Exotic quantum states of intermediate correlated fermion systems and possible realization in optical lattice

Su-Peng Kou (寇谡鹏)
Beijing Normal University

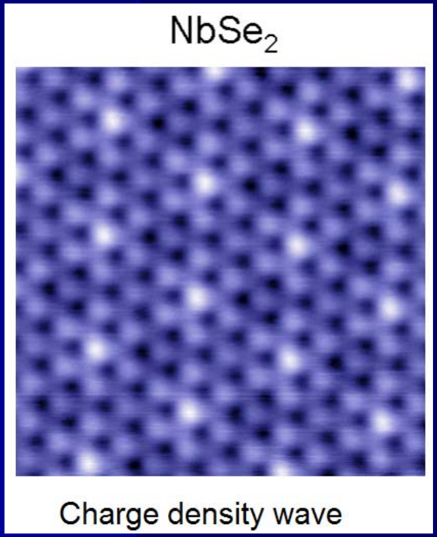
Outline

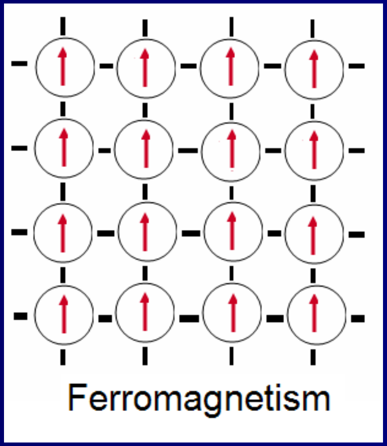
- Introduction
- Motivation
- Exotic quantum states of intermediate correlated electron systems
- Experimental realization in cold atoms
- Conclusion

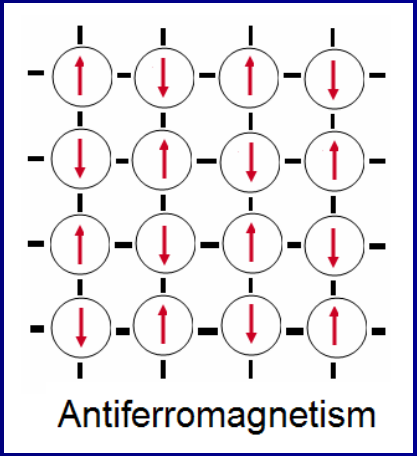
Collaborators: **Jing He, Gao-Yong Sun**, Lan-Feng Liu, Ya-Jie Wu, Yang-Hua Zong, Ying Liang, ShiPing Feng

I. Introduction 1: exotic quantum states

Traditional orders

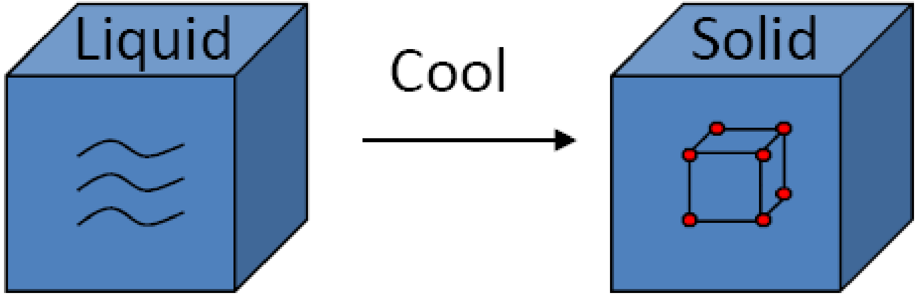




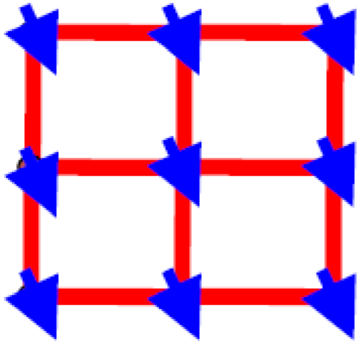


Landau paradigm

- **Solids** (broken translation symmetry)
- **Magnets** (broken spin symmetry)



order parameter - density



magnetization

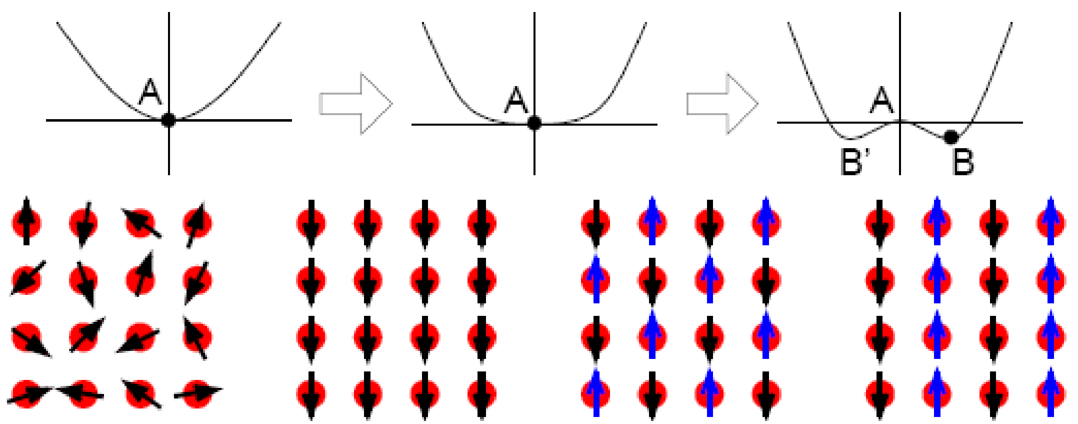
Landau paradigm

Landau symmetry breaking theory:

Different organizations = different symmetries

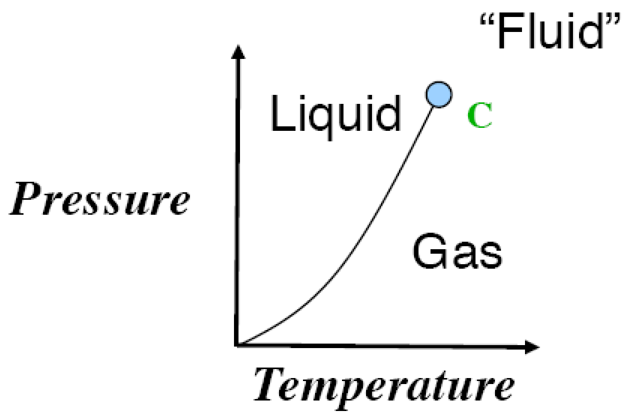
Phase transition = symmetry breaking

Landau

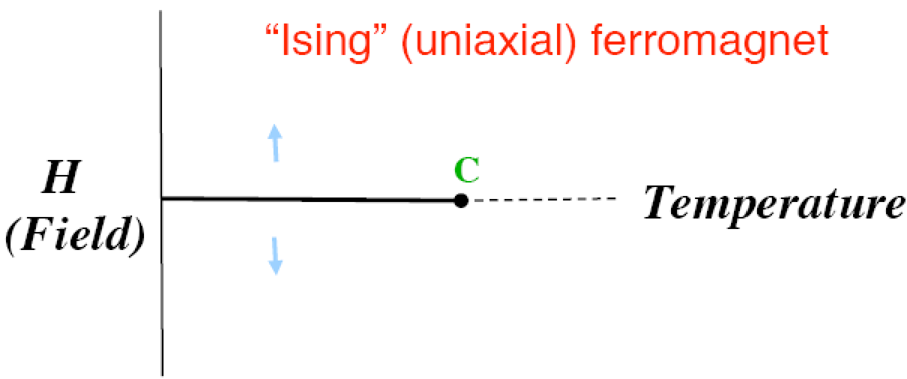


- **The critical parameters between a classical continuous phase transition are determined by the dimension of the system and the degree freedom of the order parameters.**

Universal law for traditional phase transitions



$$\rho_L - \rho_G \sim \left(\frac{T_C - T}{T_C} \right)^{\beta}$$

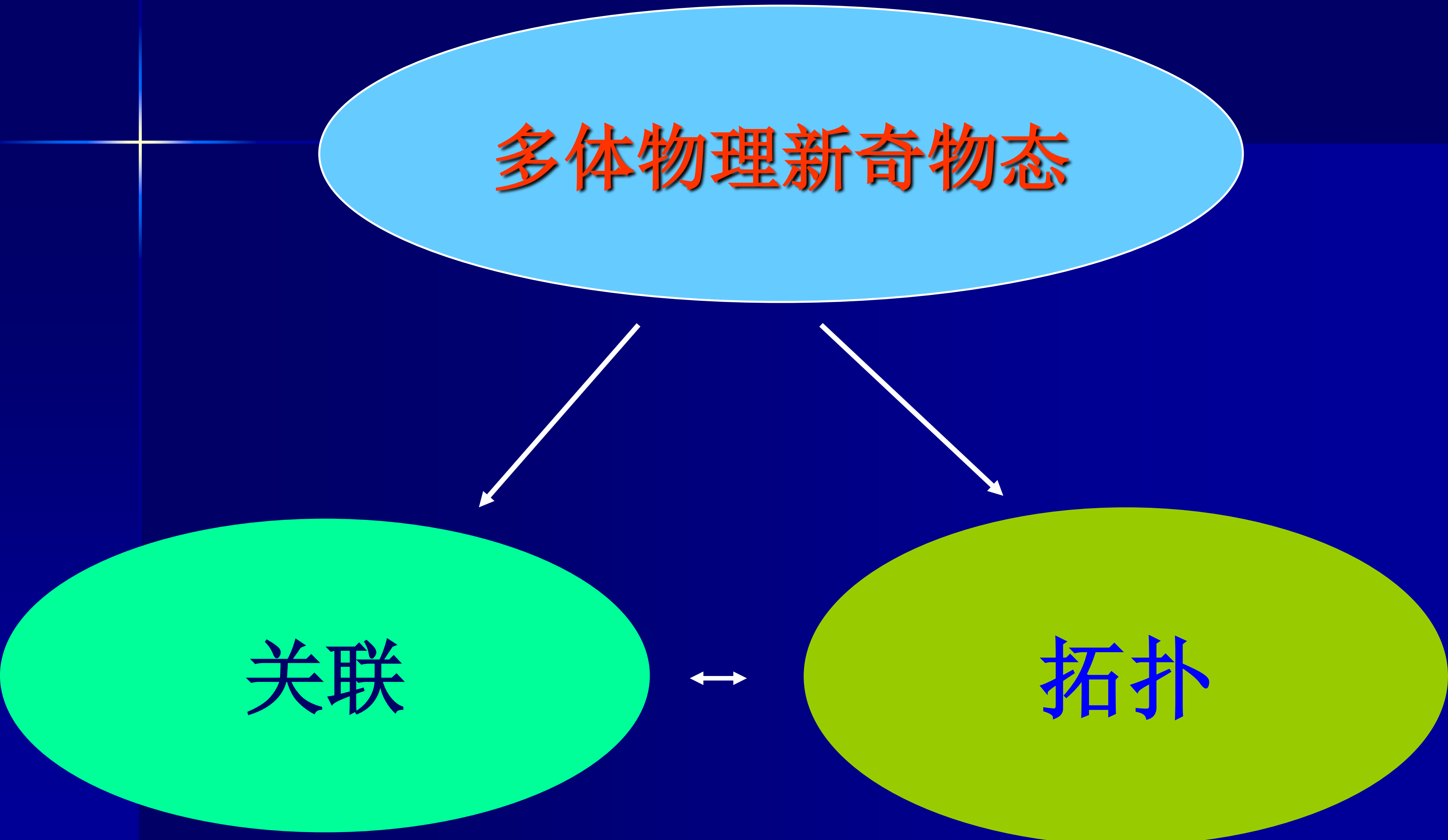


$$M_{\uparrow} - M_{\downarrow} \sim \left(\frac{T_C - T}{T_C} \right)^{\beta}$$

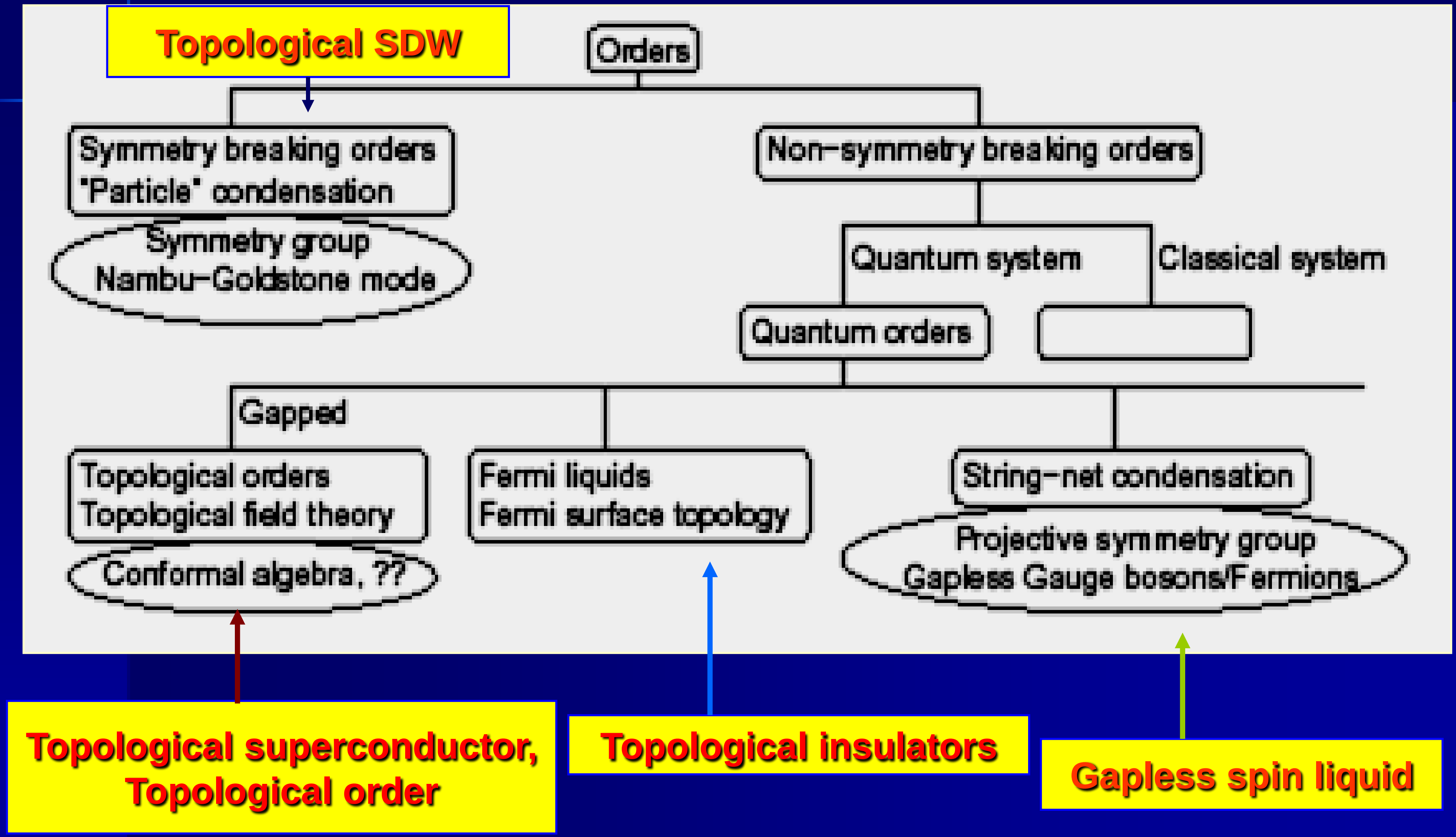
$$\text{Experiment}: \quad \beta = 0.322 \pm 0.005$$

$$\text{Theory}: \quad \beta = 0.325 \pm 0.002$$

"Universality" at continuous phase transitions (Wilson, Fisher, Kadanoff, ...)


多体物理新奇物态
关联
拓扑

Keywords : Roadmap for exotic states



X. G. Wen, Quantum Field Theory of Many-Body Systems

Topological insulators

Topological Insulators

= Bulk insulators with topologically protected delocalized states on their boundary

Well-known example: Quantum Hall Effect

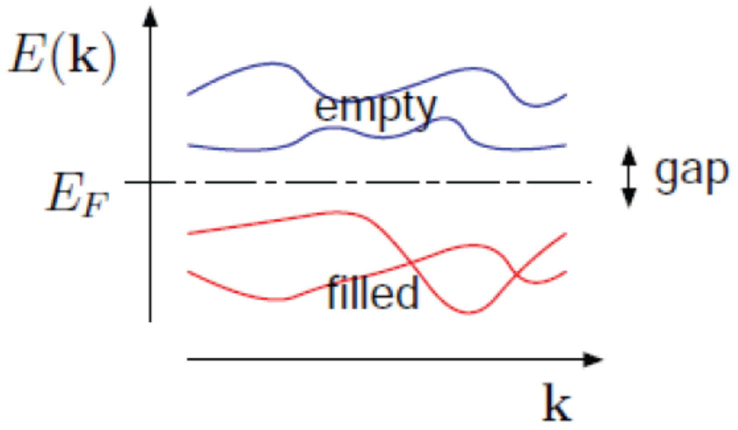
QH insulators $\longrightarrow$ $n = \ldots, -2, -1, 0, 1, 2, \ldots$ edge states

$\longrightarrow$ $\mathbb{Z}$ topological insulator

Novel class: $\mathbb{Z}_2$ TIs: $n = 0$ or $n = 1$

First realization: 2D Quantum Spin Hall Effect insulators

- "bulk" point of view



$$\pi_2\left[U(m+n)/U(m)\times U(n)\right]=\mathbb{Z}$$

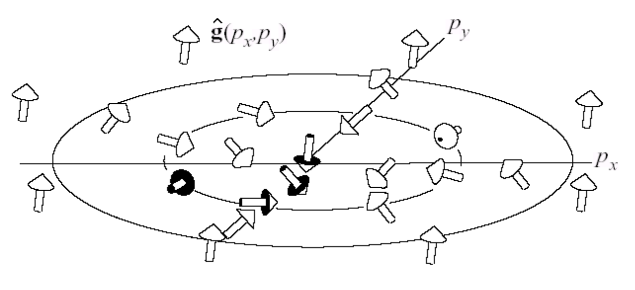
Integer quantum Hall state – Z topological insulator

Thouless-Kohmoto-Nightingale-den Nijs (1982)

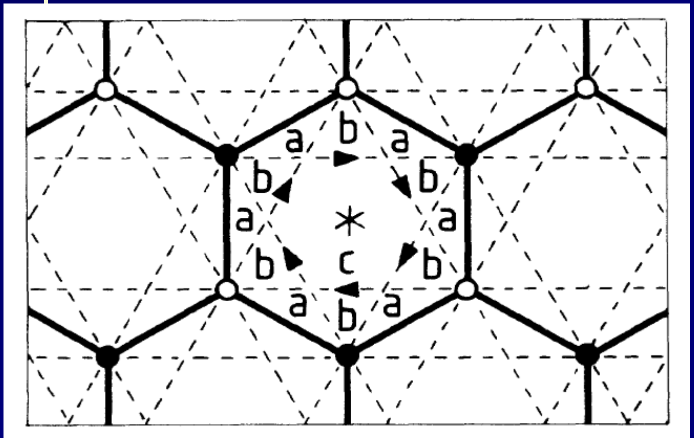
$$\sigma_{xy}=\frac{e^2}{h}\sum_{bands}\frac{i}{2\pi}\int d^2k\left(\left\langle\frac{\partial u(k)}{\partial k_x}\middle|\frac{\partial u(k)}{\partial k_y}\right\rangle-\left\langle\frac{\partial u(k)}{\partial k_y}\middle|\frac{\partial u(k)}{\partial k_x}\right\rangle\right)$$

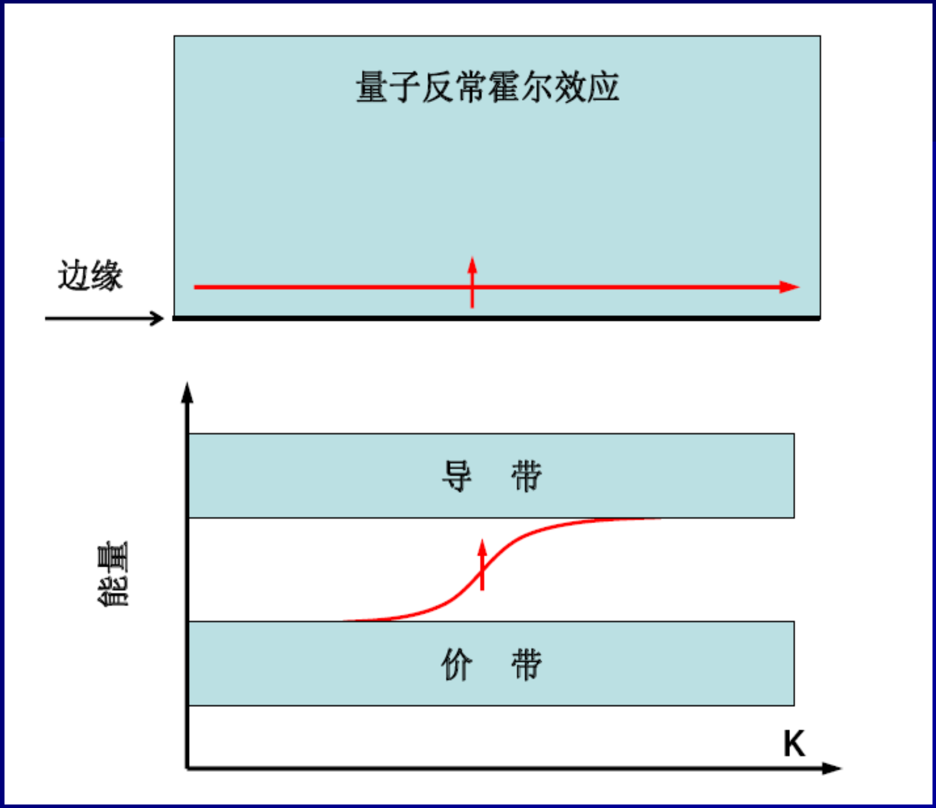
topological invariant ! "Chern number"

$$n=\sum_{bands}\frac{i}{2\pi}\int d^2k\left(\left\langle\frac{\partial u}{\partial k_1}\middle|\frac{\partial u}{\partial k_2}\right\rangle-\left\langle\frac{\partial u}{\partial k_2}\middle|\frac{\partial u}{\partial k_1}\right\rangle\right)$$



The Haldane model





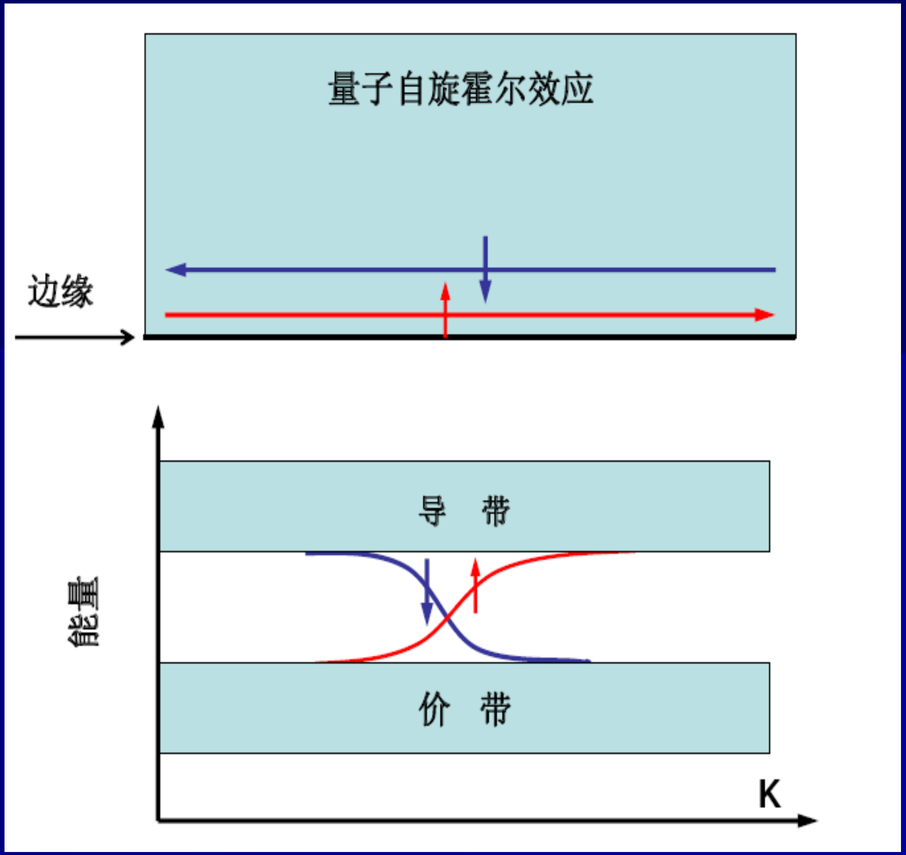
$$H_{\mathrm{H}} = -t \sum_{\langle i,j \rangle,\sigma} \left(\hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + h.c. \right) - t' \sum_{\langle\langle i,j \rangle\rangle,\sigma} e^{i\varphi_{ij}} \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma}$$

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).

