Characterization of information transfer during decoherence: dissipative dynamics and einselection

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Outline

- Background
- Main concerns

1) Information distribution between system and environment

2) Information feedback and non-Markovianity

- Model and case

1) Dephasing channel and non-Markovian channels

2) Pure and mixed input states

- Summary

Various measures of correlations

Mutual information

$$I(A:B)$$

$$= S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Entanglement:

$$\rho_{AB} \neq \sum_i p_i \rho_i^A \otimes \rho_i^B$$

Quantum discord

$$\rho_{AB} \neq \sum_i p_i |i_A\rangle\langle i_A| \otimes \rho_i^B$$

Classical correlation:

$$J_{AB} = S(\rho_B) - \min_{\{\Pi_k^A\}} \sum_k q_k S(\rho_k^B)$$

Distribution of correlations in multipartite systems

Monogamy of entanglement:

$$E(A:BC) \geq E(A:B) + E(A:C)$$

Strong subadditivity inequality of entropy:

D. Petz, *Rep. Math. Phys., 23(1):57–65, 1986.*

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$$

Mutual information: $I(A:BC) \geq I(A:B)$

Classical correlation: polygamy; Quantum discord?

Dynamics of entanglement under decoherence

Sudden Death of Entanglement

Ting Yu[1*] and J. H. Eberly[2*]

A new development in the dynamical behavior of elementary quantum systems is the surprising discovery that correlation between two quantum units of information called qubits can be degraded by environmental noise in a way not seen previously in studies of dissipation. This new route for dissipation attacks quantum entanglement, the essential resource for quantum information as well as the central feature in the Einstein-Podolsky-Rosen so-called paradox and in discussions of the fate of Schrödinger's cat. The effect has been labeled ESD, which stands for early-stage disentanglement or, more frequently, entanglement sudden death. We review recent progress in studies focused on this phenomenon.

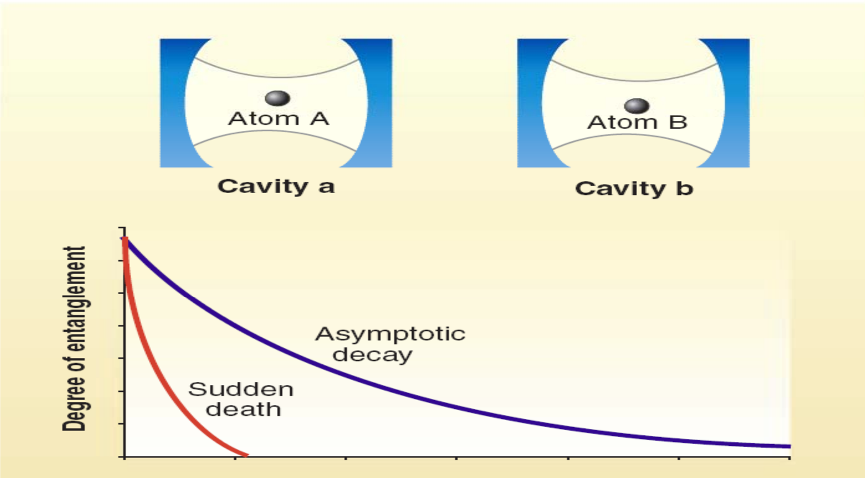
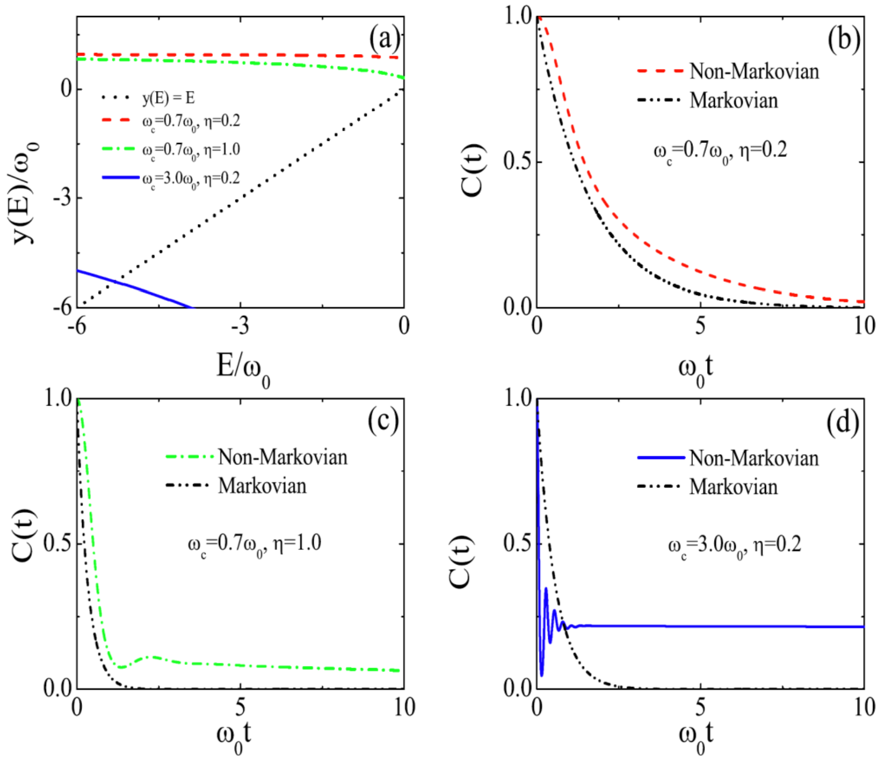


Fig. 1. Curves show ESD as one of two routes for relaxation of the entanglement, via concurrence $C(\rho)$, of qubits A and B that are located in separate overdamped cavities.

