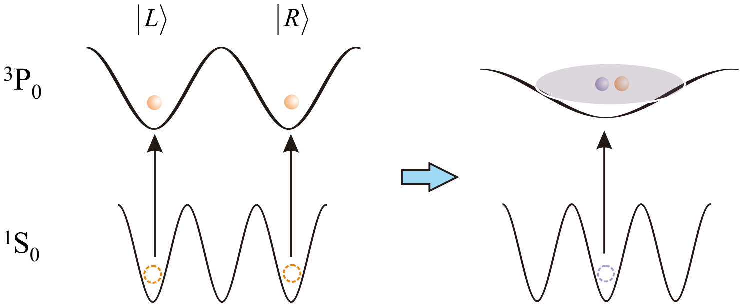


Implementation of reduced H_p



$$\left(J_i^-\right)^\dagger J_i^- = n_{i,\uparrow}(a_{i+1,\downarrow}^\dagger + a_{i-1,\downarrow}^\dagger)\left(a_{i+1,\downarrow} + a_{i-1,\downarrow}\right) + 2n_{i,\uparrow}$$

- Time evolution in a digital fashion

Summary

- Designing jump operators for fermions
- Pairing states with different symmetries
- Implementation and application
- More exotic stuff:
  topologically non-trivial state?
  Majorana zero modes?
  S. Diehl, E. Rico, M. A. Baranov, P. Zoller, arXiv:1105.5947 (2011)

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THANK YOU!

S. Diehl, W. Yi, A. J. Daley, P. Zoller, Phys. Rev. Lett. 105, 227001 (2010)
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