Kane-Mele model

Kane and Mele, Phys. Rev. Lett. 95, 146802 (2005)

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\sigma\sigma'} \nu_{ij}\, \sigma_{\sigma\sigma'}^{z}\, c_{i\sigma}^{\dagger} c_{j\sigma'}$$



$$(-1)^{\nu} = \prod_{i=1}^{4} \delta_i .$$

$$\delta_i = \frac{\sqrt{\det[w(\Gamma_i)]}}{\mathrm{Pf}[w(\Gamma_i)]} = \pm 1,$$

Z2 topological invariable : Pfaffian = zero counting

$$w_{mn}(\mathbf{k}) \equiv \langle u_{m-\mathbf{k}} | \Theta | u_{n\mathbf{k}} \rangle .$$

Topological superconductors

- All excitations have mass gaps
- Quantized vortex with induced fermion number
- Majorona edge states
- Exotic quantum statistics and nontrivial fusion rule

dx+idy-wave SC

px+ipy-wave SC

Topological Superconductors

$$K_{\text{eff}} = \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \frac{1}{2} \left(\Delta_{\mathbf{k}}^{*} c_{-\mathbf{k}} c_{\mathbf{k}} + \Delta_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{-\mathbf{k}}^{\dagger} \right) \right]$$

$$\Delta_p(\mathbf{k}) = \Delta_0 \hat{\mathbf{z}} (\sin k_x + i \sin k_y)$$

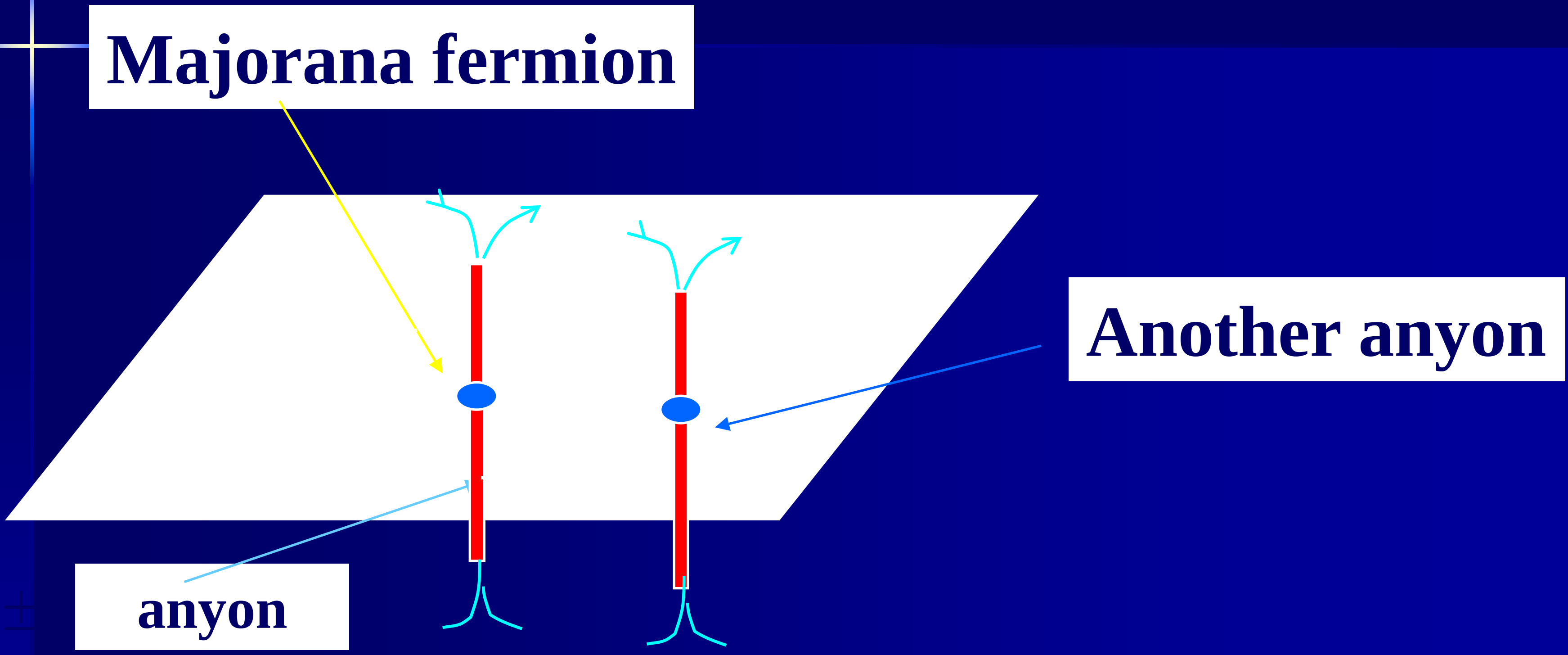
$$\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$$

- **μ>0, non-Abelian Topologial SC state**
- **μ<0, trivial SC state**

Read, Green, 2000.

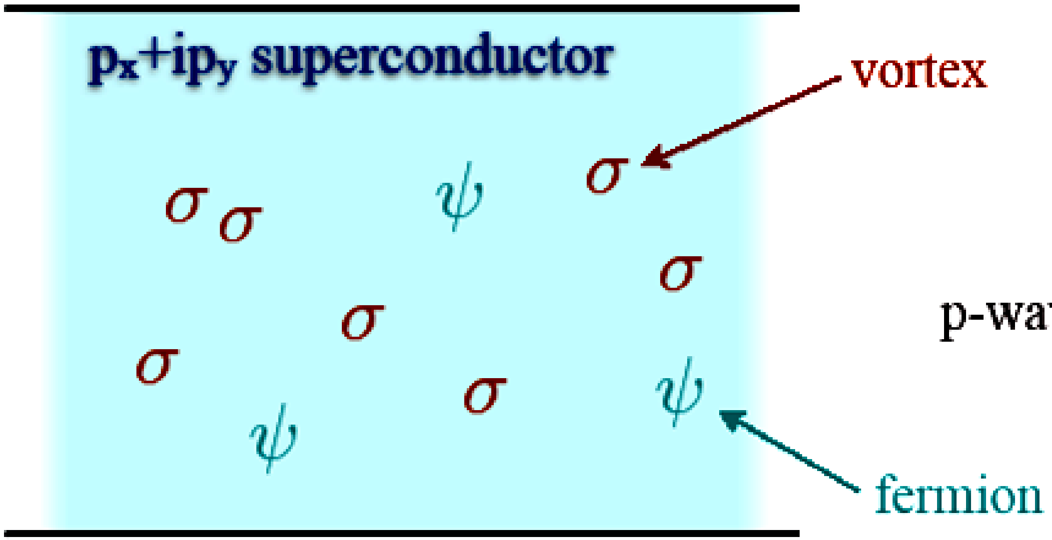
S. P. Kou and X.G. Wen, 2009.

Ising anyons :π-Flux binding a Majorana Fermion



$SU(2)_2$ non-Abelian statistics between π-flux with a trapped majorana fermion.

p_x+ip_y superconductors



possible realizations

Sr_2RuO_4

p-wave superfluid of cold atoms

A_1 phase of ^3He films

$E = n\omega$

Topological properties of p_x+ip_y superconductors

Read & Green (2000)

$SU(2)_2$

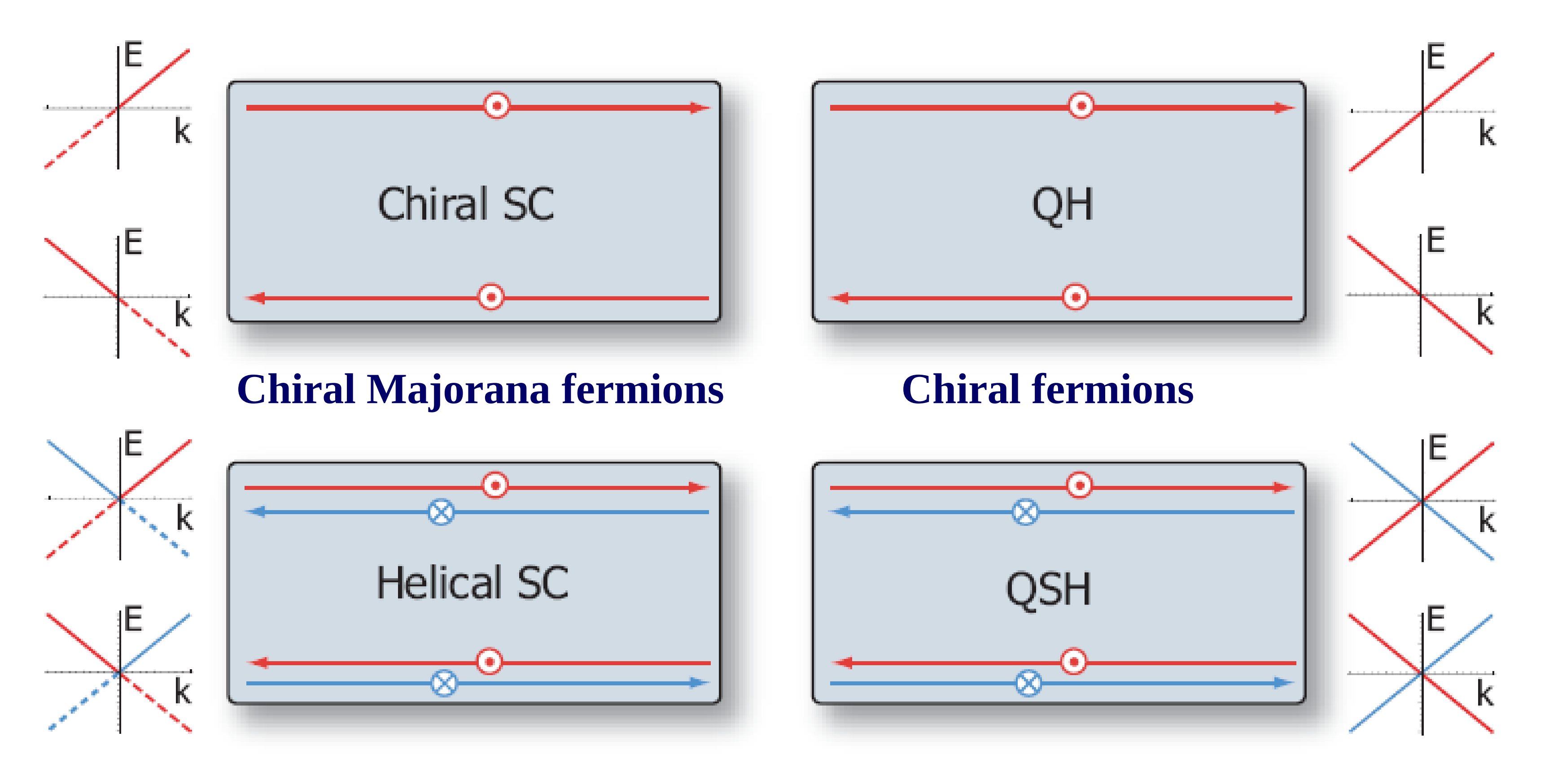
σ-vortices carry "half-flux" $\phi = \frac{hc}{2e}$

characteristic "zero mode"

2N vortices give degeneracy of 2^N.

$$\sigma \times \sigma = 1 + \psi$$

Edge state of topological superconductors and topological insulators



Chiral Majorana fermions **Chiral fermions**

Massless Majorana fermions **Massless Dirac fermions**

Qi, Hughes, Raghu and Zhang, 2009

Classification of topological states : ten-fold way of random matrix

System	Cartan nomenclature	TRS	PHS	SLS	Hamiltonian
standard (Wigner-Dyson)	A (unitary)	0	0	0	$U(N)$
	AI (orthogonal)	+1	0	0	$U(N)/O(N)$
	AII (symplectic)	−1	0	0	$U(2N)/Sp(2N)$
chiral (sublattice)	AIII (chiral unit.)	0	0	1	$U(N+M)/U(N)\times U(M)$
	BDI (chiral orthog.)	+1	+1	1	$SO(N+M)/SO(N)\times SO(M)$
	CII (chiral sympl.)	−1	−1	1	$Sp(2N+2M)/Sp(2N)\times Sp(2M)$
BdG	D	0	+1	0	$SO(2N)$
	C	0	−1	0	$Sp(2N)$
	DIII	−1	+1	1	$SO(2N)/U(N)$
	CI	+1	−1	1	$Sp(2N)/U(N)$

Altland-Zirnbauer (1997) : "ten-fold way"

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

IQHE
p+ip wave SC
polyacetylene
3He B
TMTSF
Z2 topological insulator
QSHE
d+id wave SC

SR, Schnyder, Furusaki, Ludwig (for d=1,2,3, 2008)

Kitaev (all d and periodicity, 2009)

AZ class	SU(2)	TRS	Constraints on Hamiltonians	Examples in 2D
D	×	×	$t_x \mathcal{H}^T t_x = -\mathcal{H}$	Spinless chiral $(p \pm ip)$-wave
DIII	×	◯	$t_x \mathcal{H}^T t_x = -\mathcal{H}$, $is_y \mathcal{H}^T (-is_y) = \mathcal{H}$	Superposition of $(p+ip)$- and $(p-ip)$-wave
A	△	×	no constraint	Spinful chiral $(p \pm ip)$-wave
AIII	△	◯	$r_y \mathcal{H} r_y = -\mathcal{H}$	Spinful p_x- or p_y-wave
C	◯	×	$r_y \mathcal{H}^T r_y = -\mathcal{H}$	$(d \pm id)$-wave
CI	◯	◯	$r_y \mathcal{H}^T r_y = -\mathcal{H}$, $\mathcal{H}^* = \mathcal{H}$	$d_{x^2-y^2}$- or d_{xy}-wave

Topological Orders

Topological order – an emergent world in a many-body system

- All excitations have mass gaps
- Topological excitations – anyons with fractional statistics
- Effective theory - topological field theory
- No local order parameters – string net condensation
- Edge states

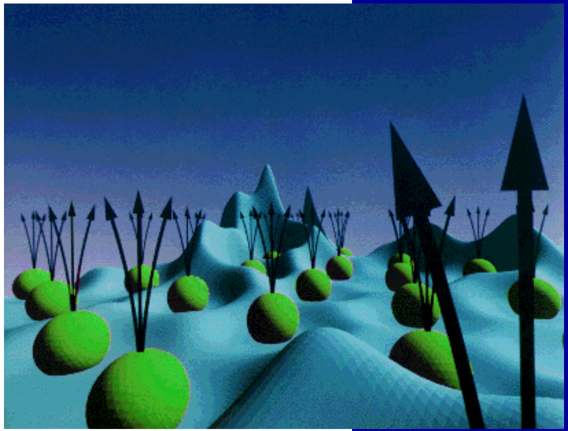
Topological orders in condensed matter physics

- **Fractional Quantum Hall (FQH) states**
- **Topological orders in spin liquids:**

  chiral spin liquid,

  Z2 topological order,

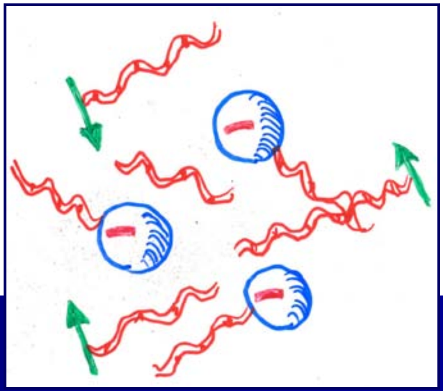
  non-Abel topological order.



- **Rotating cold atoms**
- **FQH states in flat band models**

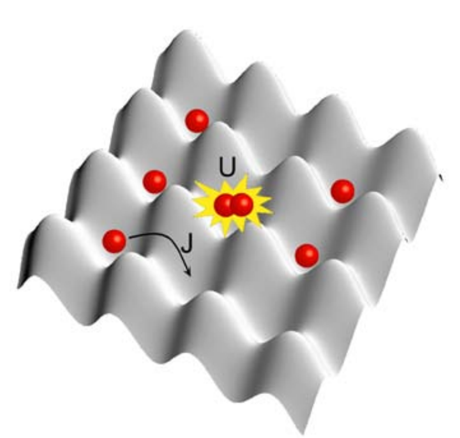
Introduction 2: strongly correlated fermion systems

- Mott Physics
- High Tc cuprates
- Non-perturbative approaches
- Spin physics – VBS, spin liquid
- …



Hubbard model – typical model for Mott physics

$$\mathcal{H} = -t\sum_{\langle ij\rangle}[c^{\dagger}_{i\alpha}c_{j\alpha} + h.c.] + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$