598 30 JANUARY 2009 VOL 323 SCIENCE www.sciencemag.org

Entanglement preservation under decoherence

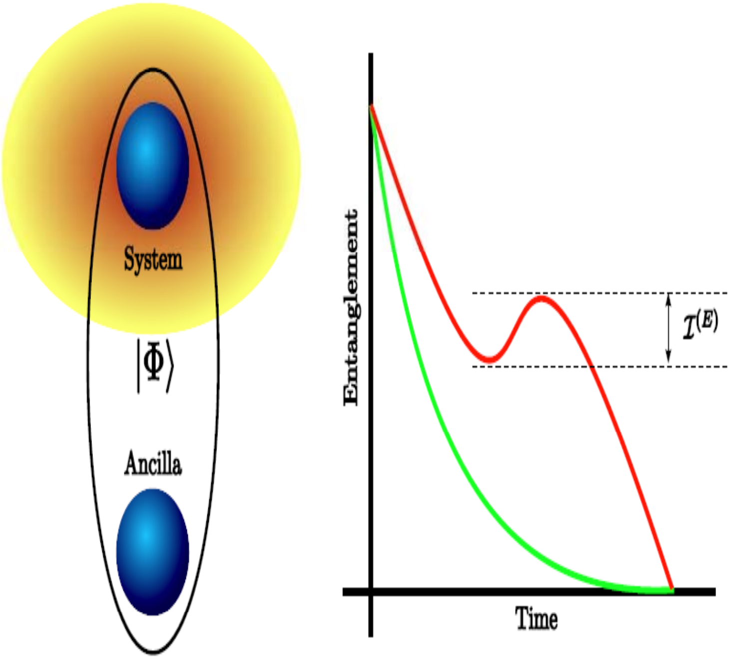


Entanglement preservation for super-Ohmic env.

Tong, An, Luo, Oh, PRA 81, 052330 (2010)

Dynamics of entanglement under decoherence (cont.)

Measure the non-Markovianity via entanglement oscillation



$$\mathcal{I}^{(E)} = \int_{t_0}^{t_{\max}} \left| \frac{dE[\rho_{SA}(t)]}{dt} \right| dt - \Delta E,$$

$$\Delta E = E[\rho_{SA}(t_0)] - E[\rho_{SA}(t_{\max})]$$

A. Rivas, S. F. Huelga, and M. B. Plenio, PRL 105, 050403 (2010)

Dynamics of quantum discord under decoherence

a) Robustness to sudden death

Werlang etc.,
PRA 80, 024103 (2009)
Maziero etc., PRA 2009

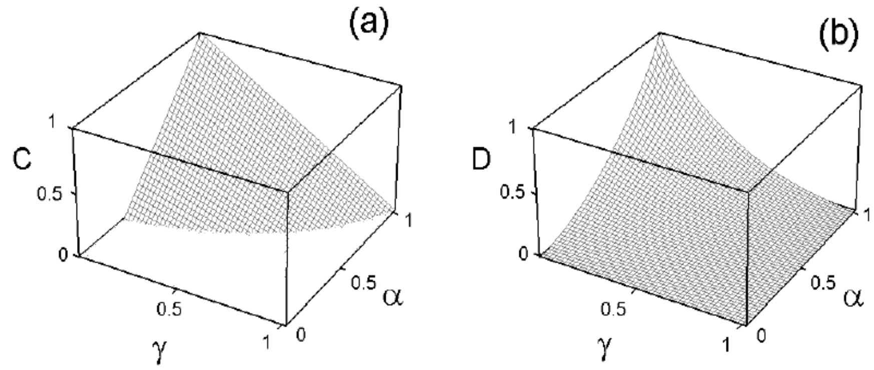


FIG. 1: Dissipative dynamics of (a) concurrence and (b) discord as functions of α and γ, assuming independent dephasing perturbative channels.

b) **Could quantum discord be exploited to detect/measure non-Markovianity?**

Always be decreasing under local trace-preserving CP map or not?

Dynamics of quantum discord under decoherence (cont.)

Creating quantum correlations through local non-unitary memoryless channels

Francesco Ciccarello and Vittorio Giovannetti
NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, Piazza dei Cavalieri 7, I-56126 Pisa, Italy
(Dated: May 30, 2011)

Behavior of Quantum Correlations under Local Noise

Alexander Streltsov,* Hermann Kampermann, and Dagmar Bruß
Heinrich-Heine-Universität Düsseldorf, Institut für Theoretische Physik III, D-40225 Düsseldorf, Germany

$$Q(\rho_{AB}) = S(\rho_A) + S(\rho_B|A) - S(\rho_{AB})$$

$$S(\rho_B|A) \equiv \min_{\{A_k\}} \sum_i p_i S(\rho_B^k|A_k)$$

$$\{|k_A\rangle\langle k_A|\} \Rightarrow S_I(\rho_B|A)$$

$$\sum_k A_k^\dagger A_k = I \Rightarrow S_{II}(\rho_B|A)$$

Adoptability?

Einselection and Quantum Darwinism

Question: How does the classical world arise from an ultimately quantum substrate?

Copenhagen interpretation

devices and observers: classical

System: quantum

"No [quantum] phenomenon is a phenomenon until it is a recorded (observed) phenomenon."

New orthodoxy:

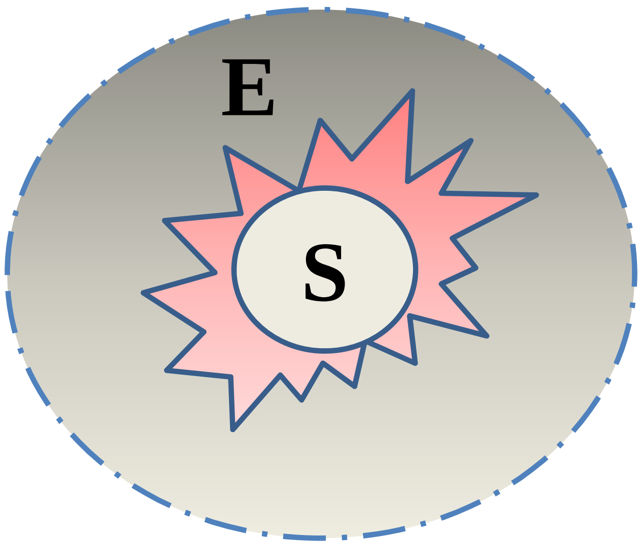
Quantum Darwinism:

einselection

(environment induced superselection)

W.H. Zurek, RMP 75, 715 (2003)

Einselection and Quantum Darwinism (cont.)



Different roles of the environment:

Role 1: decoherence

Destroys coherence of the quantum system, leading to loss of its information

Role 2: einselection

Monitor the system, leading to superselection and broadcasting of the information

⇔

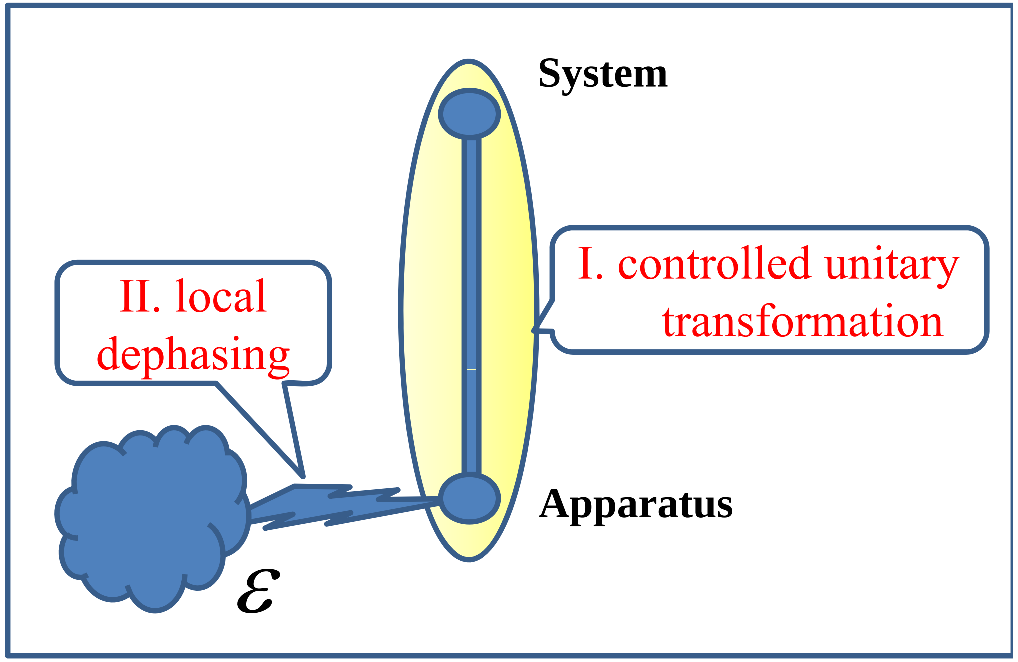
environment-induced superselection: essential element of Quantum Darwinism

Einselection and Quantum Darwinism (cont.)

I. Establish correlations between the apparatus and the system

$$|A_0\rangle\left(\sum_i \alpha_i |S_i\rangle\right)$$

$$\rightarrow \sum_i \alpha_i |A_i\rangle |S_i\rangle$$



II. Decoherence leads to einselection and information transfer:

$$\left(\sum_i \alpha_i |A_i\rangle |S_i\rangle\right)|E_0\rangle$$

$$\rightarrow \sum_i \alpha_i |A_i\rangle |S_i\rangle |E_i\rangle$$

$$\Rightarrow \quad \rho_{AS}^{D} = \sum_i |\alpha_i|^2 \, \underline{|A_i\rangle\langle A_i|} \otimes |S_i\rangle\langle S_i|$$

$$\underline{I(A:S) = I(E:S)}$$

pointer bases

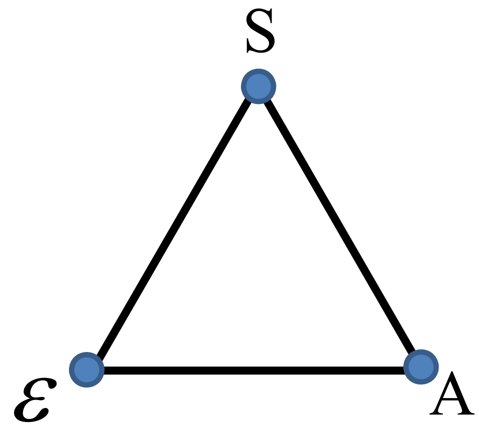
III. Correlations associated with pointer states is selectively proliferated and observers obtain the information by probing fragments of the environment

Information transfer during decoherence: motivation

1) Distribution of mutual information

Pure: $I(S:A)+I(S:\varepsilon)=I(S:A\varepsilon)$

Mixed: $I(S:A)+I(S:\varepsilon)\backsim I(S:A\varepsilon)$?