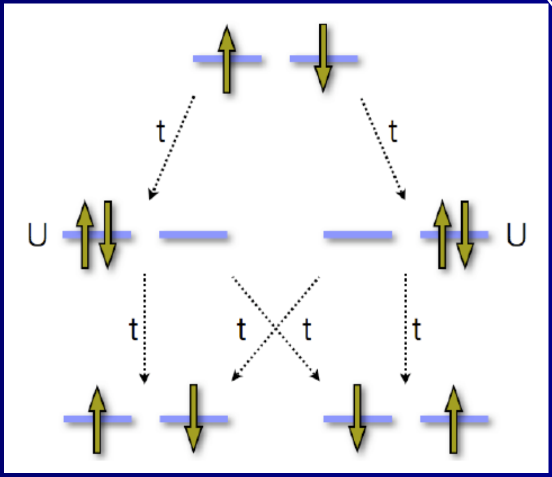
Weak coupling

Strong coupling

0 — metal — **Mott transition** — insulator → U/t

Free Fermi gas,

Fermi liquid



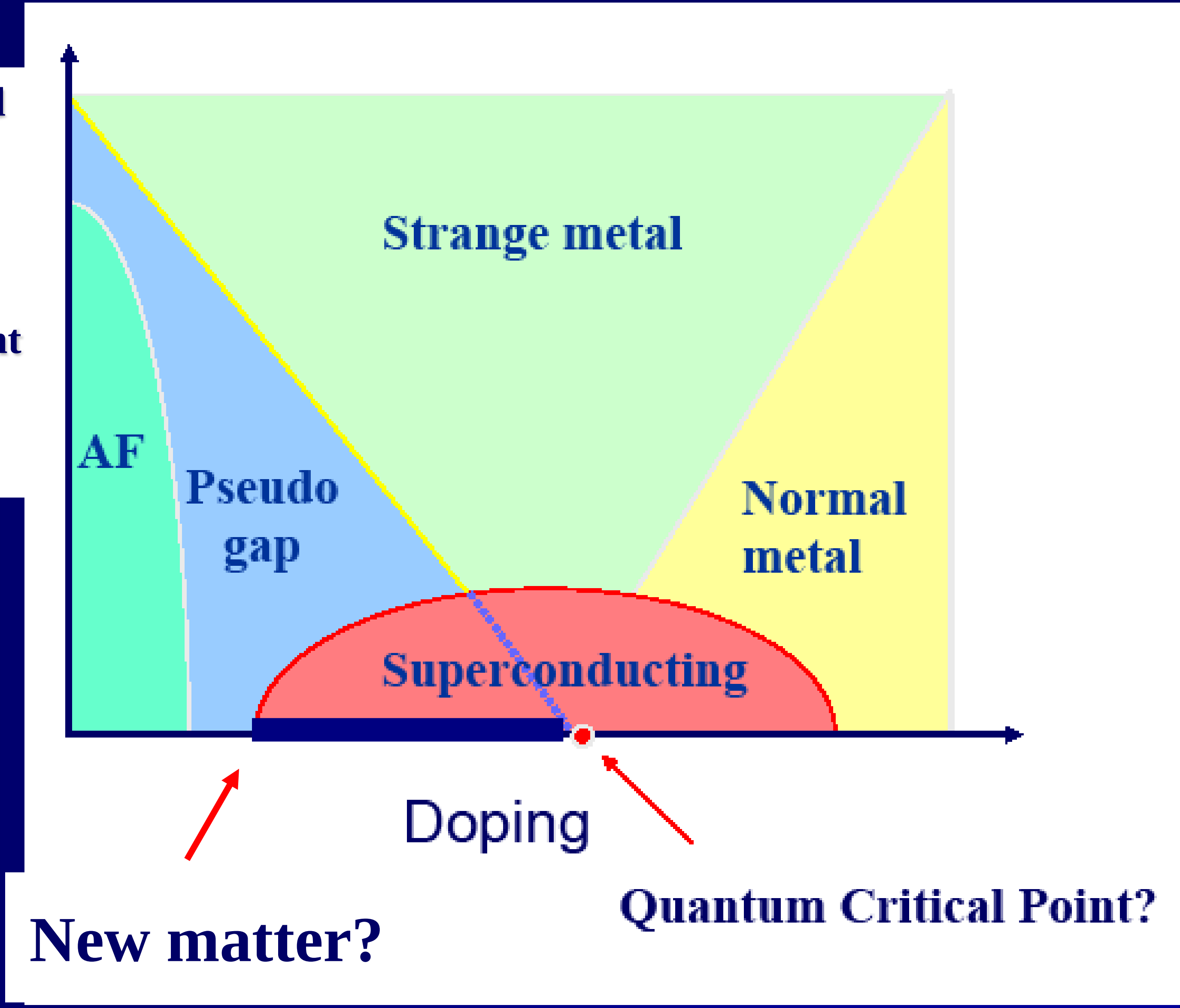
Heisenberg model

$$H_{spin} = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j$$

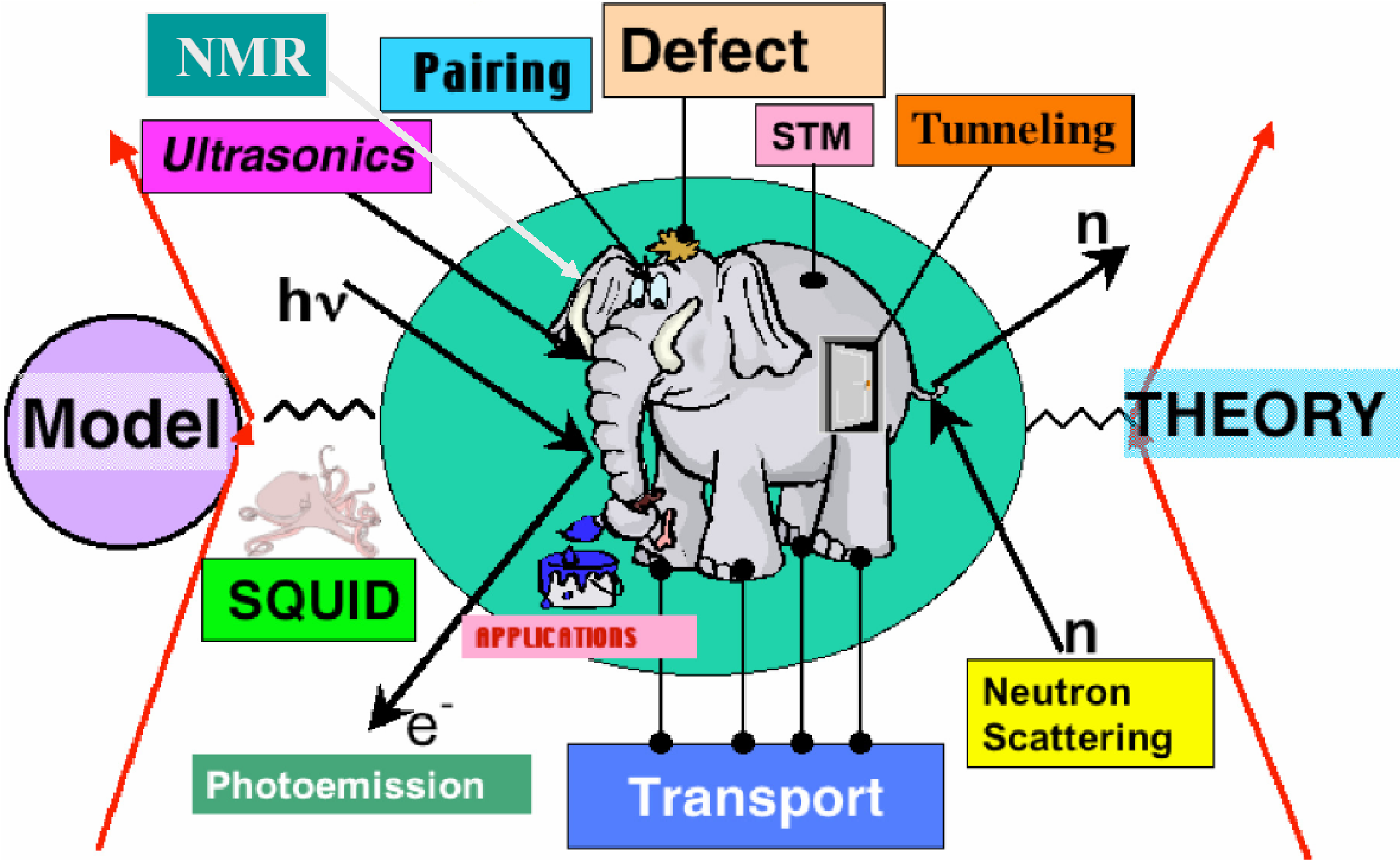
$J=4t^2/U$

Key issue: High Tc superconductor

- Quantum Spin Liquid
- phase competing
- Stripe phase,
- D-density wave state
- Quantum critical point
- …



Mott 1977
Mott insulator



NMR
Pairing
Defect
STM
Tunneling
Ultrasonics
hν
n
Model
THEORY
SQUID
APPLICATIONS
n
e-
Neutron Scattering
Photoemission
Transport

Key issue - Nonpertubative

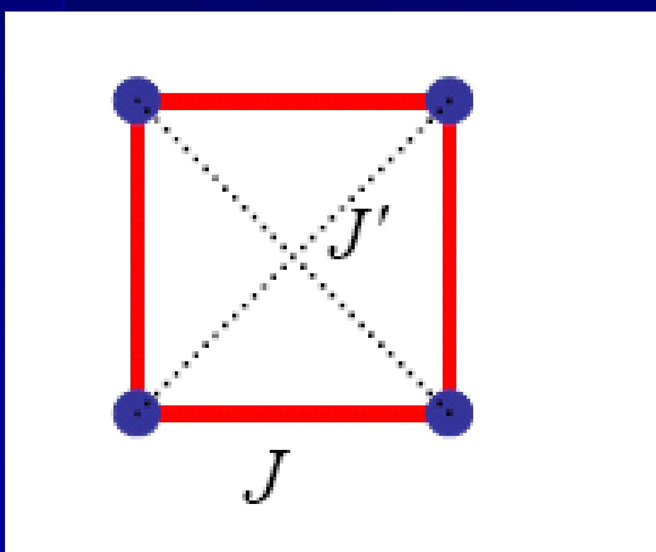
Key issue : spin physics in strongly correlated fermion system

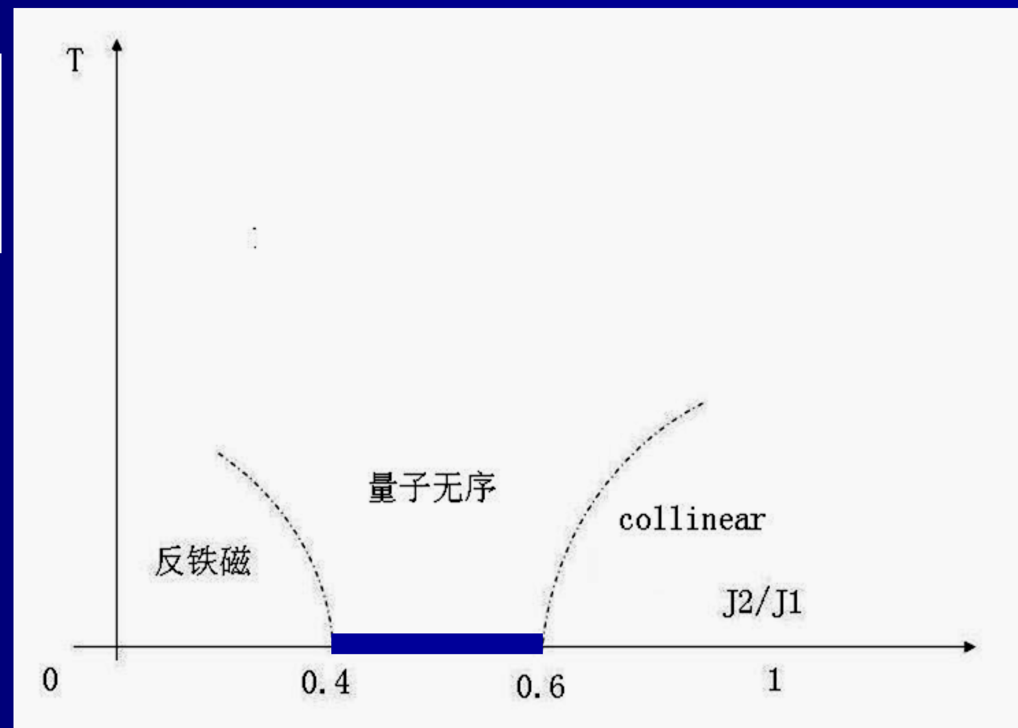
Phase	$2S$/cell	Order	Degeneracy	Broken sym.	Excitations	Thermo.	Examples
Néel AF p-sublattice	any	spin-spin LRO	$O(N^p)$	$SU(2)$ Translations Point group	Gapless magnons (spin waves)	$C_v \sim T^2$ $\chi \sim$ cst	Spin-$\frac{1}{2}$ triangular Heisenberg AF
VBC (§3) (spontaneous)	odd	singlet-singlet LRO colli. spin-spin SRO	> 1	Translations Point group	Gapped magnons	C_v and χ activated	Honeycomb J_1–J_2 Checkerboard Square J_1–J_2 ?
VBC (§3) (explicit)	even	None	1	None	Gapped magnons	C_v and χ activated	$SrCu_2(BO_3)_2$ CaV_4O_9
VBS (§3.3)	even	"String" LRO	1	None	Gapped magnons Edge excitations	C_v and χ activated	AKLT Hamiltonians $S=1$ kagome AF ?
RVB SL (§5.5 §5.6, §6.4)	odd	Topological non-colli. SRO	4 (torus)	None	Gapped spinons Gapped visons	C_v and χ activated	MSE (§6) QDM on triangular and kagome lattices
Kagome (§7) Heisenberg AF	3	None ?	$\sim 1.15^N$?	None ?	Gapped triplets ? Gapless singlets	$C_v \sim T^\alpha$? χ activated	

G. Misguich, arXiv:cond-mat/0310405

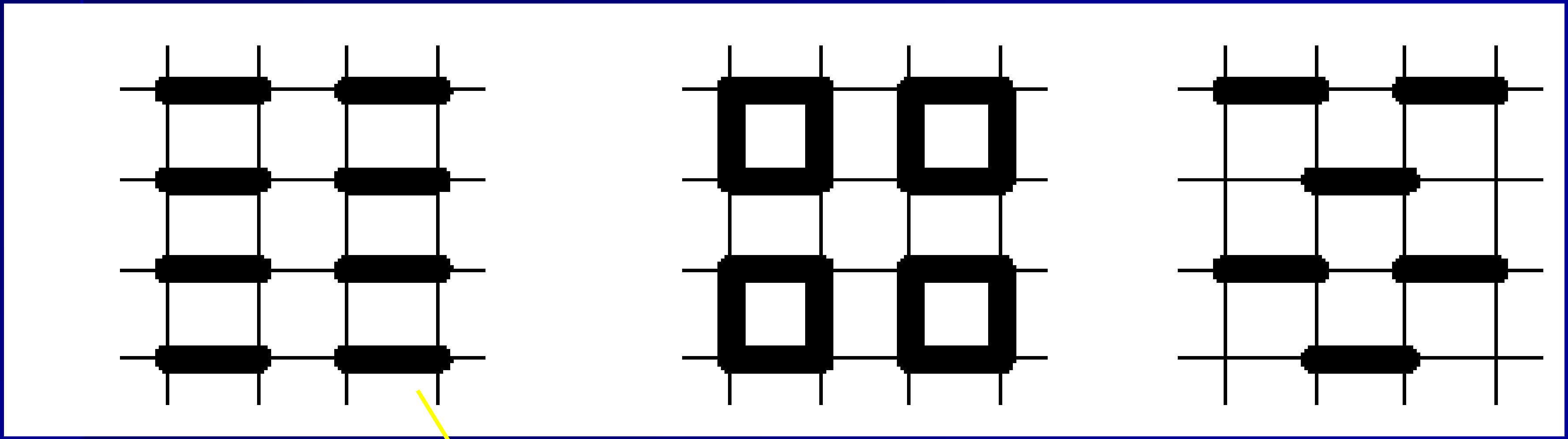
2D J1-J2 Model

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j,$$





Valence bond crystal (solid)



Spin singlet

Spin liquid – emergent in physics

Definition :
Spin liquid state-an insulator with spin-rotation symmetry and with an odd number of electrons per unit cell.

No broken symmetry

+

Deconfined spinons

+

Emergent gauge field

↓

Spin liquid

Spin liquid
- projected superconductor on lattice!

$$H_{mean} = \sum_{ij} \psi_i^\dagger u_{ij} \psi_j + \sum_i \psi_i^\dagger a_i^l \tau^l \psi_i$$

where u_{ij} is 2 by 2 matrix and $\psi^T = (\psi_1, \psi_2)$. Let $|\Psi_{\text{mean}}^{(u_{ij},a_i^l)}$ be the ground state of the above free fermion Hamiltonian, then a many-spin wave function can be obtained by projecting $|\Psi_{\text{mean}}^{(u_{ij},a_i^l)}$ into the space with only even number of fermions per site:

$$|\Psi_{\text{spin}}^{(u_{ij},a_i^l)}\rangle = P|\Psi_{\text{mean}}^{(u_{ij},a_i^l)}\rangle$$

where Here the projection operator is given by

$$P = \prod_i \frac{1 + (-1)^{n_i}}{2},$$

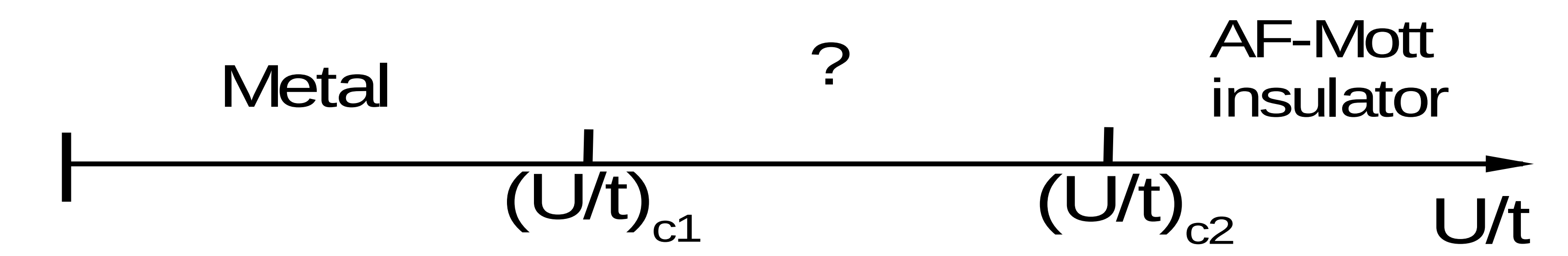
where $n_i = \psi_i^\dagger \psi_i$.

Fermionic-projective-representation (SU(2) slave-Boson representation) of S=1/2 spin models

Classification of spin liquids: classification of emergent physics

- Z2 spin liquid (topological or gapless)
- U(1) spin liquid (algebra)
- SU(2) spin liquid (algebra or chiral)
- nonAbelian spin liquid (with nonAblian anyon)
- …

II. Exotic quantum states of intermediate correlated electron systems



- Spin liquid state with spinon Fermi surface near Mott transition of the Hubbard model on the triangle lattice
- Spin liquid state near Mott transition on the honeycomb lattice
- **Spin liquid state near Mott transition on the π-flux Hubbard model**
- Spin liquid state of interacting Kane-Mele model
- **Spin liquid state and topological spin density wave of interacting spinful Haldane model**
- Spin liquid state of 3D interacting topological insulators

Starting point : The generalized Hubbard models

$$\mathcal{H} = -\sum_{\langle i,j\rangle,\sigma} t_{ij}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_{j}\hat{n}_{j\uparrow}\hat{n}_{j\downarrow} - \mu\sum_{i,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + h.c.$$

- **The Hubbard model on square lattice – Spin density wave**
- **The π-flux Hubbard model on square lattice – nodal spin liquid**
- **The Hubbard model on honeycomb lattice – puzzle of spin lquid**
- **The interacting Kane-Mele model – Z2 spin liquid**
- **The interacting spinful Haldane model – chiral spin liquid and topological spin density wave**

1. Spin density wave in the Hubbard model on square lattice

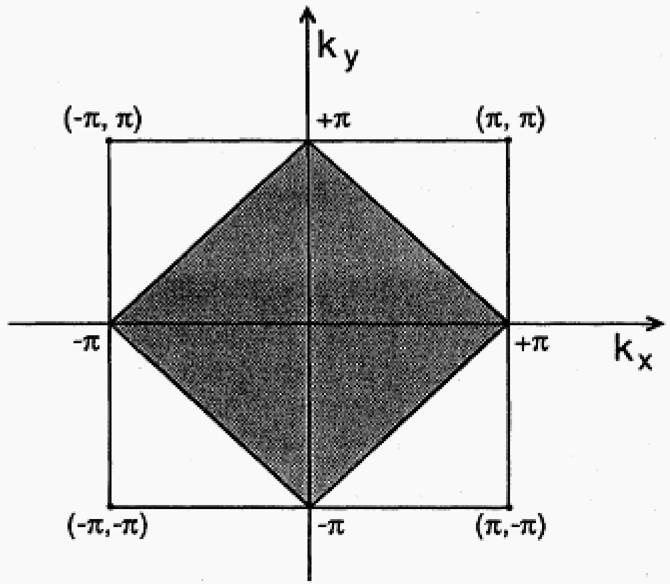
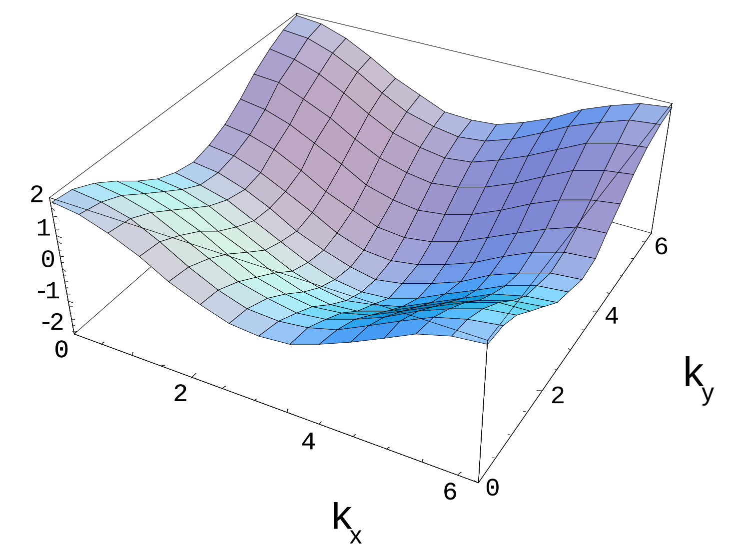
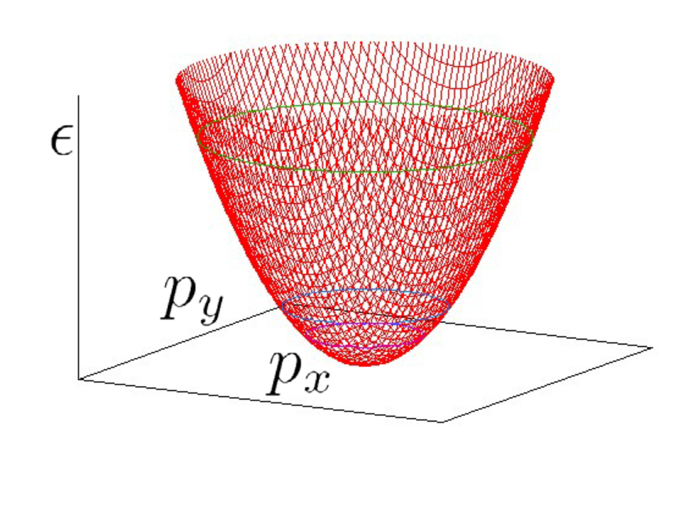


FIG. 9. The Brillouin zone of the original square lattice and the Brillouin zone (shaded area) that corresponds to the magnetic unit cell in the real space.



$$E_k = 2t(\cos k_x + \cos k_y)$$

Perfect nesting



The Hartree-Fock decoupling

$$\langle \hat{c}^{\dagger}_{i,\sigma} \hat{c}_{i,\sigma} \rangle = \frac{1}{2}(n + (-1)^i \sigma M)$$

M is the staggered magnetization.

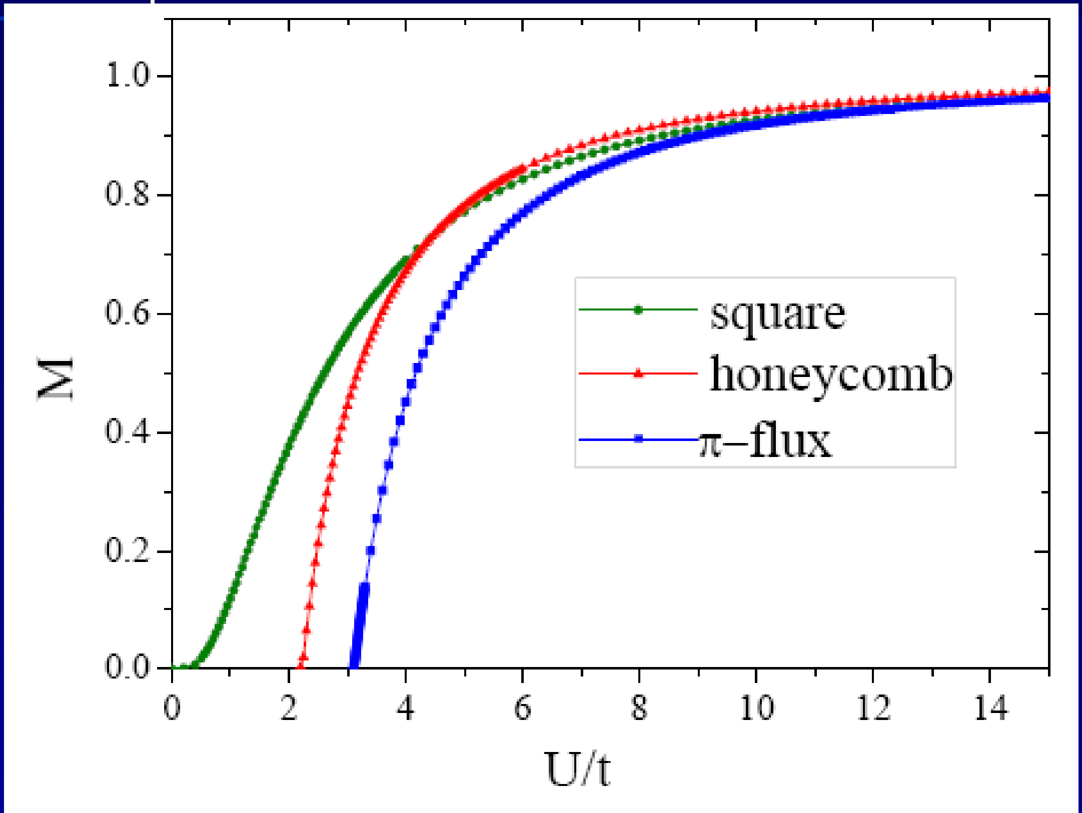
The Hamiltonian can be written as in the Momentum space at half-filling