2) Choice of pointer bases --> information transfer

mutual information **vs** **Quantum correlation**

See poster by Bo You

3) Measure non-Markovianity via information feedback

MI versus EoF; pure versus mixed

Local dephasing channel

Model:

$$H_{A\varepsilon} = \frac{1}{2}\sigma_{\mathbf{n}} \otimes h_\varepsilon, \quad h_\varepsilon \equiv \sum_{k=1}^{N} g_k \sigma_z^{(k)},$$

$$U_{A\varepsilon}(t) = \sum_{+,-} |n_\pm\rangle\langle n_\pm| \otimes u_\pm(t),$$

$$u_\pm(t) = e^{\mp i h_\varepsilon t/2}$$

Pointer bases

$$|\varepsilon_\pm(t)\rangle = u_\pm(t)|\varepsilon(0)\rangle$$

$$\langle\varepsilon_+|(t)|\varepsilon_-(t)\rangle \to 0$$

Kraus operators:

$$\rho_A(t) = \sum_{\mu=1,2} M_\mu \rho_A(0) M_\mu^+$$

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{p} \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{q} \end{pmatrix}$$

$$q = e^{\gamma t}, \; p = 1 - q$$

Output: $$\rho_{AS}(0) \to \rho_{AS}^d = \sum_{+,-} p_\pm |n_\pm\rangle\langle n_\pm| \otimes \rho_S^{(\pm)}$$

$$p_\pm \rho_S^{(\pm)} = \langle n_\pm|\rho_{SA}(0)|n_\pm\rangle$$

Information transfer under local dephasing channels

Pure initial state; zero-temperature environment

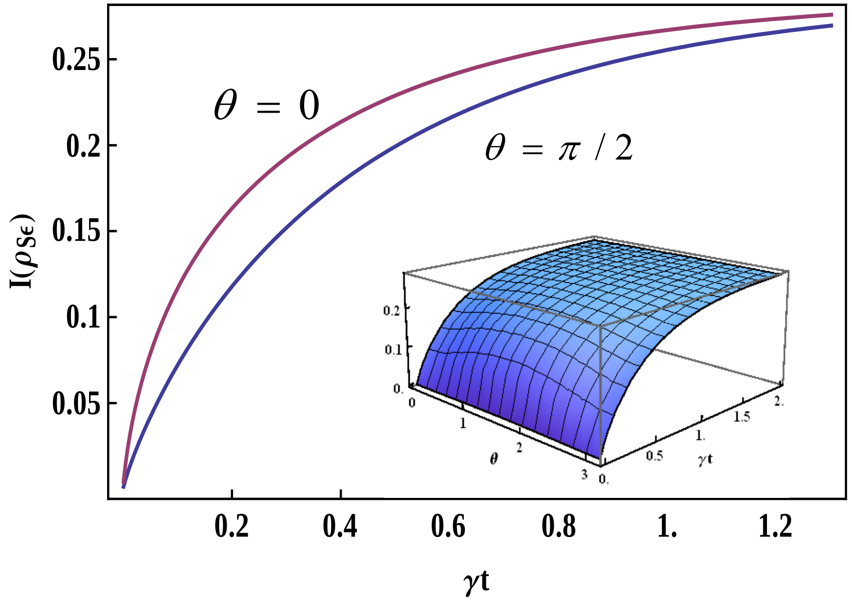
(SAE) joint system: $\left|\psi_{A\varepsilon S}(t)\right\rangle = \sum_{+,-} \left|n_\pm\right\rangle \left|\varepsilon_\pm(t)\right\rangle \left|\varphi_\pm\right\rangle, \quad \left|\varphi_\pm\right\rangle = \left\langle n_\pm \middle| \psi_{SA}(0)\right\rangle$

$$\boxed{I(S:A) = I(S:\varepsilon) = \tfrac{1}{2} I(S:A\varepsilon) = S(\rho_S)}$$

$$\left|\psi_{AS}(0)\right\rangle = \sqrt{\tfrac{1+r}{2}}\left|A_0 S_0\right\rangle + \sqrt{\tfrac{1-r}{2}}\left|A_1 S_1\right\rangle$$

$$\Leftrightarrow \rho_A(0) = \frac{1}{2}(I_2 + \vec{r}\cdot\vec{\sigma})$$

θ : angle between $\vec{r}$ and pointer vectors



r = 0.9, information flow in relation to θ

Information transfer under local dephasing channels: mixed initial states

$$\rho_{SA}(0) \otimes |\varepsilon_0\rangle\langle\varepsilon_0| \to \rho^{d}_{SA\varepsilon}$$

$$\rho^{d}_{AS} = \sum_{+,-} p_{\pm}|n_{\pm}\rangle\langle n_{\pm}| \otimes \rho_S^{(\pm)}$$

$$\rho_{S\varepsilon}(t) = \sum_{+,-} p_{\pm}|\varepsilon_{\pm}(t)\rangle\langle\varepsilon_{\pm}(t)| \otimes \rho_S^{(\pm)}$$

$t \to \infty$

$$I(S:A) = I(S:\varepsilon) \quad \& \quad I(S:A\varepsilon) = I(\rho_{SA}(0))$$

Optimal pointer state maximizing $I(S:A)$ $\Rightarrow$ Measurement responsible for quantum discord

Information transfer under local dephasing channels: mixed initial states (cont.)

$$\rho_{AS}(0) = p^2(\alpha|01\rangle + \beta|10\rangle)(\alpha\langle 01| + \beta\langle 10|) + \frac{1-p^2}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

$$\rho_{AS}(0) \Rightarrow \rho_{AS}^{d}$$

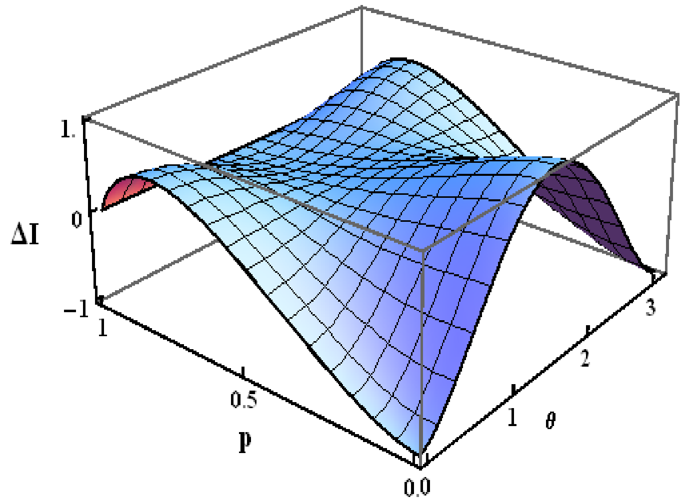
Pointer bases:

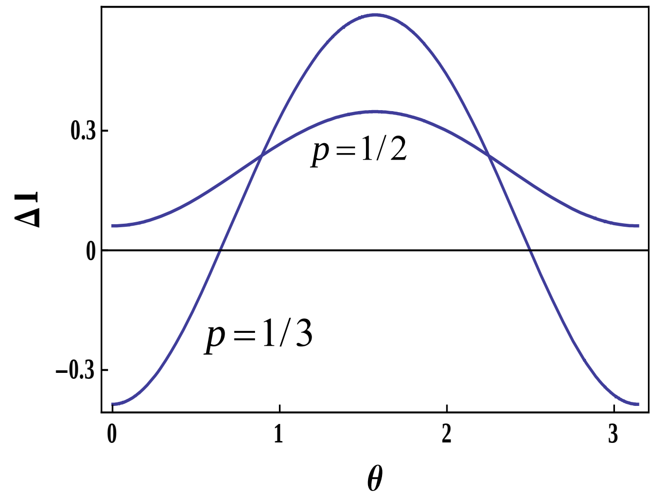
$$|n_+\rangle = \cos\tfrac{\theta}{2}|0\rangle + \sin\tfrac{\theta}{2}e^{i\varphi}|1\rangle,$$
$$|n_-\rangle = \sin\tfrac{\theta}{2}|0\rangle - \cos\tfrac{\theta}{2}e^{i\varphi}|1\rangle$$

$$\Delta I \equiv I(S:A\varepsilon) - I(S:A) - I(S:\varepsilon)$$

If $\Delta I > 0$ for the whole region $\Rightarrow$

QD is larger than classical correlation





$\alpha = \sqrt{2}/2$

Information transfer under local dephasing channels: mixed environment

$$I(S:A\varepsilon) \geq I(S:A) + I(S:\varepsilon)$$

Proof:

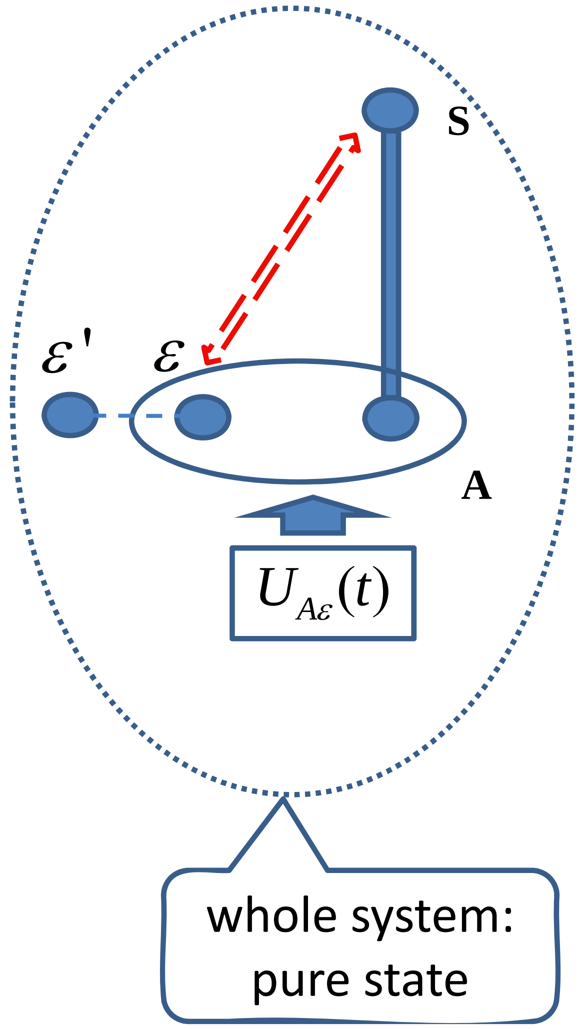
Step 1:

$$\begin{aligned} I(S:A\varepsilon) &= I(S:A\varepsilon\varepsilon') + \underbrace{I(S:\varepsilon')}_{\Rightarrow\, 0} \\ &= I(S:A\varepsilon\varepsilon') \\ &= I(S:A) + I(S:\varepsilon\varepsilon') \end{aligned}$$

Step 2:

Strong subadditivity inequality

$$\Rightarrow \quad I(S:\varepsilon\varepsilon') \geq I(S:\varepsilon)$$



Divisible condition and non-Markovianity

Dynamical map of dissipative process:

$$\rho_{AB}(t) = \Phi(t,0)\rho_{AB}(0)$$

Φ: CP maps preserving trace

Divisible: $\Phi(t+\tau, 0) = \Phi(t+\tau, t)\Phi(t, 0)$

$$\begin{aligned}\rho_{AB}(t+\delta t) &= \Phi(t+\delta t, t)\rho_{AB}(t) \\ &= \sum_{\mu} (M_\mu \otimes I_B)\rho_{AB}(t)(M_\mu^\dagger \otimes I_B)\end{aligned}$$

Markovian

$$E(\rho_{AB}(t+\delta t)) \leq E(\rho_{AB}(t))$$

Entanglement: monotonically decreasing

Information flow and non-Markovianity

Monotonicity of mutual information under divisible map:

$$\begin{aligned}\rho_{AB}(t+\delta t) &= \Phi(t+\delta t, t)\rho_{AB}(t) \\ &= \sum_{\mu}(M_\mu \otimes I_B)\rho_{AB}(t)(M_\mu^\dagger \otimes I_B) \\ &= Tr_C[\underbrace{U_{AB} \otimes I_C \rho_{AB}(t) \otimes \rho_C U_{AB}^\dagger \otimes I_C}_{\rho_{ABC}}]\end{aligned}$$

Strong subadditivity inequality:

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$$

$$\Updownarrow$$

$$I(A:B) \leq I(A:BC)$$

$$\Rightarrow$$

$$\begin{aligned} I_t(A:B) &= I(A:BC) \\ &\geq I_{t+\delta t}(A:B) \end{aligned}$$

monotonically decreasing

Information transfer in non-Markovian dynamics: model

Model and dynamics:

$$H = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k b_k^\dagger b_k + (\sigma_+ B + \sigma_- B^\dagger),$$

$$B = \sum_k g_k b_k$$

zero-temperature environment

$$\rho^A(t) = \sum_{\mu=1,2} M_\mu(t) \rho^A(0) M_\mu^\dagger(t)$$

$$= \begin{pmatrix} \rho_{11}^A(0) P_t & \rho_{10}^A(0)\sqrt{P_t} \\ \rho_{01}^A(0)\sqrt{P_t} & \rho_{00}^A(0) + \rho_{11}^A(0)(1-P_t) \end{pmatrix}$$

$$M_1 = \sqrt{1-P_t}|0\rangle\langle 1|,$$

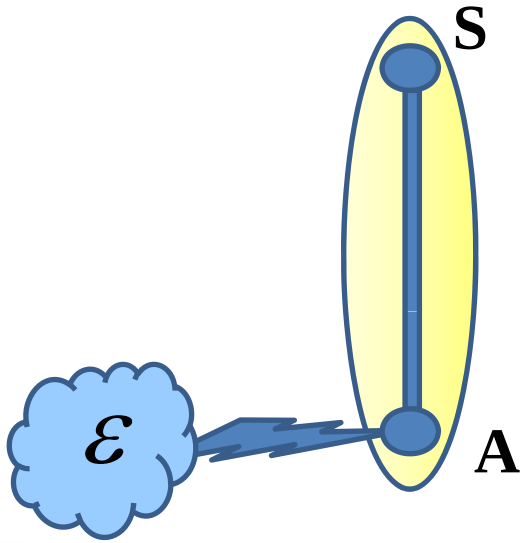
$$M_2 = \sqrt{P_t}|1\rangle\langle 1| + |0\rangle\langle 0|$$

$$\dot{P}_t = -\int_0^t dt_1 f(t-t_1) P_{t_1},$$

$$f(t-t_1) = \int d\omega J(\omega) \exp[i(\omega_0 - \omega)(t-t_1)]$$

Information feedback and non-Markovianity

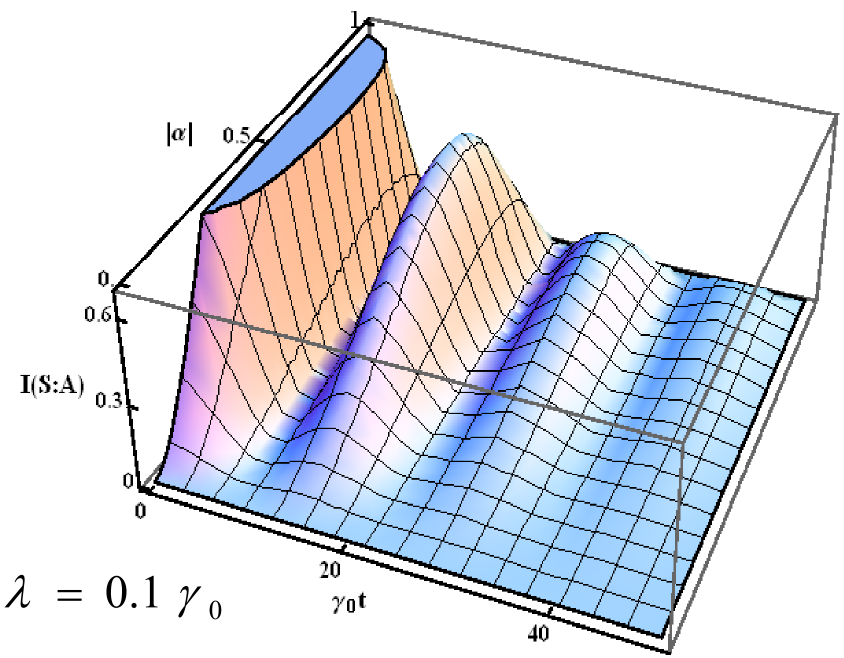
$$\left|\psi_{AS}(0)\right\rangle = \alpha\left|10\right\rangle + \beta\left|01\right\rangle$$



Information feedback:

1) $I(S{:}A)$ increases; 2) $I(S:\varepsilon)$ decreases

$$\text{pure state}: \frac{dI(S:A)}{dt} = -\frac{dI(S:\varepsilon)}{dt}$$



$\lambda = 0.1\,\gamma_0$

$I(S:A)$

Lorentzian spectrum

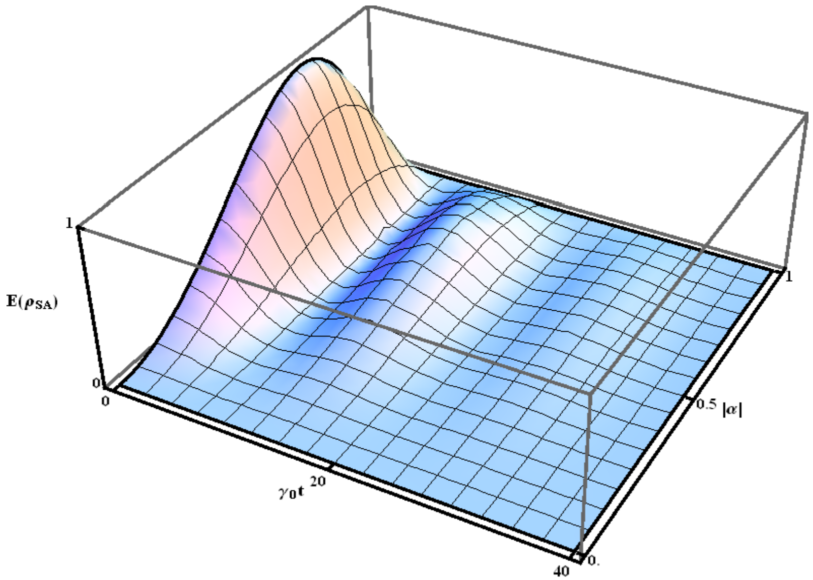
$$J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda^2}{(\omega_0-\omega)^2+\lambda^2}$$

$$P_t = e^{-\lambda t}\left[\cos\left(\frac{dt}{2}\right) + \frac{\lambda}{d}\sin\left(\frac{dt}{2}\right)\right]^2$$

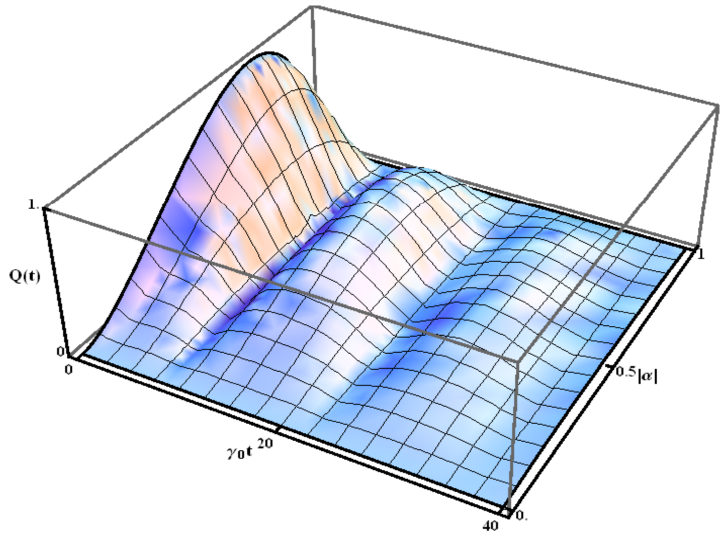
$$d = \sqrt{2\gamma_0\lambda - \lambda^2}$$

Information feedback and non-Markovianity (cont.)

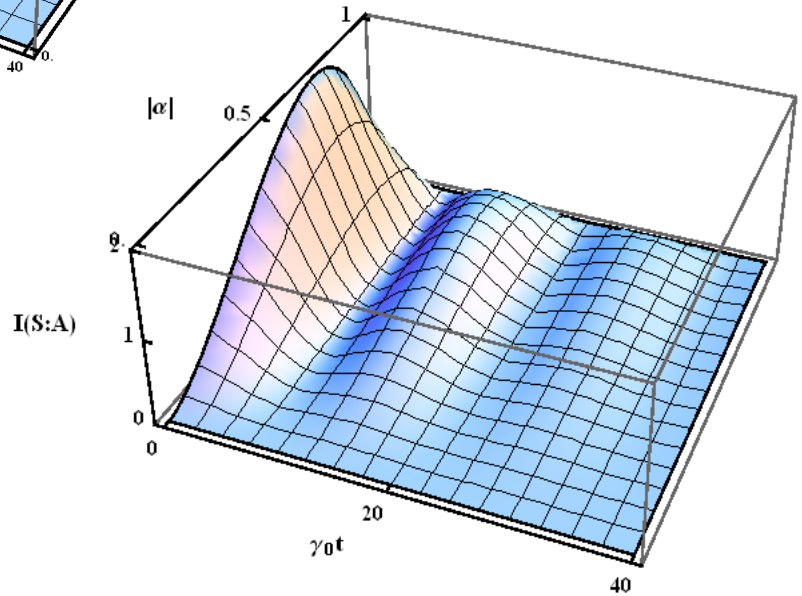
comparison of MI, EoF, and QD



EoF



QD



MI

Information feedback and non-Markovianity: mixed initial states

Werner states:

$$\rho_{SA}(0) = \frac{1-z}{4} I_4 + z|\psi\rangle\langle\psi|,$$

$$|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$$

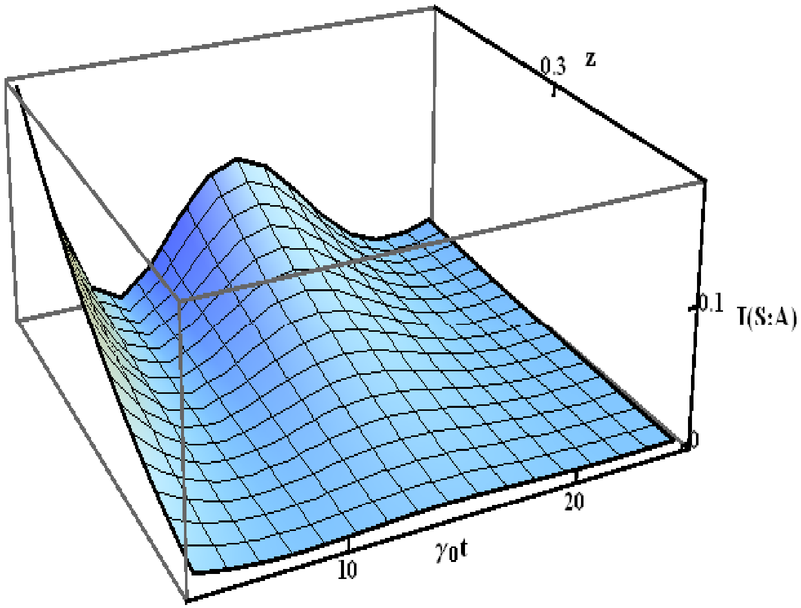
Separable when $z < 1/3$

Non-Markovianity without entanglement oscillation

$$I(S:A) = 1 + S(\rho_A) - S(\rho_{SA})$$

$$\rho_A(t) = \sum_{\mu=1,2} M_\mu(t)\rho_A(0)M_\mu^\dagger(t)$$

$$\rho_{SA}(t) = \sum_{\mu=1,2} I_S \otimes M_\mu(t)\rho_{SA}(0)I_S \otimes M_\mu^\dagger(t)$$



Information feedback and non-Markovianity: mixed initial states (cont.)