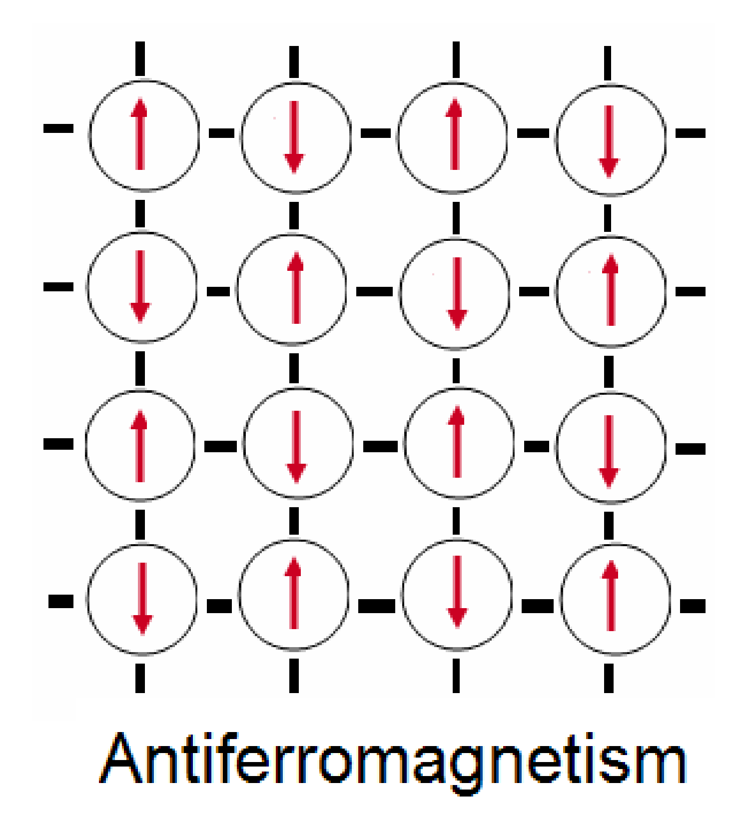
$$\hat{H} = \sum_{\mathbf{k},\sigma} [\, \hat{a}^{\dagger}_{k,\sigma} \;\; \hat{b}^{\dagger}_{k,\sigma} \,] \begin{bmatrix} -\sigma^z \Delta & \xi_k \\ \xi^*_k & \sigma^z \Delta \end{bmatrix} \begin{bmatrix} \hat{a}_{k,\sigma} \\ \hat{b}_{k,\sigma} \end{bmatrix}$$

where $\xi_k = -\sum_{\delta} t_{k,\delta} e^{i\mathbf{k}\cdot\delta}$

and $\Delta = \frac{UM}{2}$ denotes the energy band gap

The staggered magnetization: M



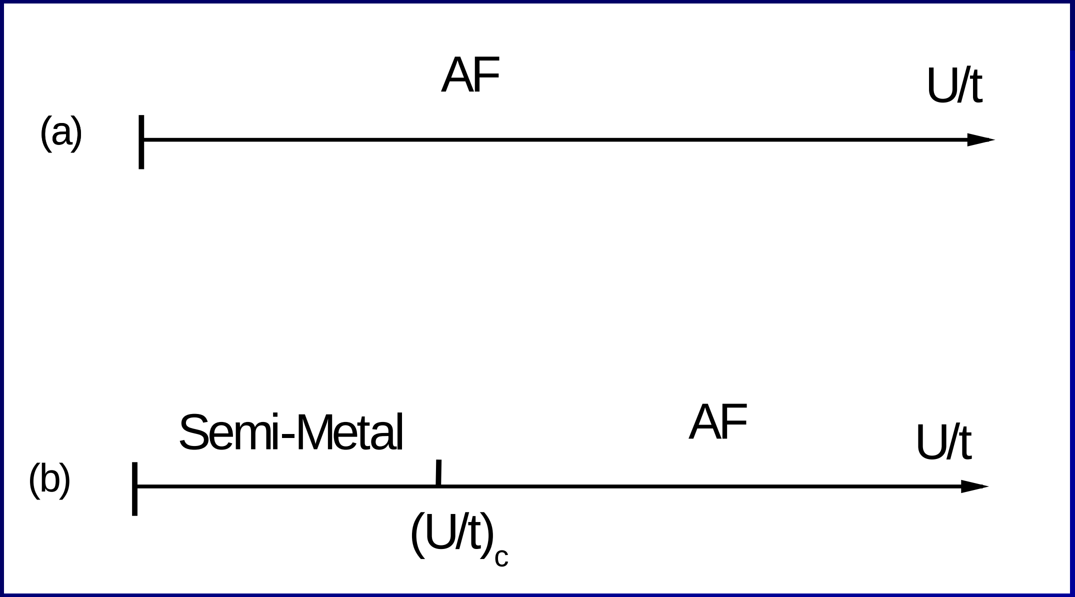


$M \neq 0$

Ground states of the Hubbard models?

Note : M≠0 not necessarily imply that there is long range AF order, for in mean field theory, the direction of the spin is chosen to be fixed along z-axis.

One need to examine stability of magnetic order against quantum fluctuations of effective spin moments based on a formulation keeping spin rotation symmetry.



No real Mott transition

$$\sigma_z \rightarrow n \cdot \sigma$$

The NLσM is obtained as by integrating out canting field L_i and fermion field

$$\mathcal{S}_{\mathrm{NL}\sigma\mathrm{M}} = \frac{1}{2g}\int_0^\beta d\tau \int d^2r [\frac{1}{c}\dot{\mathbf{n}}^2 + c(\nabla\mathbf{n})^2]$$

Transverse spin susceptibility is $\frac{1}{\chi^\perp} = \frac{1}{\chi_0^\perp} - 2U$

Spin stiffness $\rho_s = \frac{c}{g}$

Spin wave velocity $c^2 = \rho_s/\chi^\perp$

One obtains spin stiffness and the transverse spin susceptibility:

$$\chi_0^\perp = \frac{1}{N}\frac{\partial^2 E_0}{\partial B_y^2}|_{B_y=0}, \ (\rho_s^x, \rho_s^y) = \frac{1}{N}\frac{\partial^2 E_0}{\partial \mathbf{a}^2}|_{\mathbf{a}=0},$$

X. G. Wen, Quantum Field Theory of Many-Body Systems, (Oxford Univ. Press, Oxford, 2004)

Quantum phase diagram of NLσM

If $g < g_c$ the ground state is renormalized classical (RC) , the spin order parameter is

$$n_0 = \langle \mathbf{n} \rangle = 1 - g/g_c$$

If $g > g_c$ the ground stateis the quantum disordered, and the order parameter is

$$n_0 = \langle \mathbf{n} \rangle = 0$$

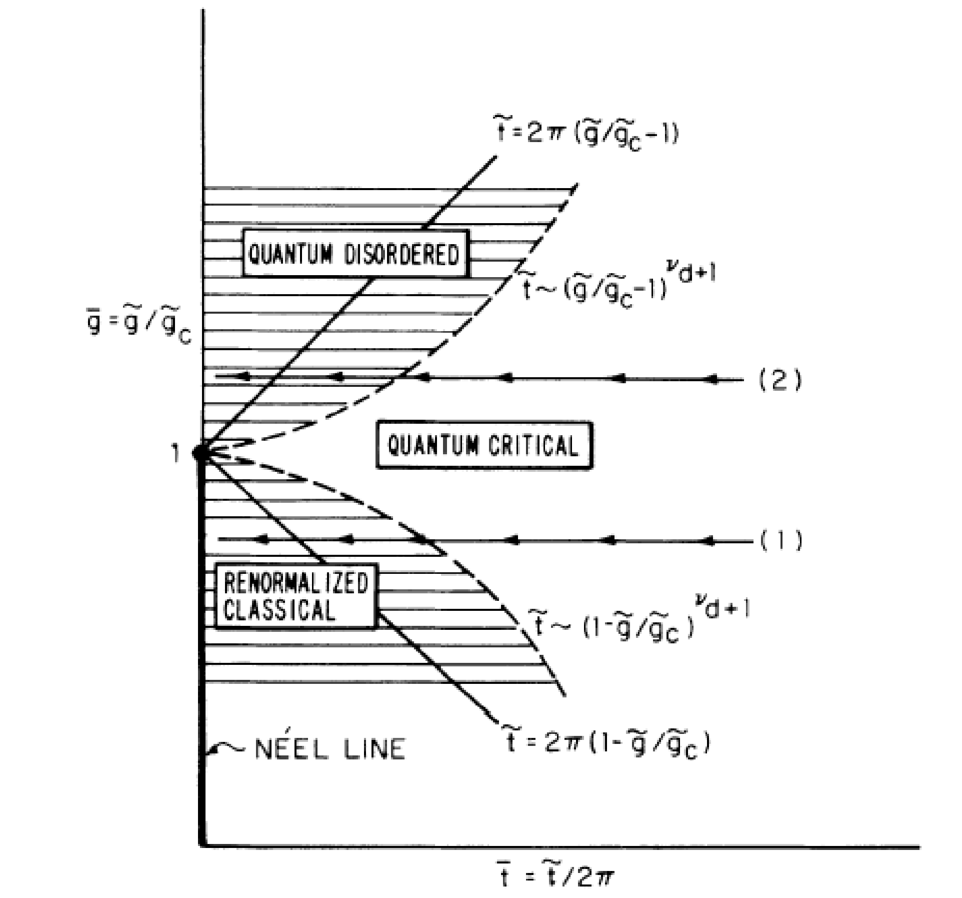
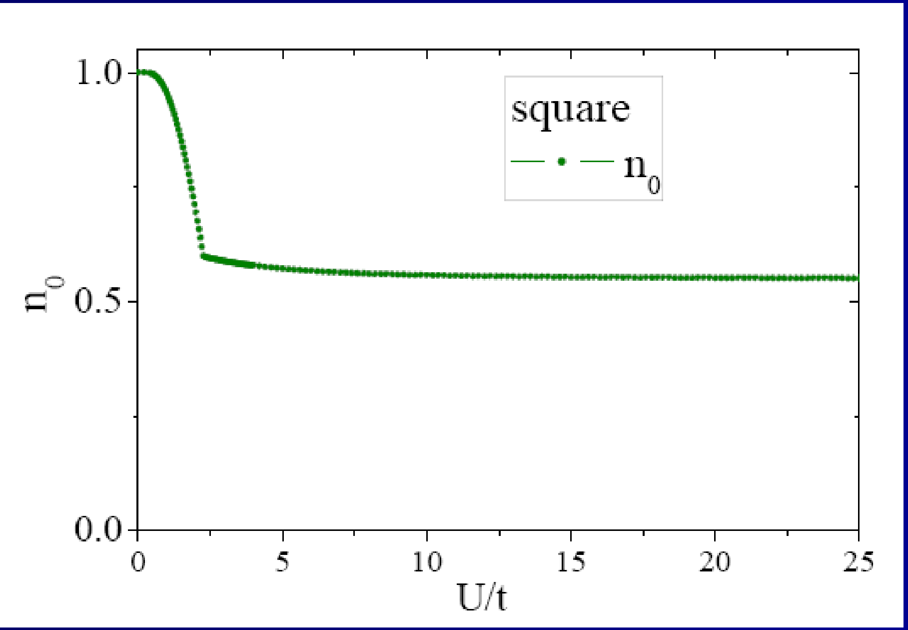


FIG. 1. Crossover phase diagram for $d=2$. $\nu_{d+1}=0.7$ for $d=2$. $\tilde{g}_c$ is the critical point of the $(d+1)$-dimensional nonlinear σ model.

S. Chakravarty, et al., Phys. Rev. B 39, 2344 (1989).

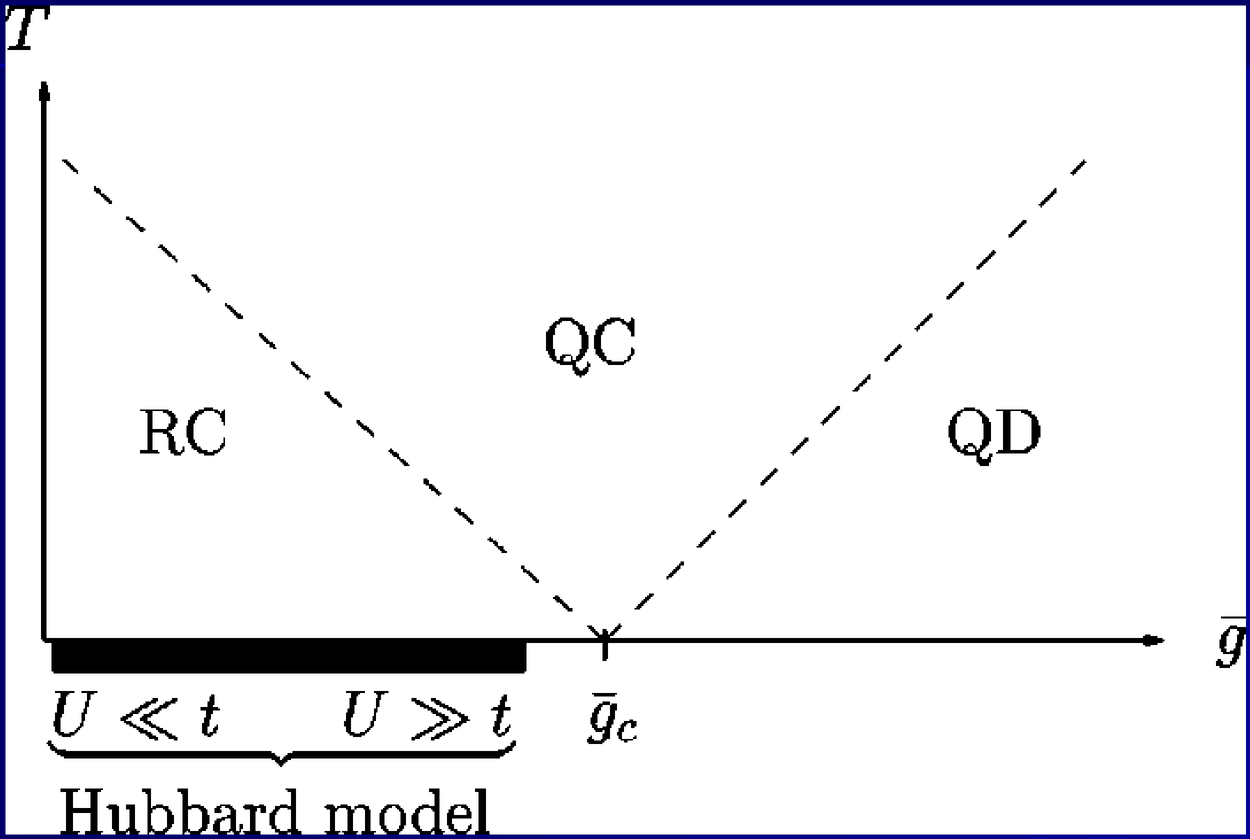
Results of the Hubbard model on square lattice of spin-fluctuation theory

The fraction condensed spin moments:



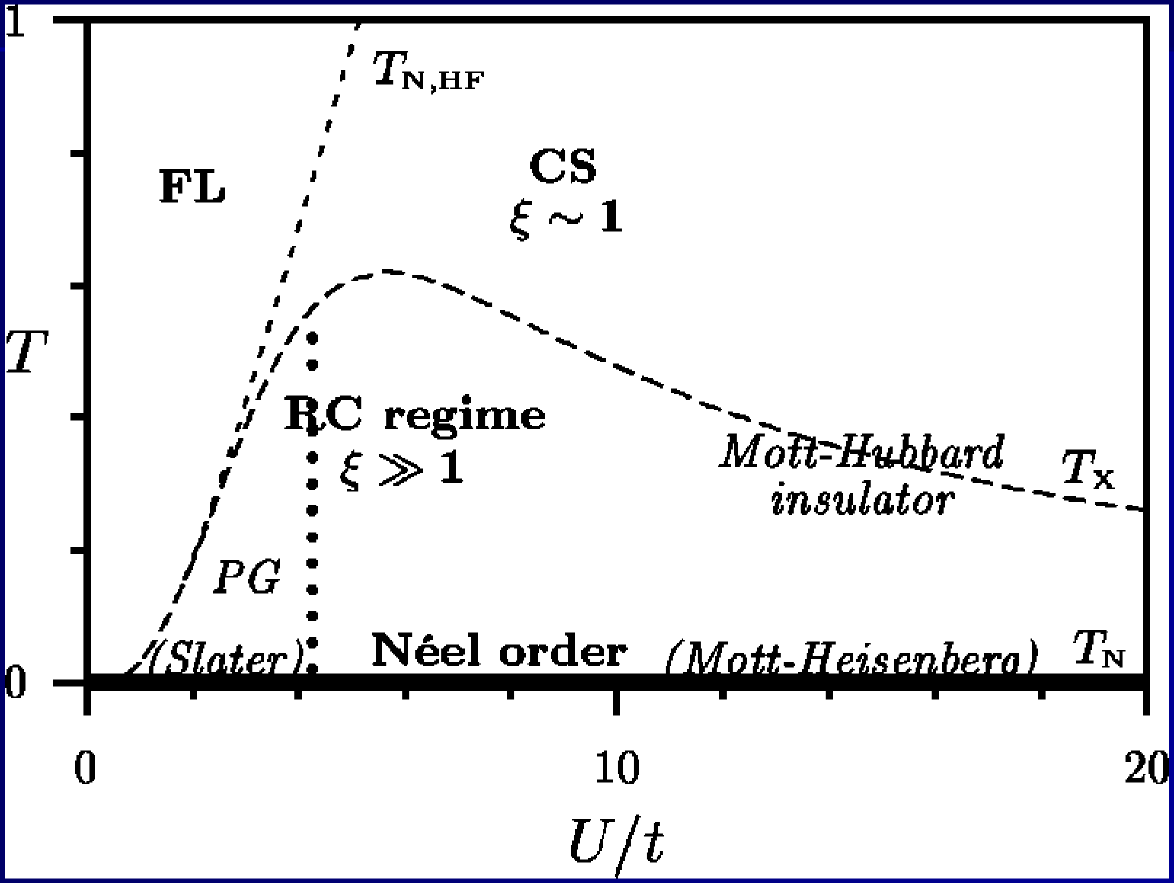
Note: all are renormalized classical (RC) regime because $n_0 \neq 0$

The scheme of the coupling constant



K. Borejsza, N. Dupuis, Euro Phys. Lett. 63, 722 (2003).
K. Borejsza and N. Dupuis Phys. Rev. B 69, 085119 (2004).

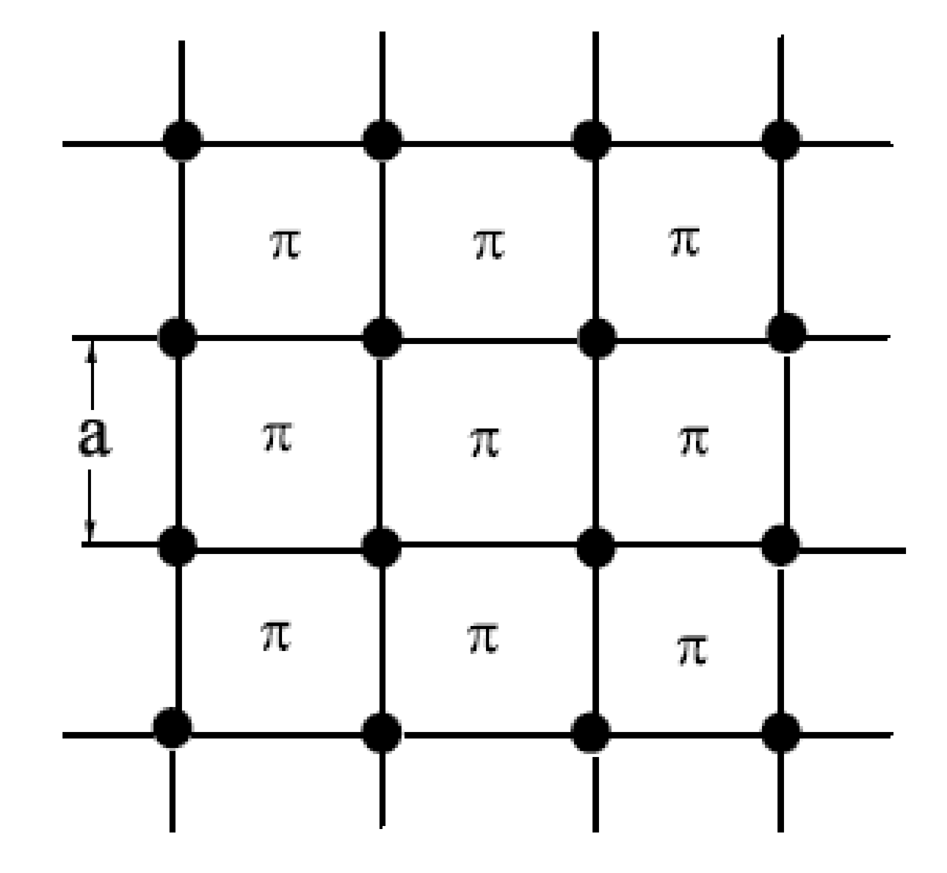
The global phase diagram



K. Borejsza, N. Dupuis, Euro Phys. Lett. 63, 722 (2003).
K. Borejsza and N. Dupuis Phys. Rev. B 69, 085119 (2004).

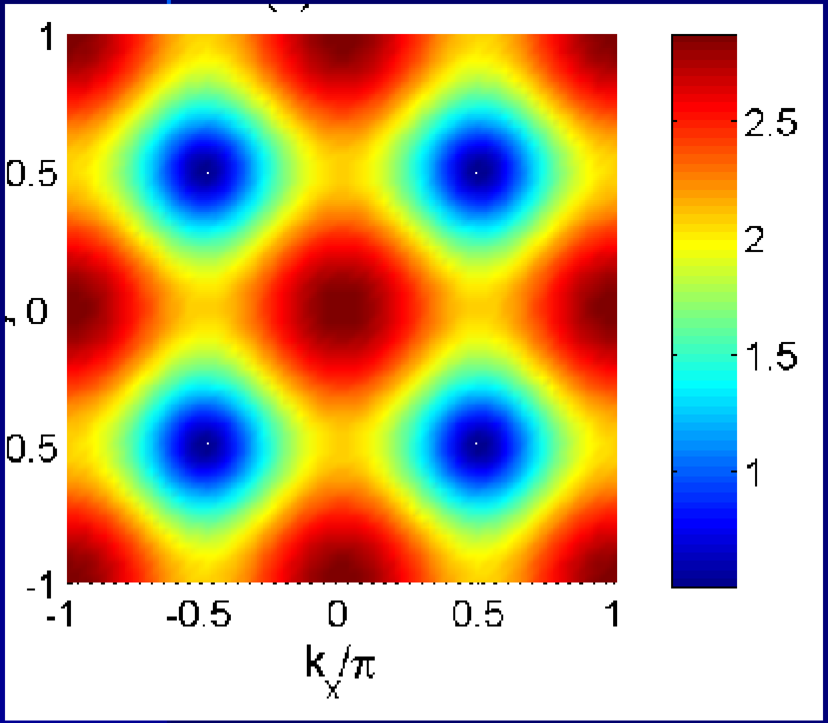
2. Nodal spin liquid in π-flux Hubbard model on suqare lattice

$$\mathcal{H} = -\sum_{\langle i,j\rangle,\sigma} t_{ij}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_{j}\hat{n}_{j\uparrow}\hat{n}_{j\downarrow} - \mu\sum_{i,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + h.c.$$