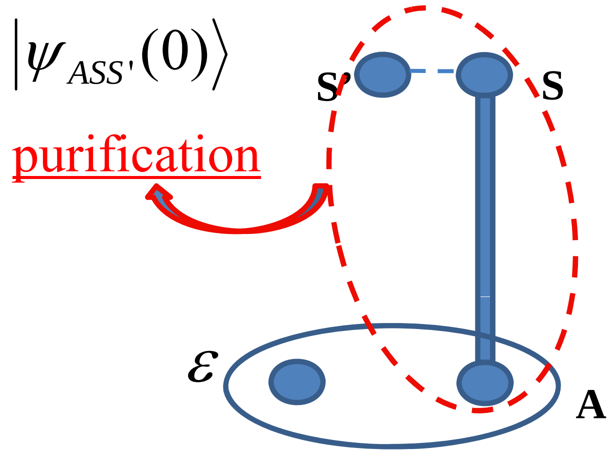
$$\frac{dI(S:A)}{dt} \neq -\frac{dI(S:\varepsilon)}{dt}$$

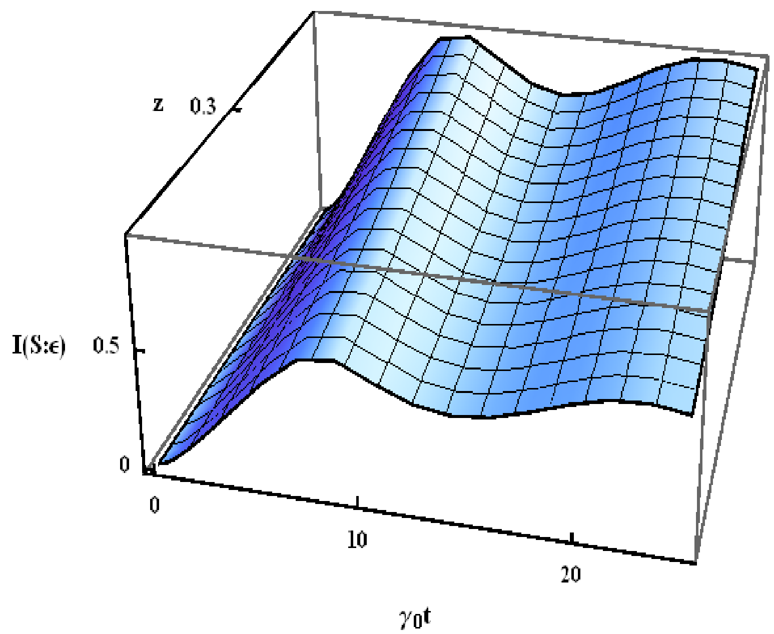
$$I(S:\varepsilon) = 1 + S(\rho_\varepsilon) - S(\rho_{S\varepsilon})$$

$$|\psi_{ASS'}(0)\rangle \to \rho_{ASS'}(t)$$

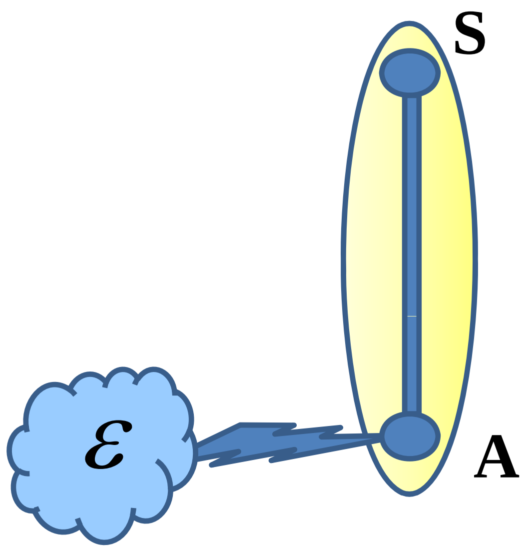
$$S(\rho_\varepsilon(t)) = S(\rho_{ASS'}(t))$$

$$S(\rho_{S\varepsilon}(t)) = S(\rho_{AS'}(t))$$





Information feedback and non-Markovianity: open issue



Question:

Does decreasing of $I(S:\varepsilon)$ and $E(\rho_{S\varepsilon})$

⇒ **Non-Markovianity?**

In the case of mixed environment:

$$I(S:A\varepsilon) \geq I(S:A) + I(S:\varepsilon)$$

⇒ **Seems to be not.**

Information feedback and non-Markovianity: open issue (cont.)

Measure non-Markovianity via information feedback with optimal pointer bases:

$$\eta(t) = \frac{d}{dt} I(S:A,t),$$

$$F = \max_{\{\text{pointer bases}\}} \int_{\eta>0} \eta(t)\,dt$$

?

Summary

- Information transfer and distribution under dephasing channels

  1) Pure initial states & mixed initial states;

  2) Zero temperature & mixed environment

- Information transfer during non-Markovian dynamics

  1) Detection of non-Markovianity: EoF vs. MI

  2) Information distribution and non-Markovianity

Thanks for your attention!