$$t_{i,i+e_x} = t, \quad t_{i,i+e_y} = \pm it$$

Dispersion of free fermion in π-flux phase on a square lattice



$$E_k = 2t\sqrt{\cos^2 k_x + \cos^2 k_y}$$

No nesting

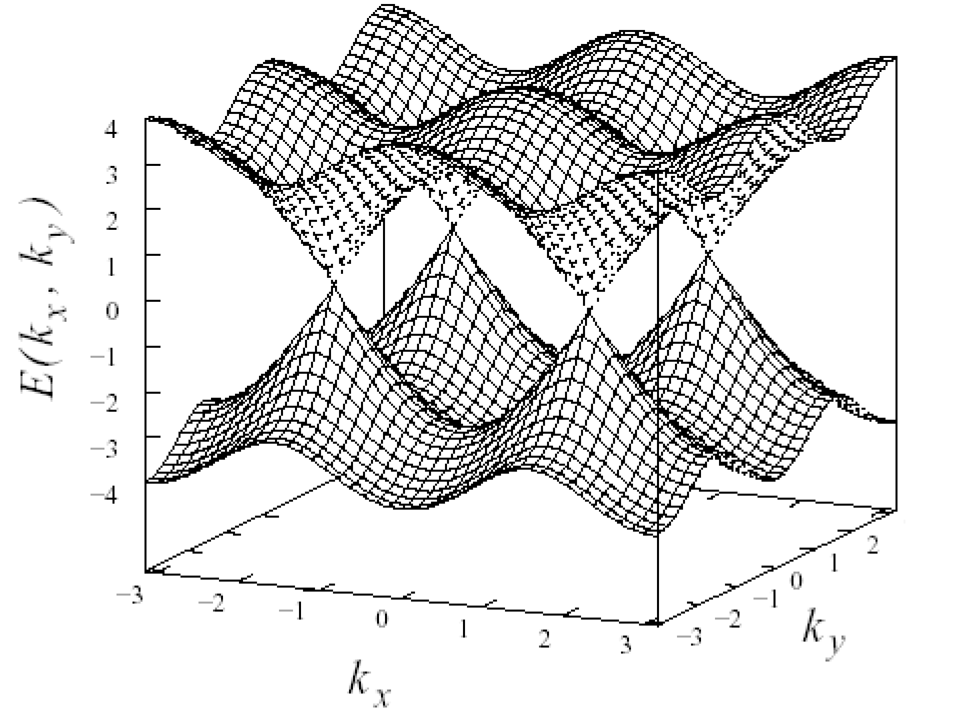
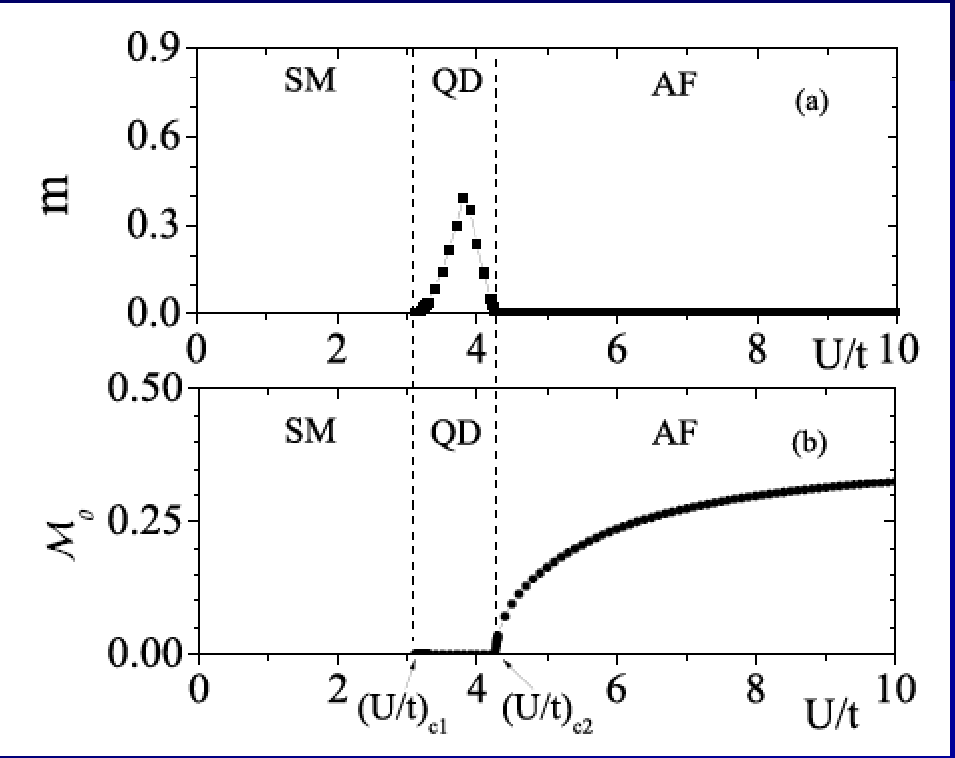


FIG. 19 The energy dispersion of the π flux phase. Note the massless Dirac spectrum at the nodal points at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$.

Results of the π-flux Hubbard model on square lattice

The fraction condensed spin moments:

Sun GY and Kou SP, EPL, 87 67002 (2009).



Note: 0<U/t<3.11, metal

3.11<U/t<4.26, quantum disordered (QD), $n_0 = 0$

U/t>4.26, renormalized classical (RC) $n_0 \neq 0$

Question?

What is the nature of the quantum non-magnetic states?

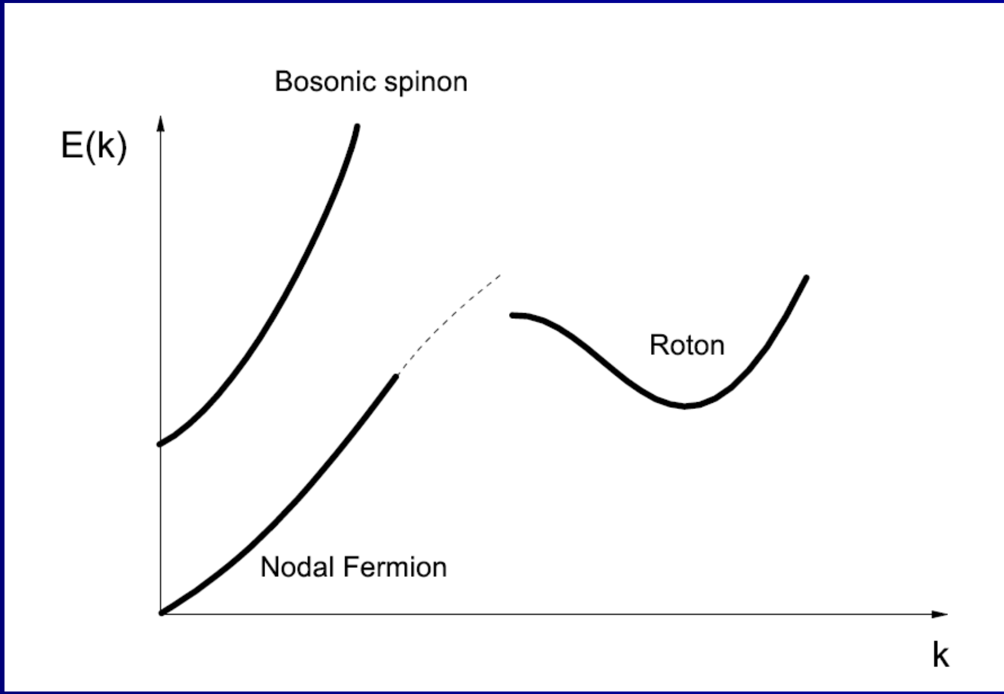
$$\mathcal{L}_{\mathbf{n}} = \frac{\Lambda}{2g}\left[(\partial_\mu \mathbf{n})^2 + m^2\mathbf{n}^2\right]$$

$$\mathcal{L}_{\mathbf{n}} = \frac{2\Lambda}{g}\left[|(\partial_\mu - ia_\mu)\mathbf{z}| + m_z^2\mathbf{z}^2\right]$$

Nodal spin liquid

$$\mathcal{L}_{\rm eff} = i\sum_a \bar{\Psi}_a \gamma_\mu \partial_\mu \Psi_a + \frac{1}{2g}\left[(\partial_\mu \mathbf{z})^2 + m_z^2 \mathbf{z}^2\right] - \frac{1}{4e_a^2}(\partial_\nu a_\mu)^2 + \frac{\sqrt{p^2}}{\pi^2} a_\mu^2.$$

There are three types of quasi-particles : *gapless fermionic spinons, gapped bosonic spinons and the roton-like gauge field*.



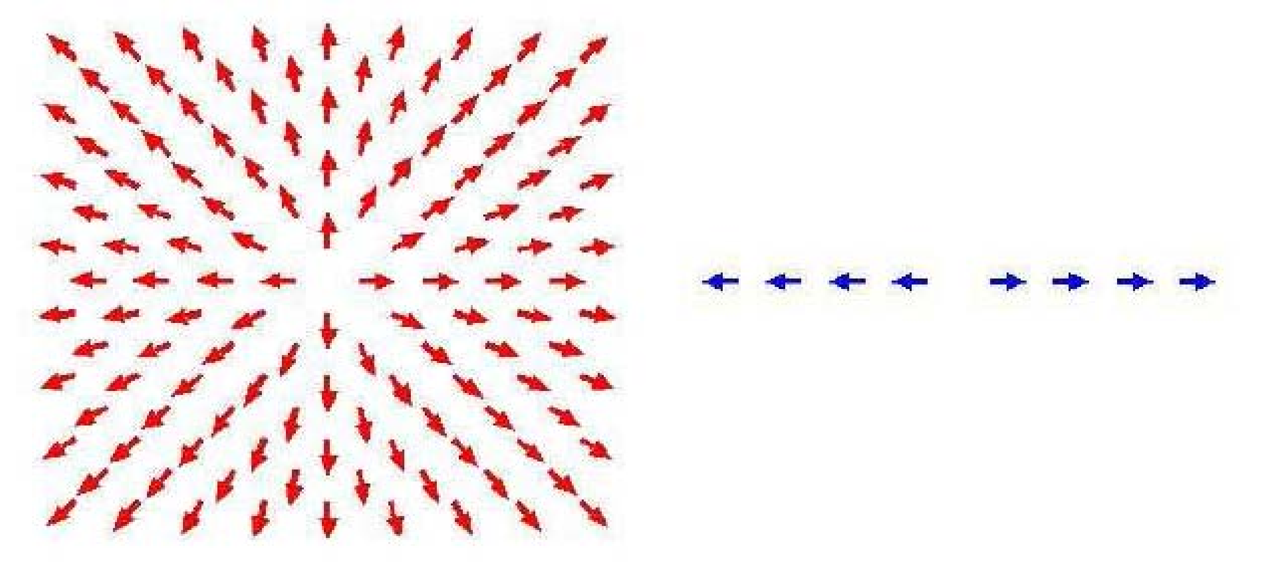
Nodal spin liquid is not algebra spin liquid.

Gapless spin-vortex as quasi-particle

- With trapping a ½ staggered spin moment,
- a π spin vortex becomes a fermion!

$$S^z_{(\pi,\pi)} = \pm\frac{1}{2}$$

- The quantum disordered state becomes a spin liquid state with emergent (deconfined) fermionic excitations.

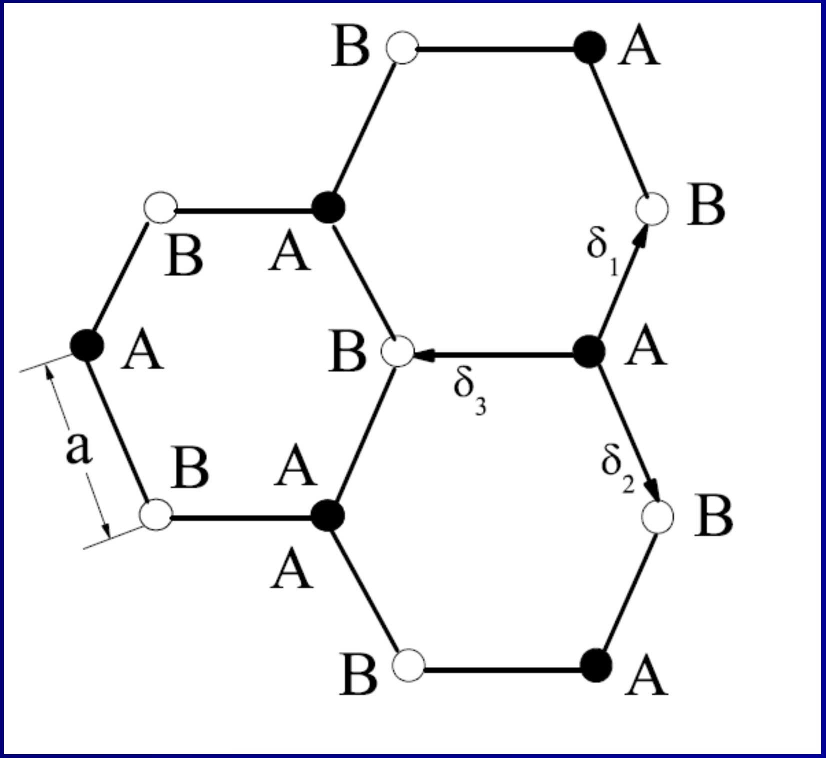


S.P. Kou, PHYS. REV. B 78, 233104 (2008).

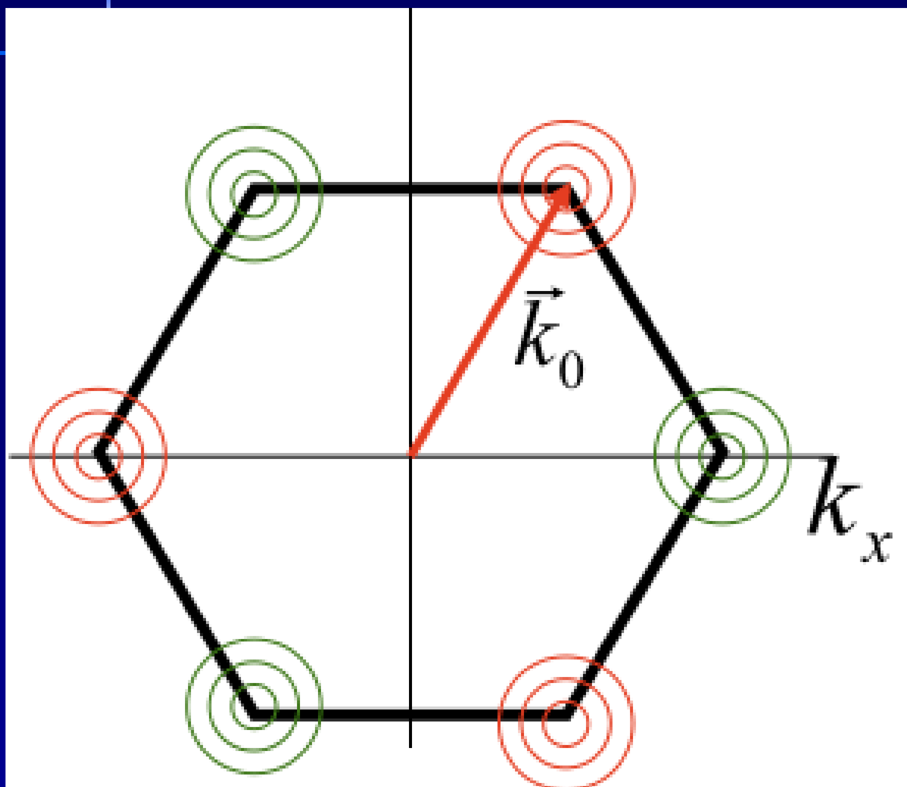
3. Spin liquid in the Hubbard model on honeycomb lattice

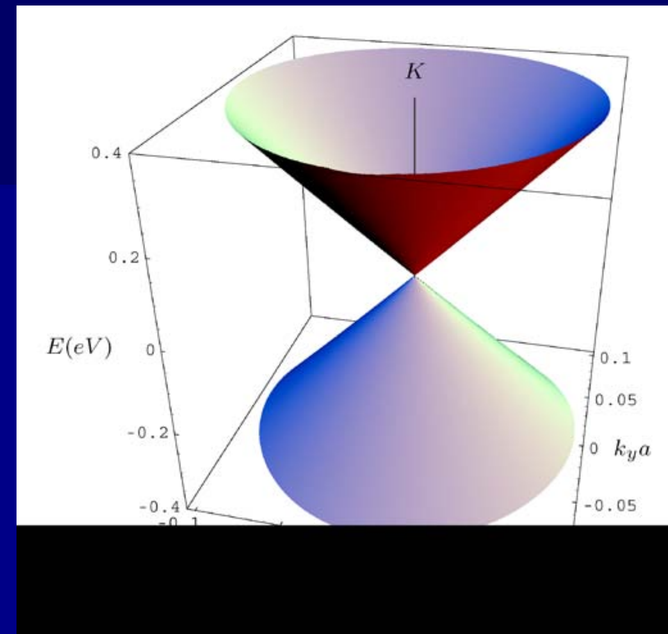
$$\mathcal{H} = -\sum_{\langle i,j\rangle,\sigma} t_{ij}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_{j}\hat{n}_{j\uparrow}\hat{n}_{j\downarrow} - \mu\sum_{i,\sigma}\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + h.c.$$

$$t_{i,j} = t$$



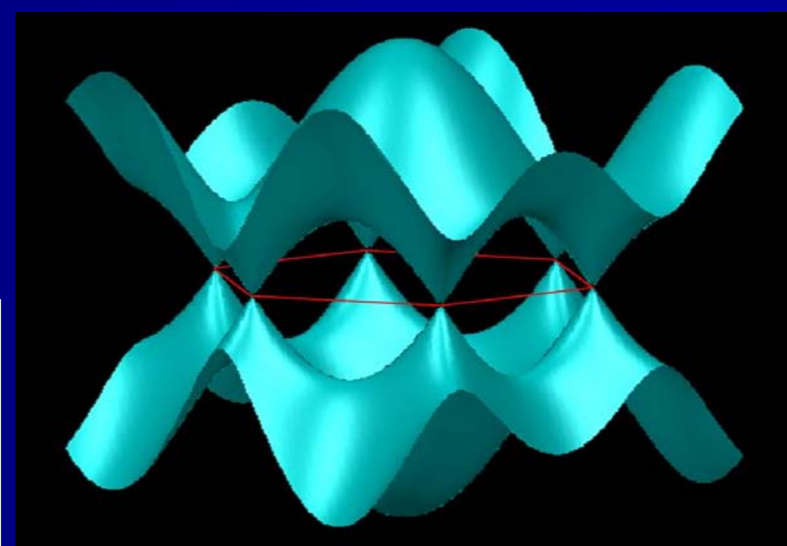
Dispersion of free fermion on a honeycomb lattice



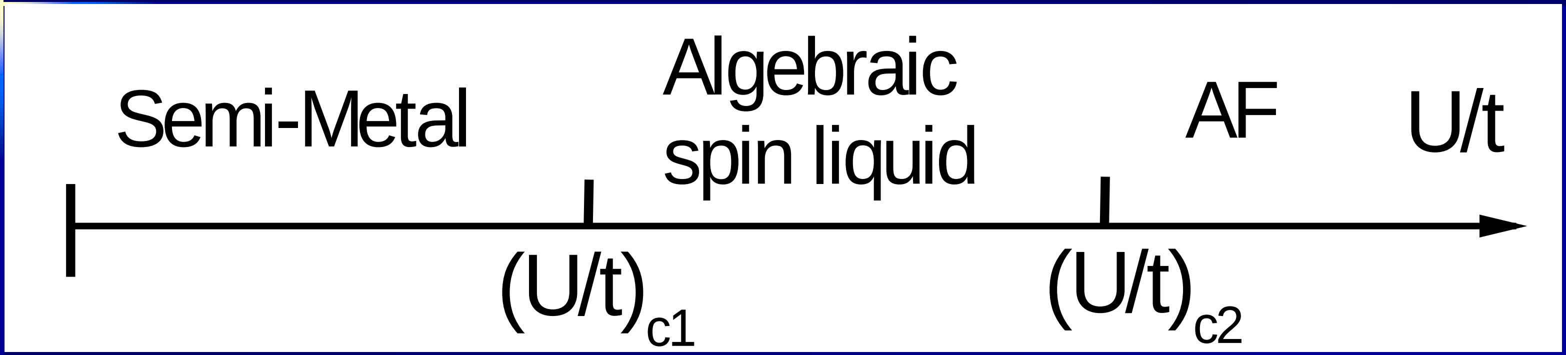


$$|\xi_{\mathbf{k}}| = \left| -t \sum_{\delta} e^{i\mathbf{k}\cdot\delta} \right|$$

$$= t\sqrt{3 + 2\cos\left(\sqrt{3}k_y\right) + 4\cos\left(3k_x/2\right)\cos\left(\sqrt{3}k_y/2\right)}$$



Algebraic spin liquid by slave-rotor theory



Slave-roton approach

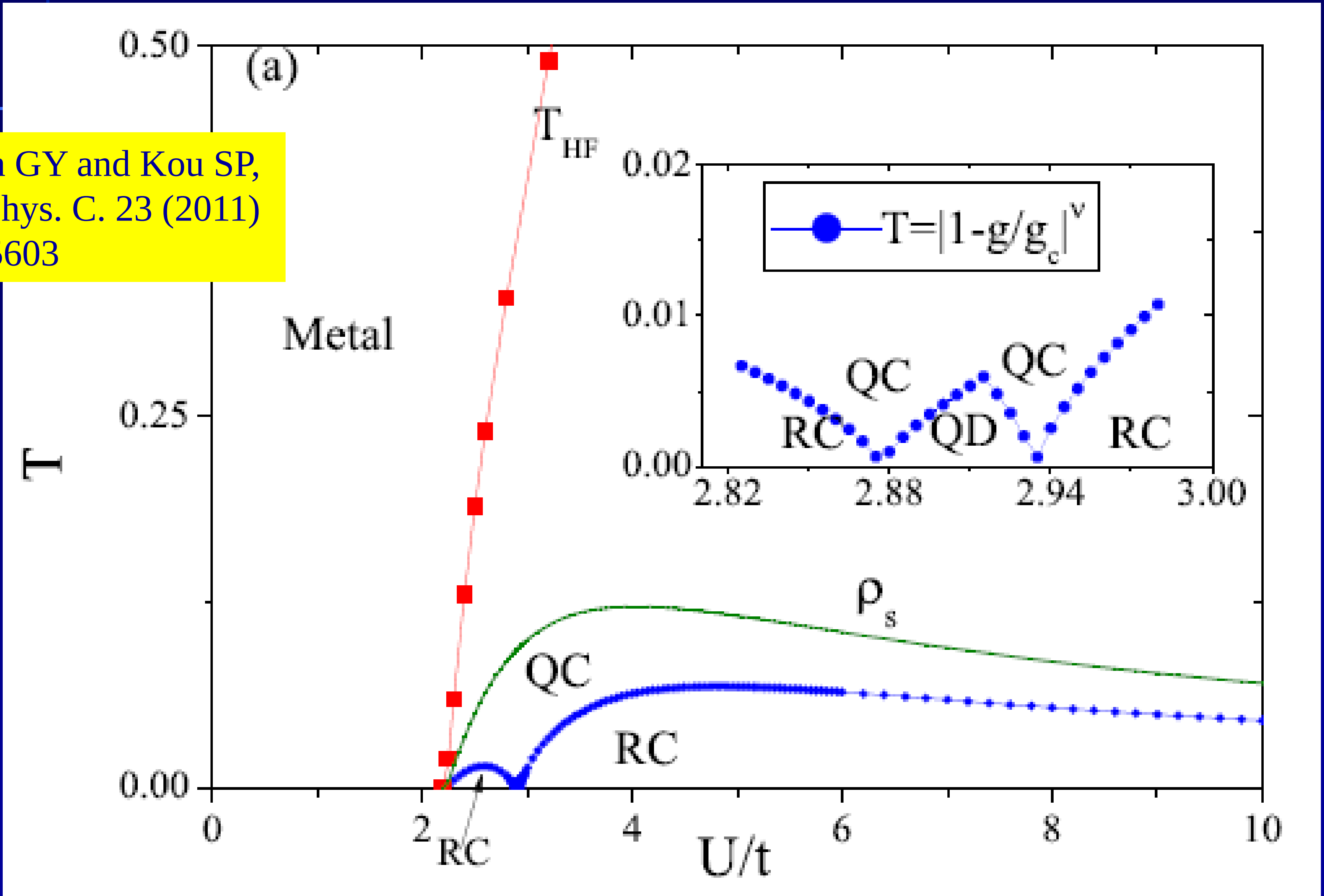
$$S_{\mathrm{ASL}} = \int d^2r d\tau \left[\overline{\Psi}\gamma^{\mu}(\partial_{\mu} - iA_{\mu})\Psi \right]$$

$$\hat{c}_{j\sigma} = e^{i\theta_j} \hat{f}_{j\sigma}$$

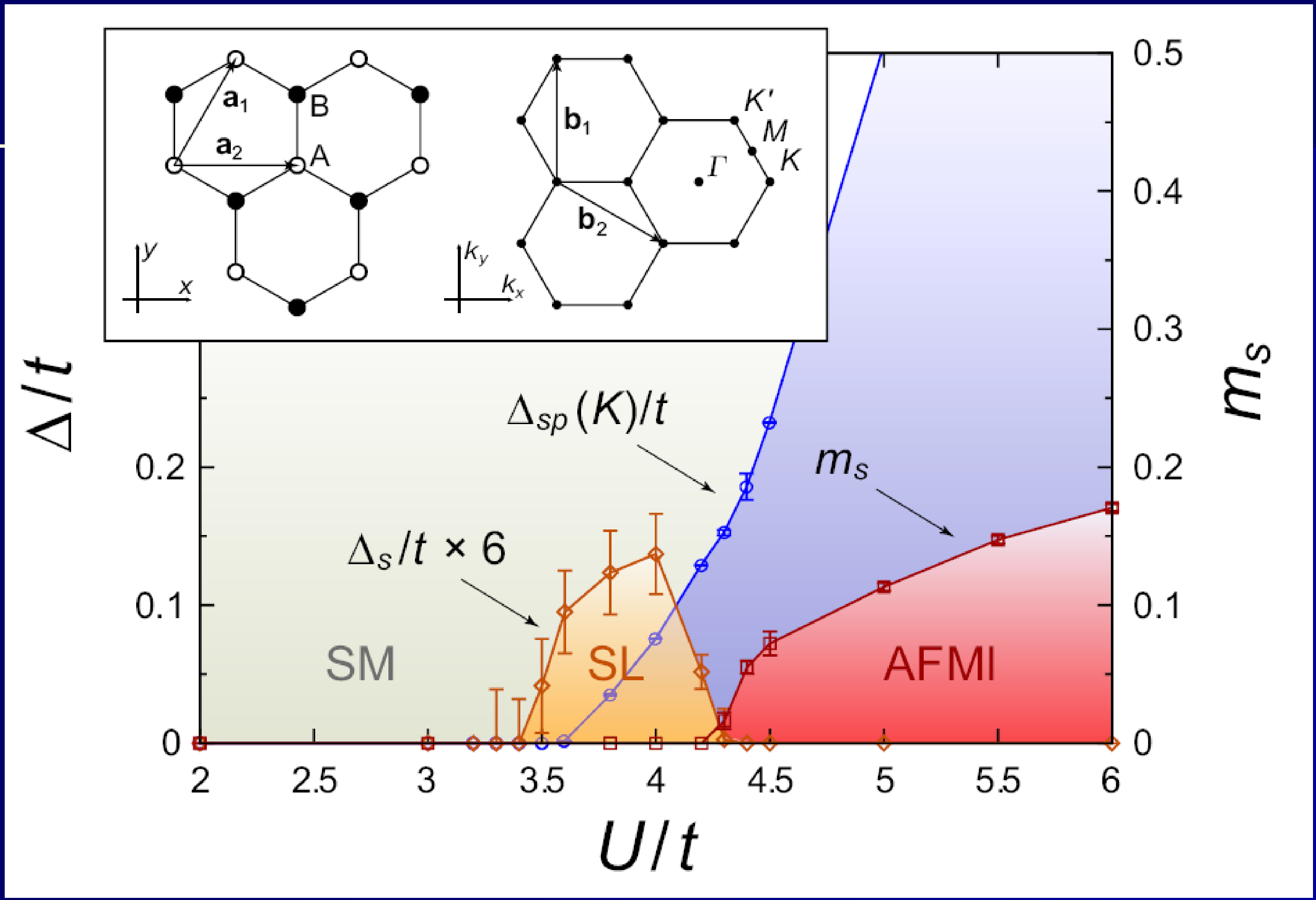
M. Hermele, *Phys. Rev. B* **76**, 035125 (2007).

Global Phase diagram by spin-fluctuation theory

Sun GY and Kou SP, J. Phys. C. 23 (2011) 045603

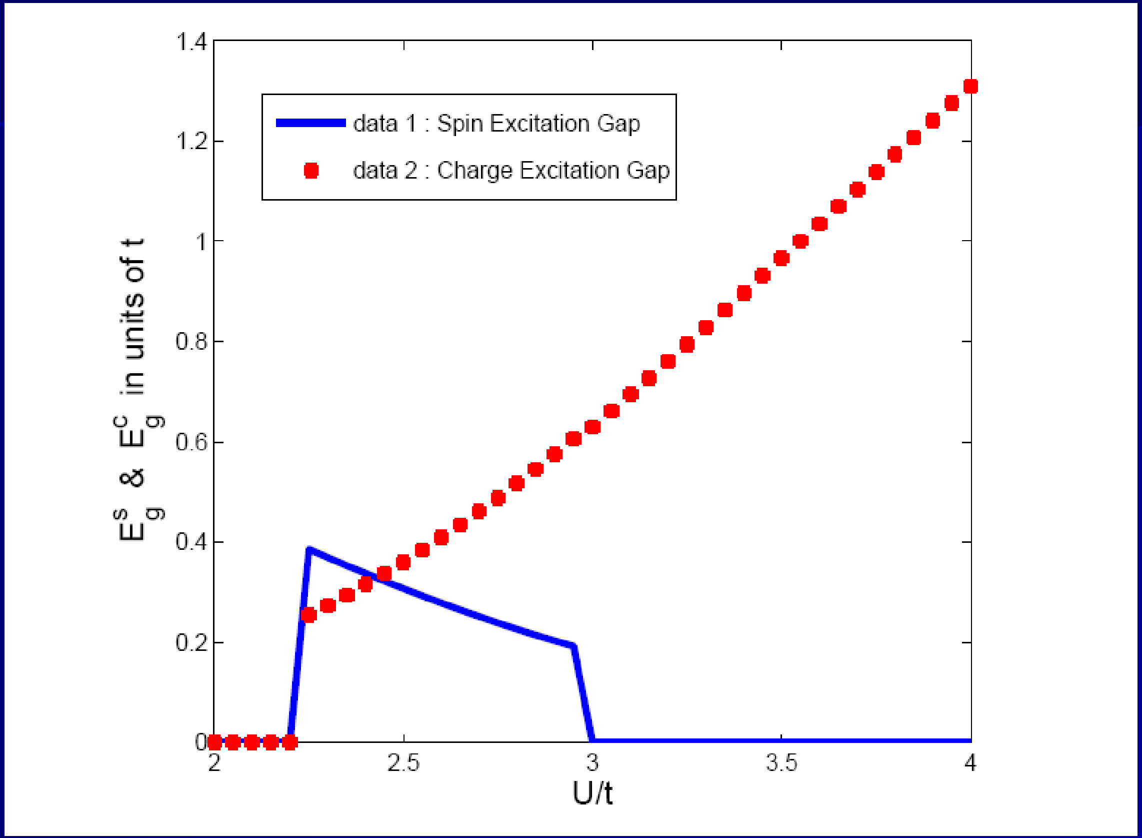


Quantum spin liquid from QMC



Z. Y. Meng, T. C. Lang, S. Wessel, F. F. Assaad & A. Muramatsu
Nature 464, 847 (2010)

Slave-particle approach

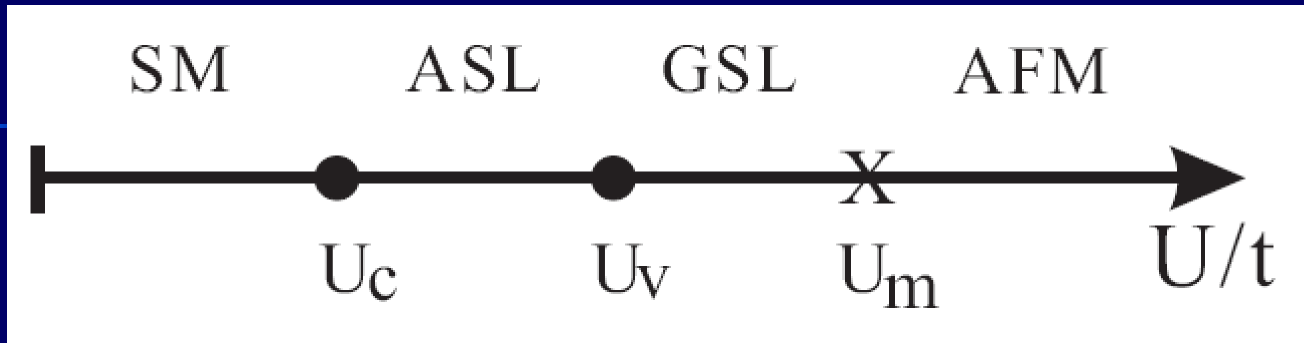


A. Vaezi and Xiao-Gang Wen, arXiv:1010.5744

electron creation operator as: $C_{i,\sigma}^{\dagger} = f_{i,\sigma}^{\dagger} h_i + \sigma\, d_i^{\dagger} f_{i,-\sigma} =$

$$\begin{bmatrix} h_i & d_i^{\dagger} \end{bmatrix} \begin{bmatrix} f_{i,\sigma}^{\dagger} \\ \sigma f_{i,-\sigma} \end{bmatrix}.$$

The SU(2) slave-rotor theory



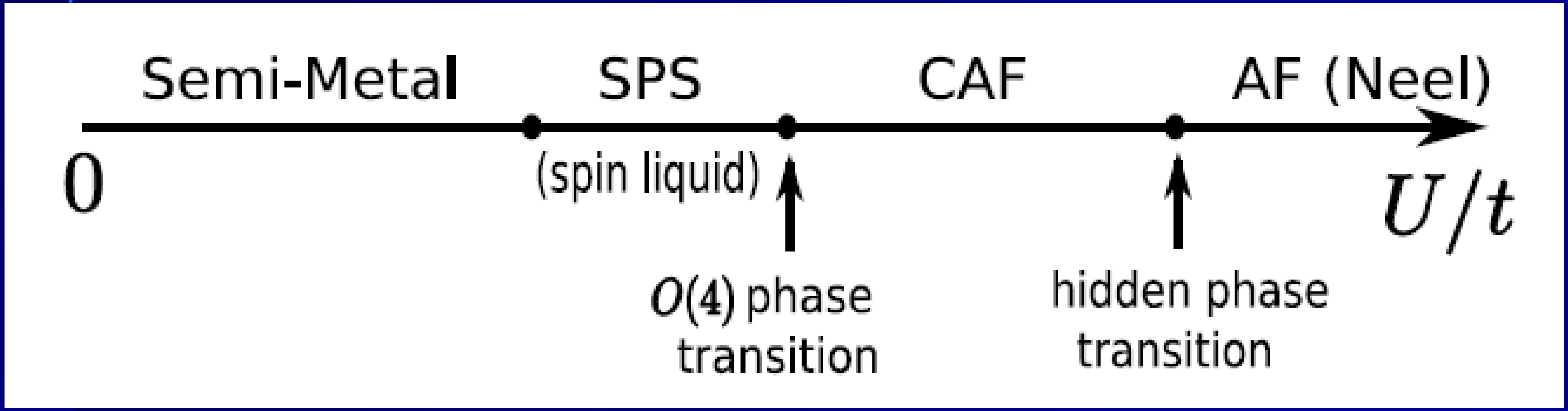
$$\psi_i = Z_i^\dagger F_i, \tag{3}$$

where $F_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$ is a fermion operator in the Nambu representation, and Z_i is an SU(2) matrix

$$Z_i = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^\dagger \\ z_{i\downarrow} & z_{i\uparrow}^\dagger \end{pmatrix}. \tag{4}$$

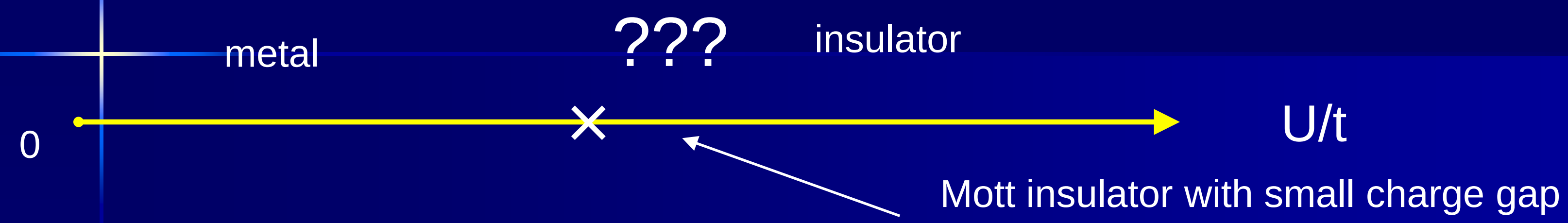
Minh-Tien Tran, Ki-Seok Kim, PhysRevB.83.125416 (2011)

PSG theory



Yuan-Ming Lu and Ying Ran, PhysRevB. 84. 024420 (2011)

Mechanism of spin liquid near Mott transition – Intrinsic frustrated spin model



$$\hat{H}_{\text{Hubbard}} = -t \sum_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Insulator --> effective spin model

$$H_{\text{spin}} = \sum_{<ij>} \left(\frac{4t^2}{U} - \frac{16t^4}{U^3} \right) \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{<<ij>>} \frac{4t^4}{U^3} \mathbf{S}_i \cdot \mathbf{S}_j + \ldots$$

Mechanism of spin liquid near Mott transition – strongly fluctuated spin-density-wave

- The spin liquid states near the MIT come from strongly quantum spin fluctuations of a relatively small effective spin-moments.

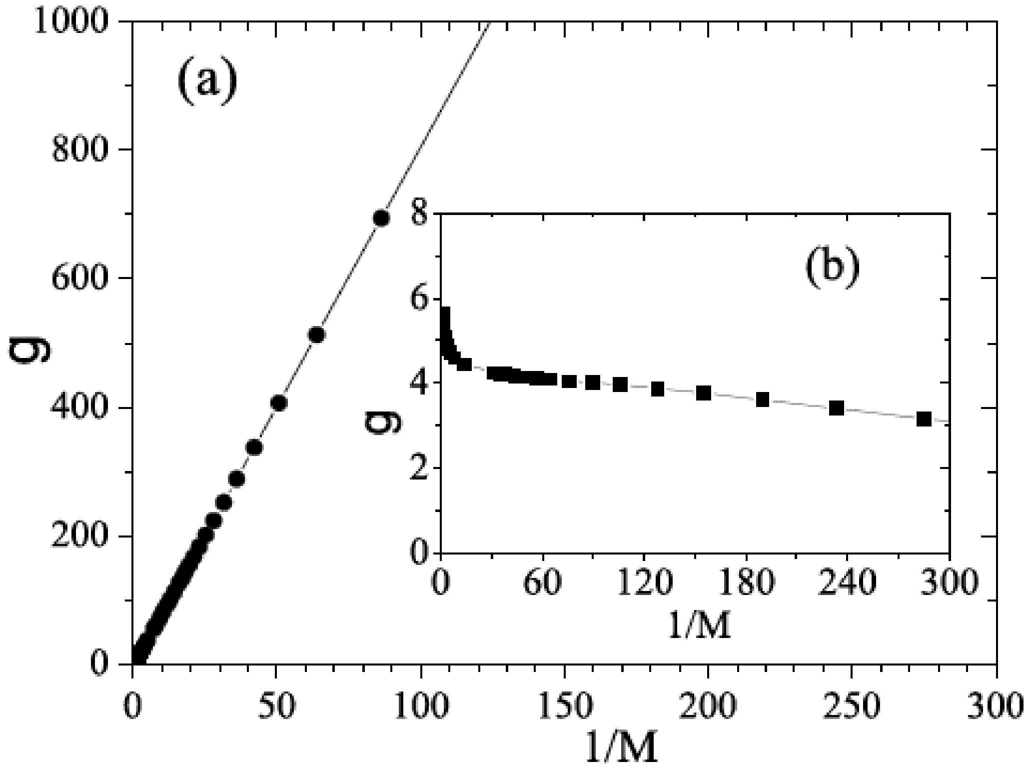


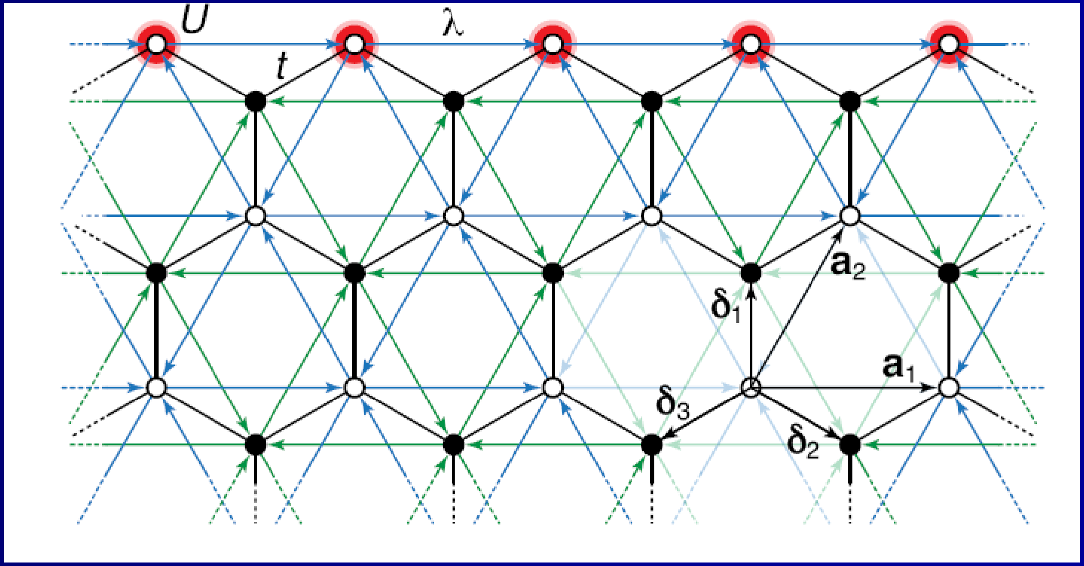
FIG. 6: Illustrations of the relations between the coupling constant g and the staggered magnetization M of the π-flux Hubbard model(cicle solid line) and the traditional Hubbard model (square solid line in inset).

Sun GY and Kou SP, EPL, 87 67002 (2009).

5. Z2 Spin liquid in interacting topological insulator – Interacting Kene-Mele model

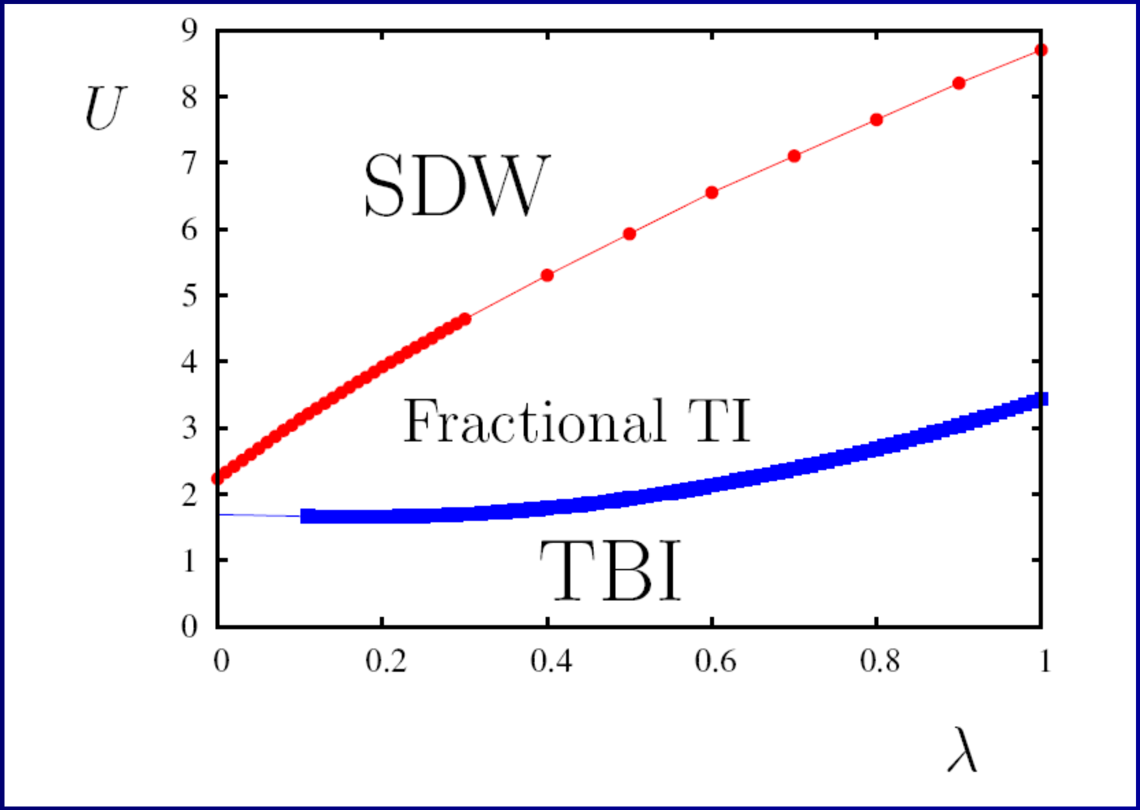
$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\sigma\sigma'} \nu_{ij}\, \sigma^{z}_{\sigma\sigma'}\, c_{i\sigma}^{\dagger} c_{j\sigma'}$$

$$\mathcal{H}_I = \frac{U}{2} \sum_{i} \left(\sum_{\sigma} n_{i\sigma} - 1 \right)^2$$

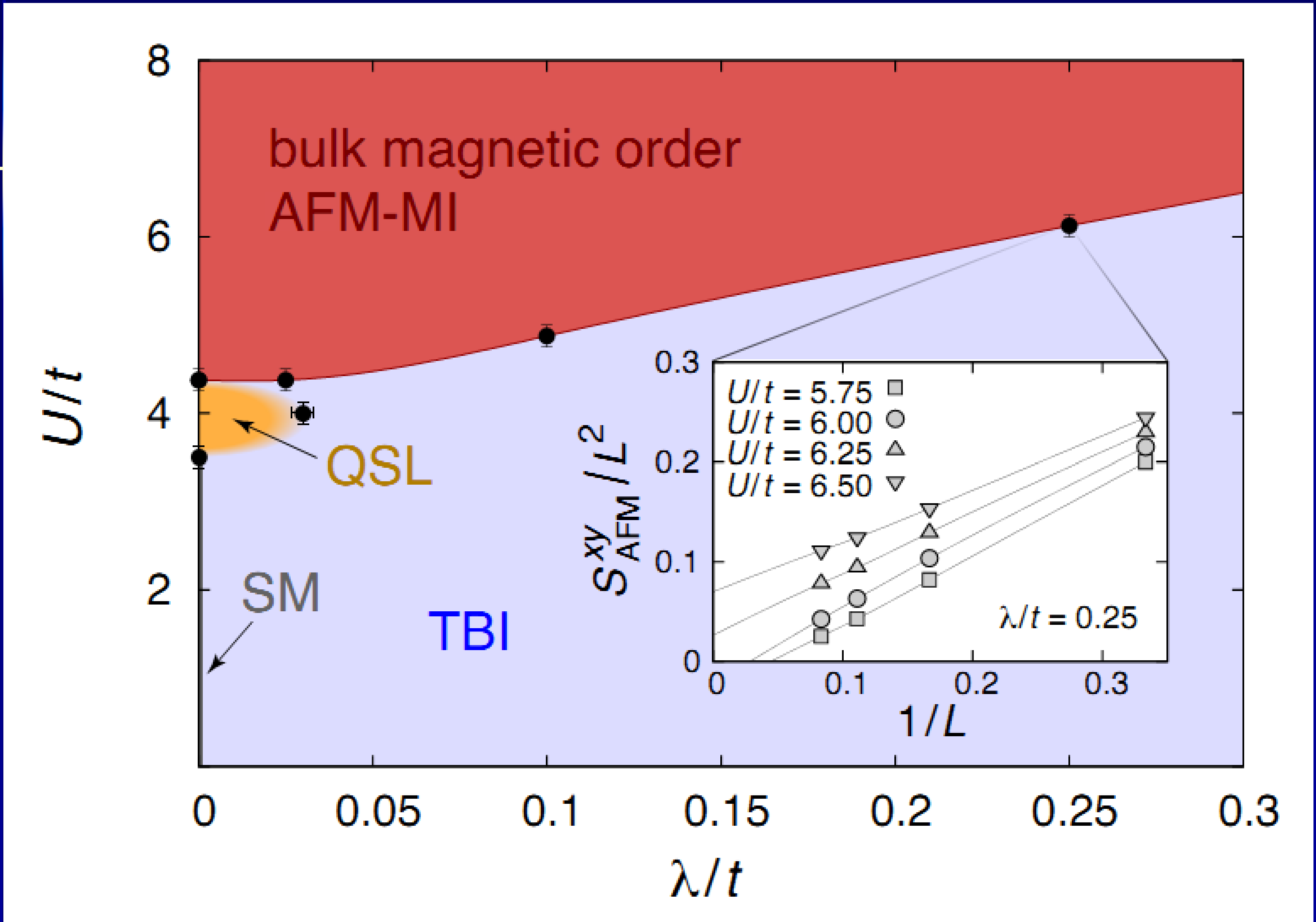


Fractional topological insulator – quantum spin liquid from slave-rotor theory

Stephan Rachel and Karyn Le Hury, PhysRevB.82. 075106 (2010)

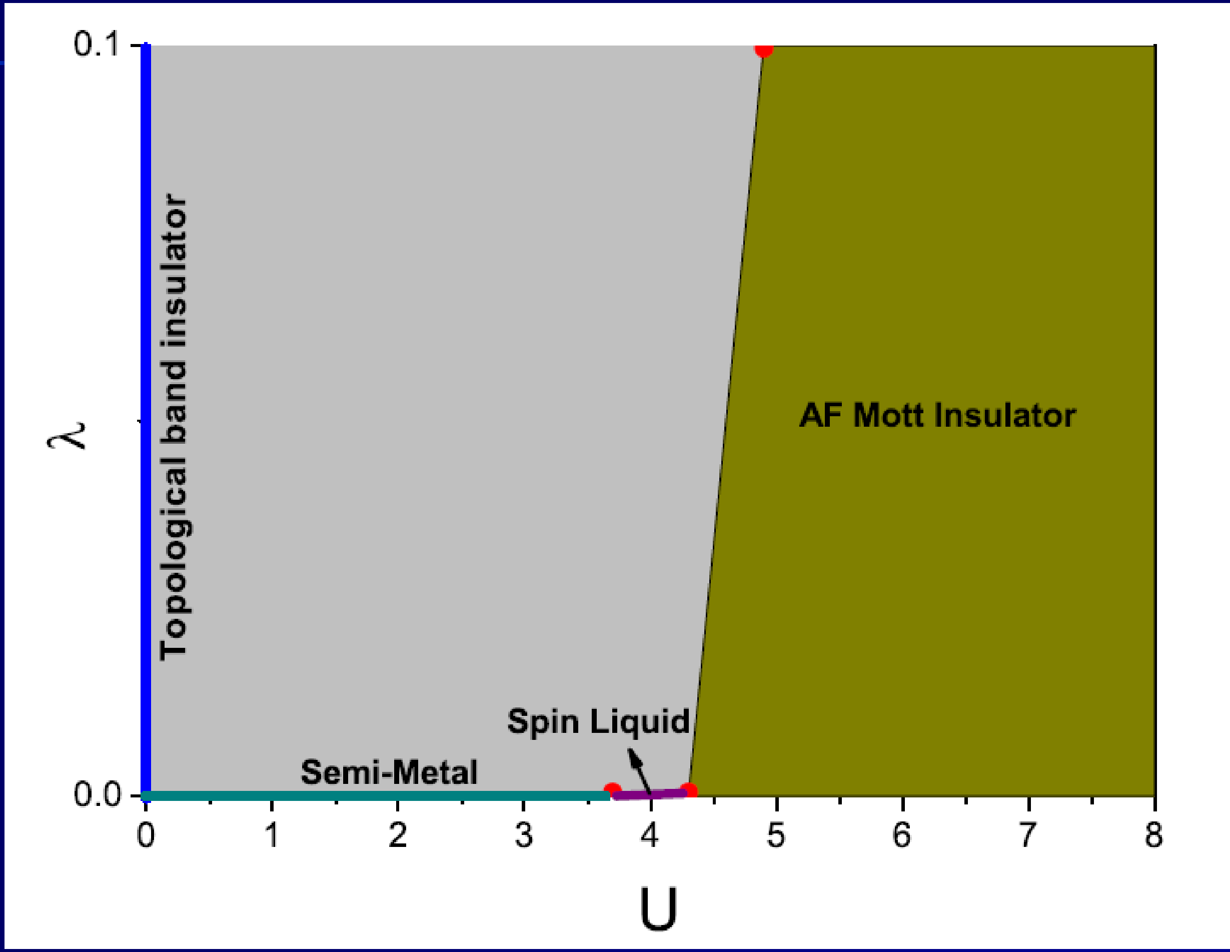


Results from QMC



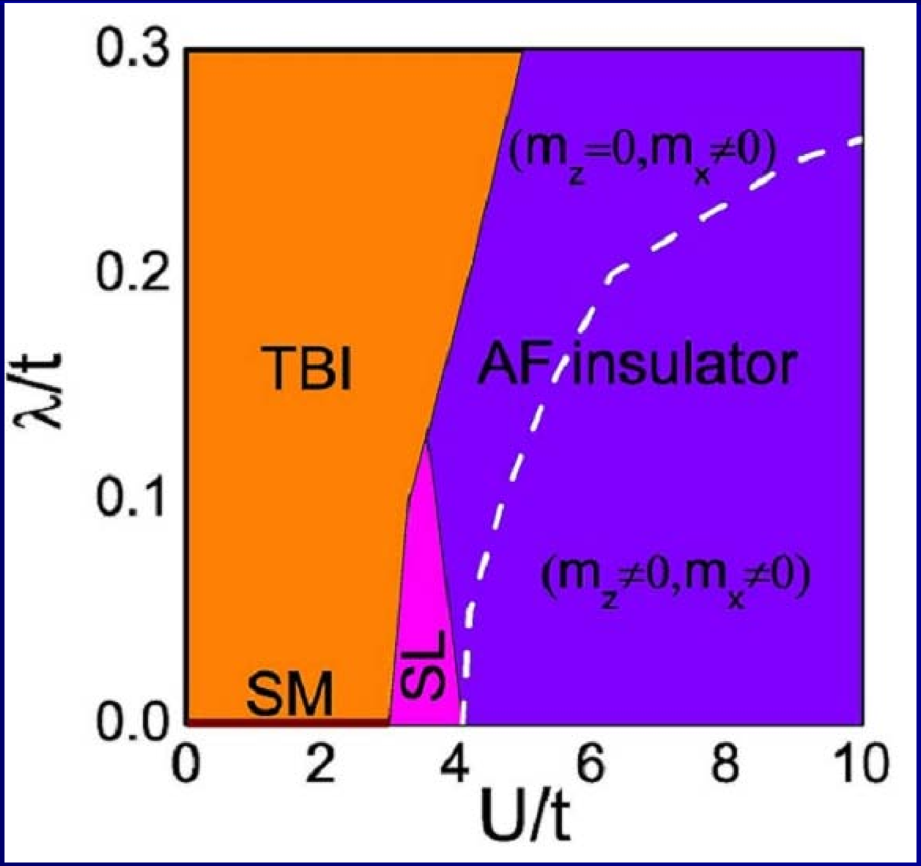
M. Hohenadler, T. C. Lang, F. F. Assaad, Phys. Rev. Lett. 106, 100403 (2011)

Results from QMC



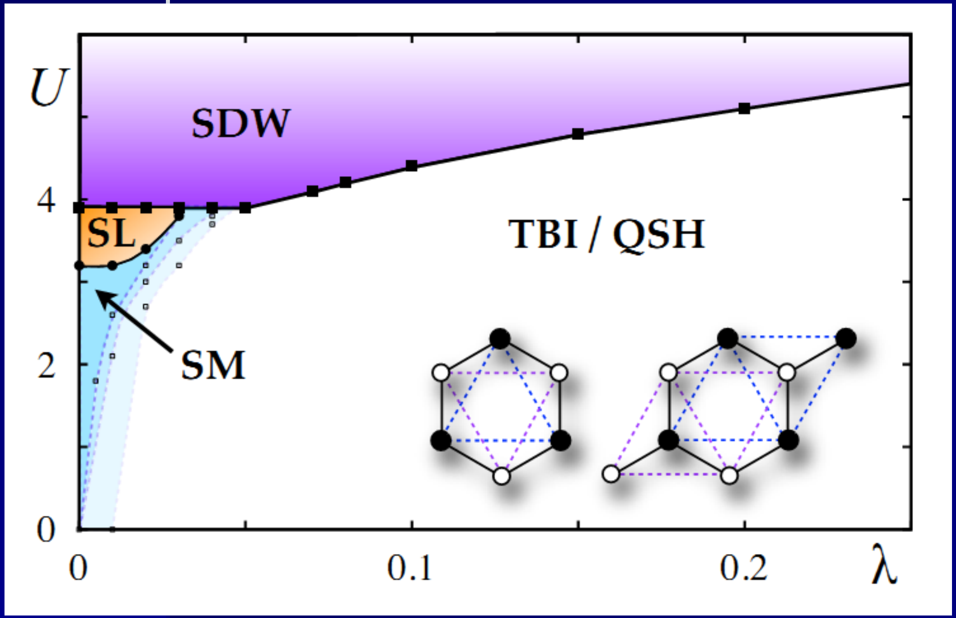
Dong Zheng, Congjun Wu and Guang-Ming Zhang, arXiv:1011.5858

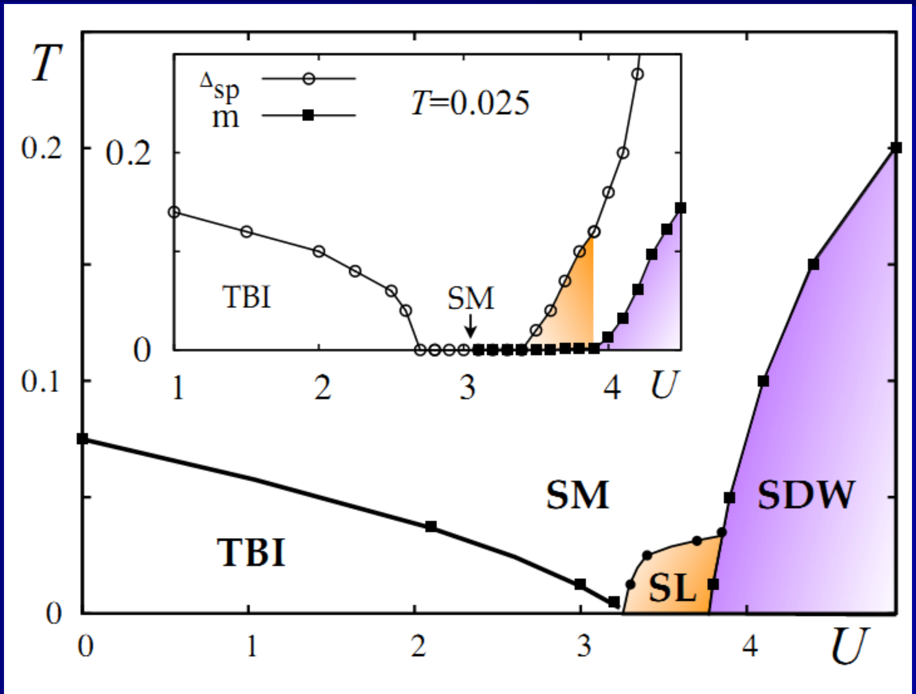
Variational cluster approach (VCA) approach



Shun-Li Yu, X.C. Xie, Jian-Xin Li, Phys. Rev. Lett. 107, 010401 (2011)

CDMFT + QMC





Wei Wu, S. Rachel, Wu-Ming Liu, K. Le Hur, **arXiv:1106.0943**

Z2 topological spin liquid – anisotropic case $\kappa > 0$

$$\mathcal{L}_{\rm eff} = i\sum_a \overline{\Psi}_a \left(\gamma_\mu\partial_\mu + i\frac{\sigma^z}{2}\gamma_\mu A_\mu + m_{hs}\right)\Psi_a + \frac{1}{2g}\left[\left|(\partial_\mu - ia_\mu)\mathbf{z}\right|^2 + m_z^2\mathbf{z}^2\right] + \frac{i}{\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda. \quad (37)$$

There are three types of quasi-particles : ***gapped fermionic spinons, gapped bosonic spinons and the gapped gauge field.***

$$\mathcal{L}_{\rm eff} = -\frac{1}{4e_A^2}(\partial_\nu A_\mu)^2 - \frac{1}{4e_a^2}(\partial_\mu a_\nu)^2 + \frac{i}{\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$$

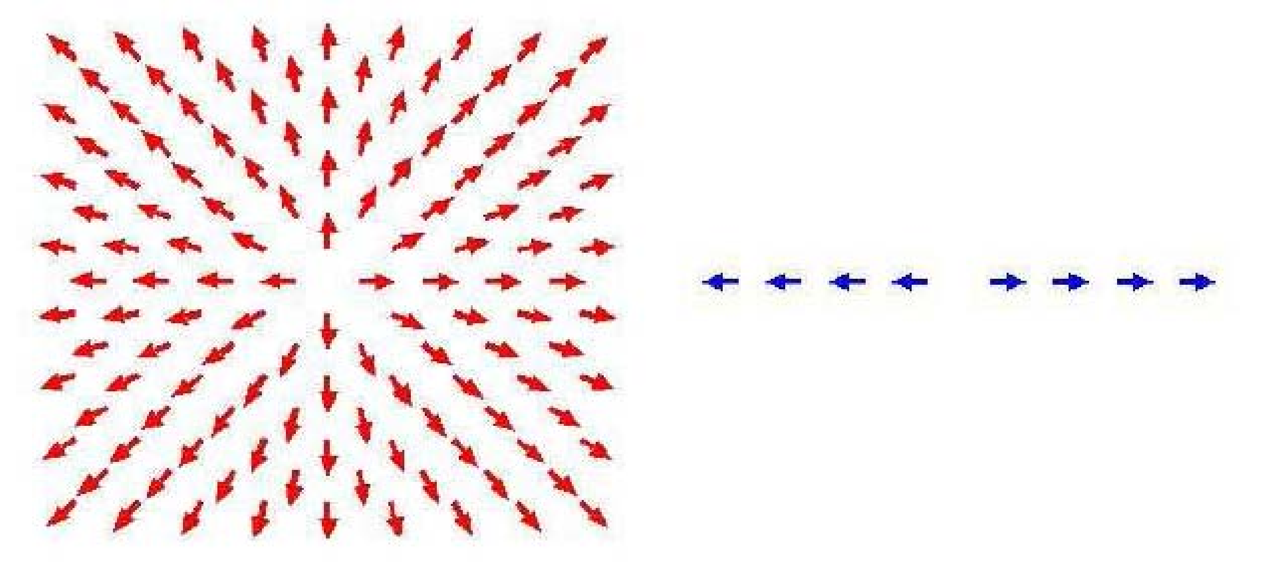
$$m_a = m_A = \frac{e_A e_a}{2\pi}$$

Gapped spin-vortex as quasi-particle

- With trapping a ½ staggered spin moment,
- a π spin vortex becomes a fermion!

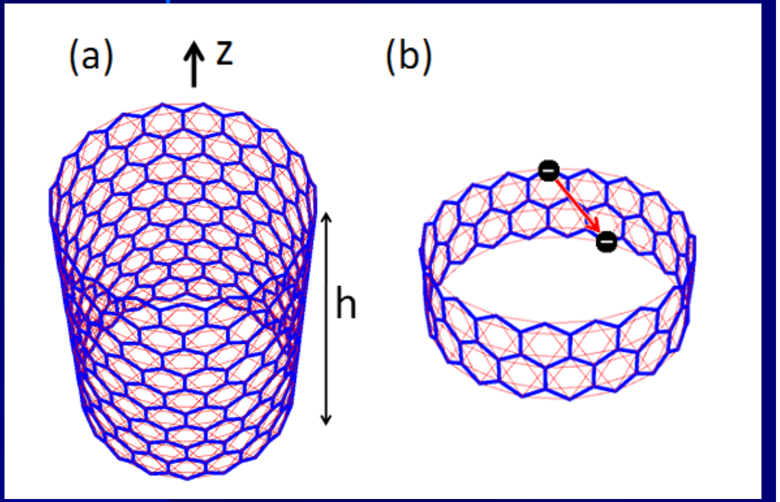
$$S^{z}_{(\pi,\pi)} = \pm \frac{1}{2}$$

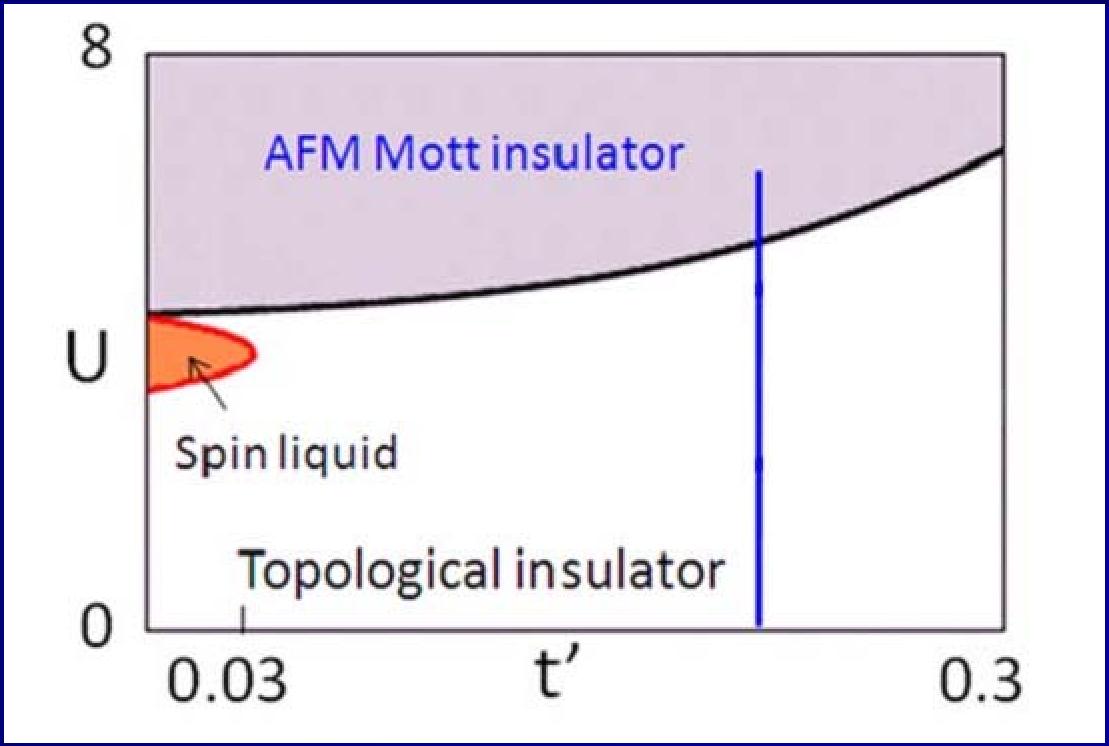
- The quantum disordered state becomes a spin liquid state with emergent (deconfined) fermionic excitations.



S.P. Kou, PHYS. REV. B 78, 233104 (2008).

Quantum phase transition of edge states





KT transition of the edge state

Dung-Hai Lee , arXiv:1105.4900

6. Chiral spin liquid in correlated topological insulator - interacting Spinful Haldane model

$$H = H_{\mathrm{H}} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \mu \sum_{i,\sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma}$$

$$H_{\mathrm{H}} = -t \sum_{\langle i,j \rangle,\sigma} \left(\hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + h.c. \right) - t' \sum_{\langle\langle i,j \rangle\rangle,\sigma} e^{i\varphi_{ij}} \hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma}$$

He J, Kou SP, Liang Y, Feng SP, PHYS. REV. B 83, 205116 (2011) .

Slave-rotor approach

$$\hat{c}_{j\sigma} = e^{i\theta_j} \hat{f}_{j\sigma}$$

$$X_i = e^{i\theta_i}$$

$$H_{eff} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{f}^\dagger_{i\sigma} \hat{f}_{j\sigma} X^\dagger_i X_j + h.c.) - \mu \sum_{i,\sigma} \hat{f}^\dagger_{i\sigma} \hat{f}_{i\sigma} \quad (2)$$
$$- t' \sum_{\langle\langle i,j \rangle\rangle, \sigma} e^{i\varphi_{ij}} \hat{f}^\dagger_{i\sigma} \hat{f}_{j\sigma} X^\dagger_i X_j + \frac{U}{2} \sum_i L_i^2$$
$$+ \sum_i h_i \Big(\sum_\sigma \hat{f}^\dagger_{i\sigma} \hat{f}_{i\sigma} + L_i - 1\Big) + \sum_i \rho_i (|X_i|^2 - 1).$$

Mean field approach

$$Q_X = \frac{1}{3tN_s} \sum_k \frac{Q_f \left|\xi_{\mathbf{k}}\right|^2}{E_f}, \; Q'_X = \frac{1}{3t'N_s} \sum_k \frac{Q'_f (\xi'_{\mathbf{k}})^2}{E_f}, \tag{3}$$

$$Q_f = \frac{1}{N_s} \sum_k \frac{|\xi_{\mathbf{k}}|}{12t} \frac{U}{\sqrt{U\left(\rho + \varepsilon_k\right)}}, \; 1 = \frac{1}{N_s} \sum_k \frac{U}{2\sqrt{U\left(\rho + \varepsilon_k\right)}},$$

$$Q'_f = \frac{1}{N_s} \sum_k \frac{g_{\mathbf{k}}}{24} \frac{U}{\sqrt{U\left(\rho + \varepsilon_k\right)}},$$

where

$$g_{\mathbf{k}} = 4\cos\left(3k_x/2\right)\cos(\sqrt{3}k_y/2) + 2\cos(\sqrt{3}k_y) \tag{4}$$

and $\varepsilon_k = -Q_X \left|\xi_{\mathbf{k}}\right| - t'Q'_X g_{\mathbf{k}}$. N_s denoting the number of unit cells.

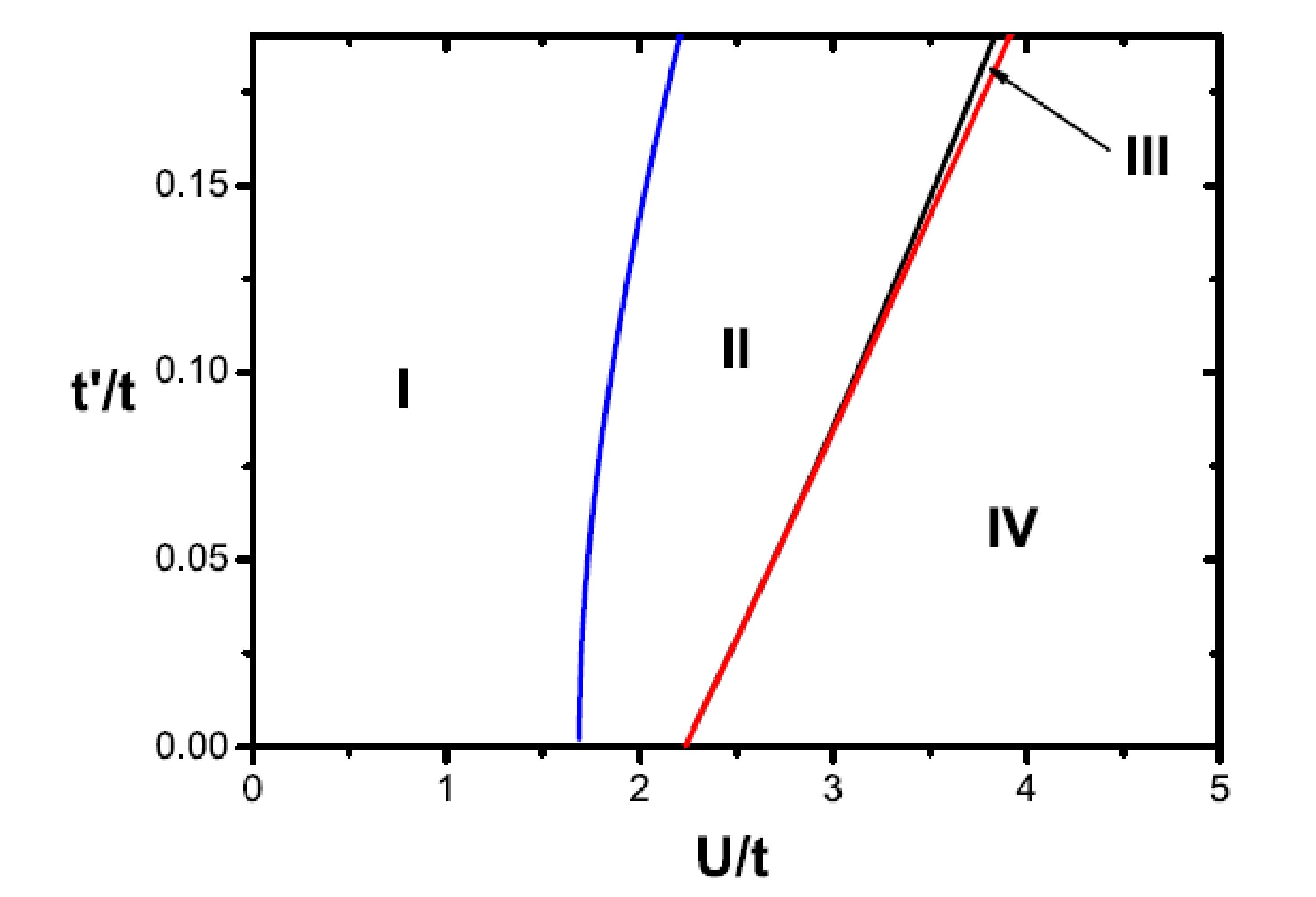
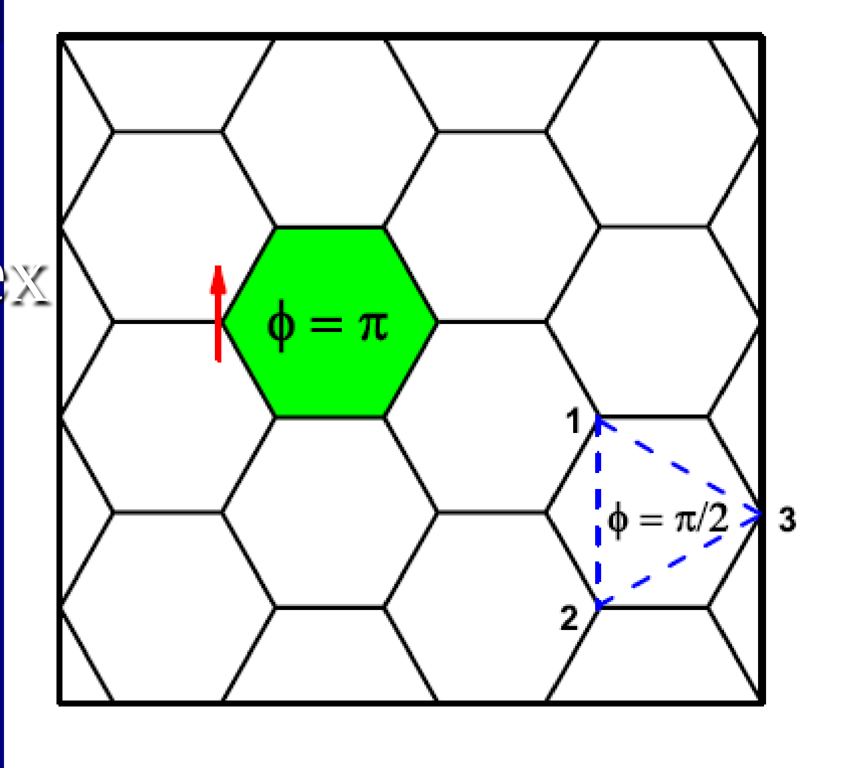


FIG. 1: (color online) Phase diagram at $T = 0$. There are four regions : I is TI state, II is the quantum spin liquid, III is an AF order with QAH effect, IV is the trivial AF order. The blue line and black line are $(\frac{U}{t})_{c1}$ and $(\frac{U}{t})_{c2}$, respectively.

π- vortex as anyons

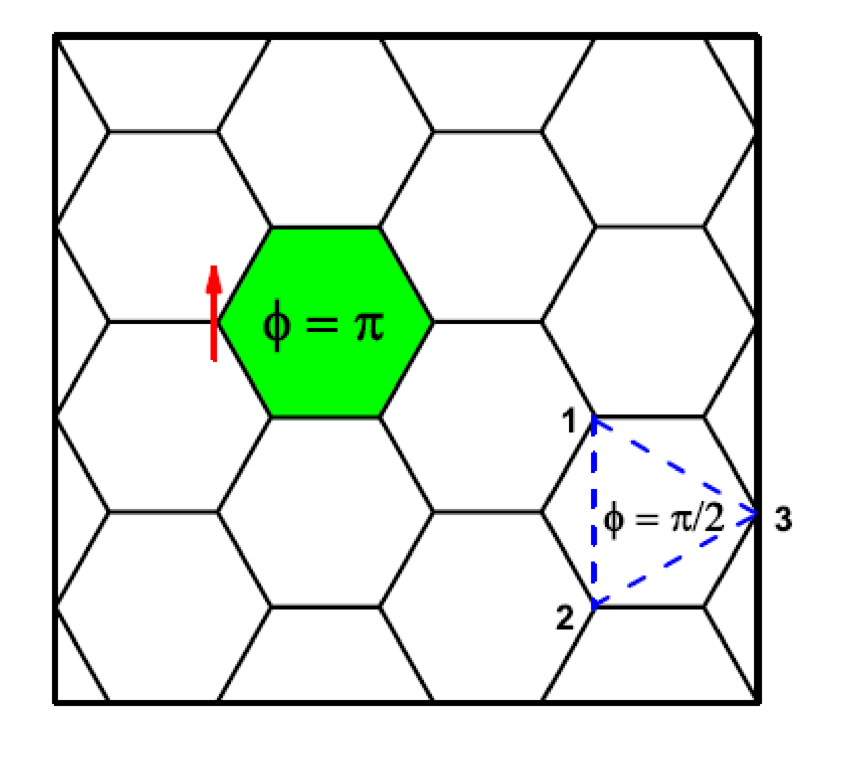
With induced fermion number $\langle N^f \rangle = 1$, π-vortex becomes anyon.

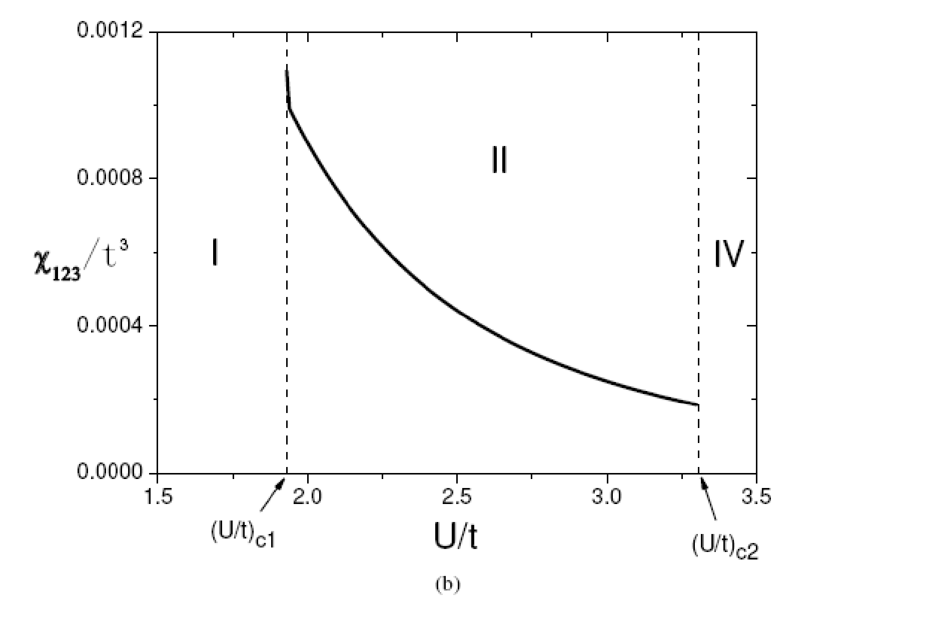


- **Statistics angle $\theta=\pi/2$**

$$\langle N^{\rm f} \rangle = -\frac{1}{2}\int_{-\infty}^{\infty} dE \, \frac{1}{\pi} {\rm Im}\ {\rm Tr}\,(\frac{1}{H_f - E - i\epsilon})\ {\rm sign}(E)$$

Chiral order parameter





$$\chi_{\langle 123\rangle} = \langle \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3)\rangle \tag{9}$$

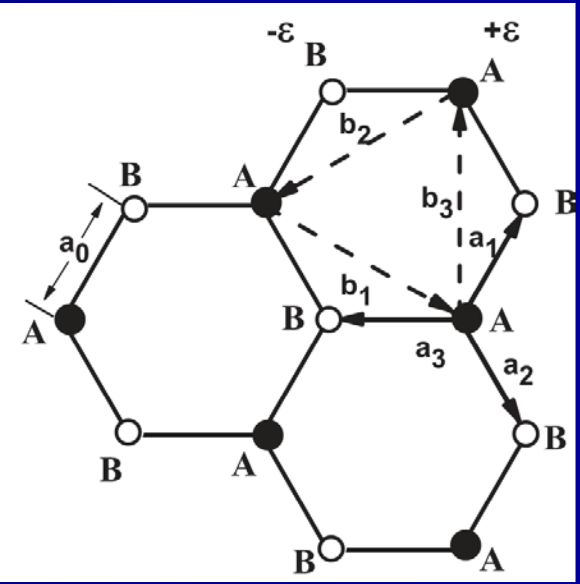
$$= \frac{1}{4i}\left\langle \hat{f}^\dagger_{1\alpha}\hat{f}_{2\alpha}\hat{f}^\dagger_{2\beta}\hat{f}_{3\beta}\hat{f}^\dagger_{3\gamma}\hat{f}_{1\gamma} - \hat{f}^\dagger_{1\alpha}\hat{f}_{3\alpha}\hat{f}^\dagger_{3\beta}\hat{f}_{2\beta}\hat{f}^\dagger_{2\gamma}\hat{f}_{1\gamma}\right\rangle$$

7. Topological spin-density-wave in correlated topological insulator - interacting Spinful Haldane model

$$H = H_{\rm H} + H' + U\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} - \mu\sum_{i,\sigma}\hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma}.$$

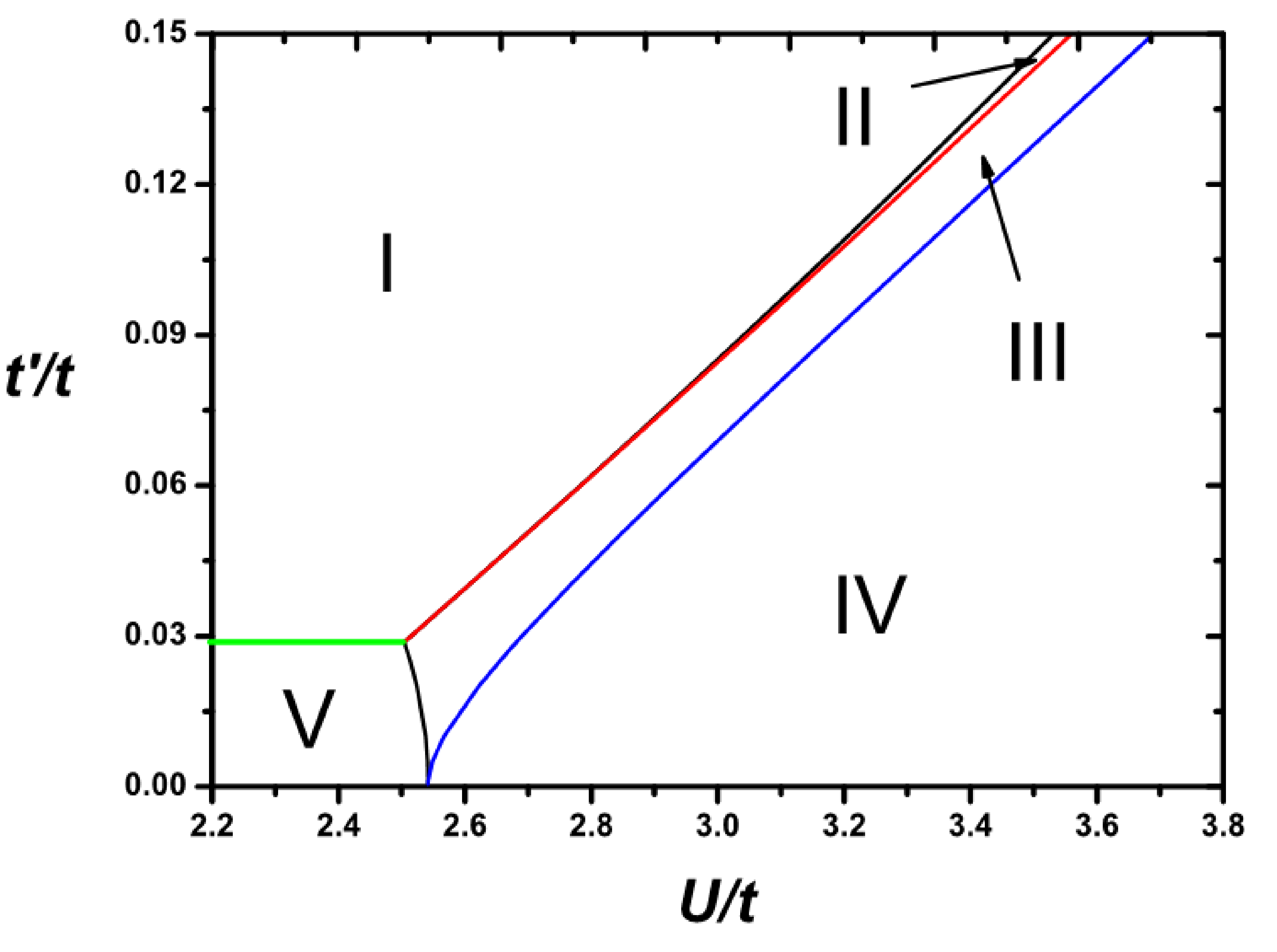
$$H_{\rm H} = -t\sum_{\langle i,j\rangle,\sigma}(\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + {\rm H.c.}) - t'\sum_{\langle\langle i,j\rangle\rangle,\sigma} e^{i\phi_{ij}}\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma}$$

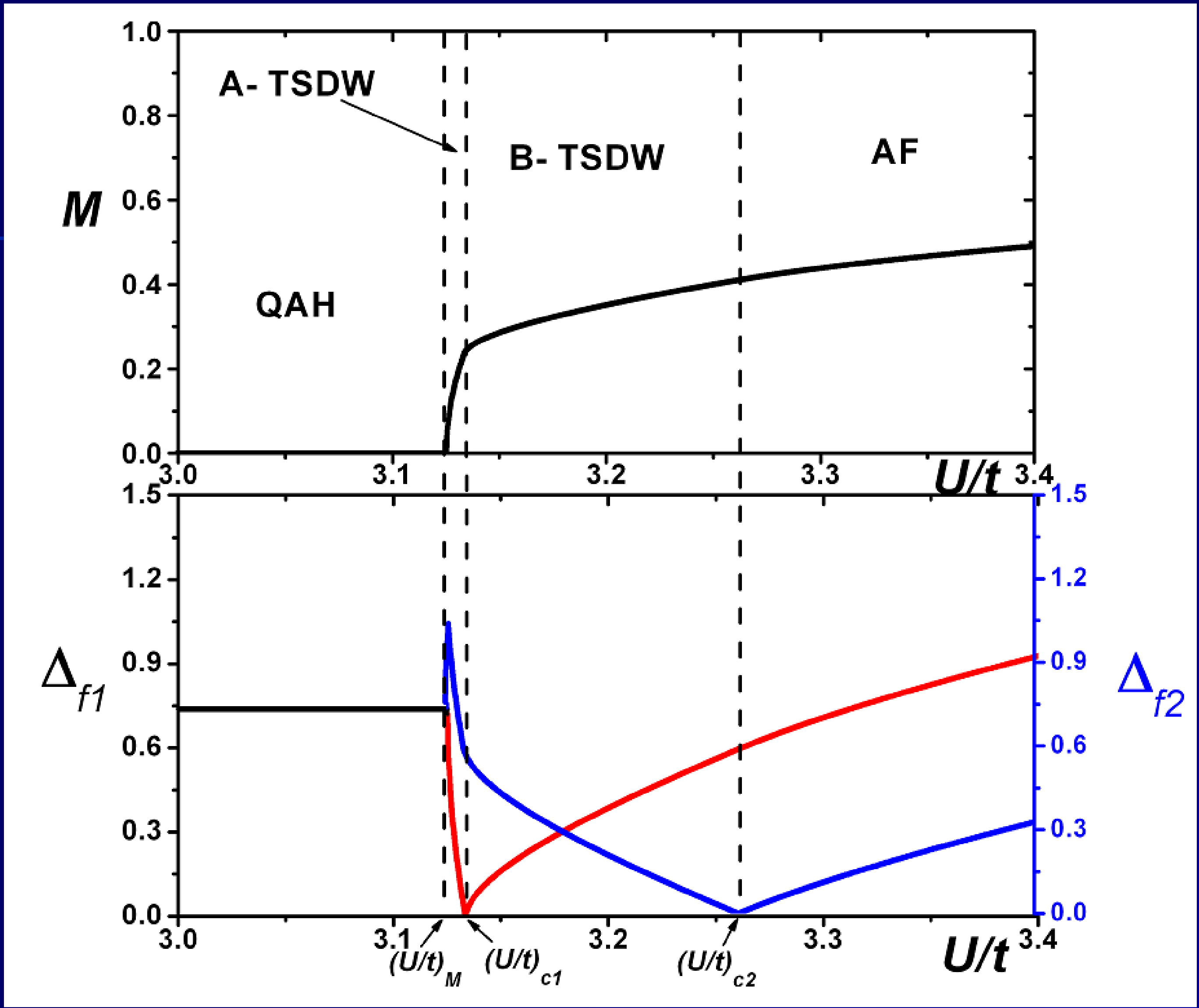
$$H' = \varepsilon\sum_{i\in A,\sigma}\hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma} - \varepsilon\sum_{i\in B,\sigma}\hat{c}^\dagger_{i\sigma}\hat{c}_{i\sigma}$$



He J, Zong YH, Kou SP, Liang Y, Feng SP, PHYS. REV. B 84, 035127 (2011)

Phase diagram





A- TSDW
B- TSDW
AF
QAH
M
U/t
Δf1
Δf2
(U/t)M
(U/t)c1
(U/t)c2
U/t

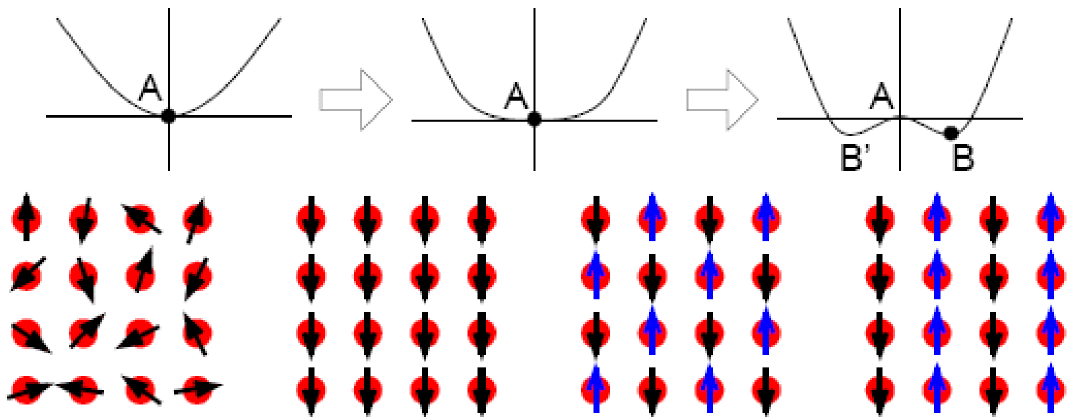
Landau paradigm

Landau symmetry breaking theory:

Different organizations = different symmetries

Phase transition = symmetry breaking

Landau



How to characterize topological SDW?

- **Different spin-density-wave with same order parameter : topological spin-density wave**

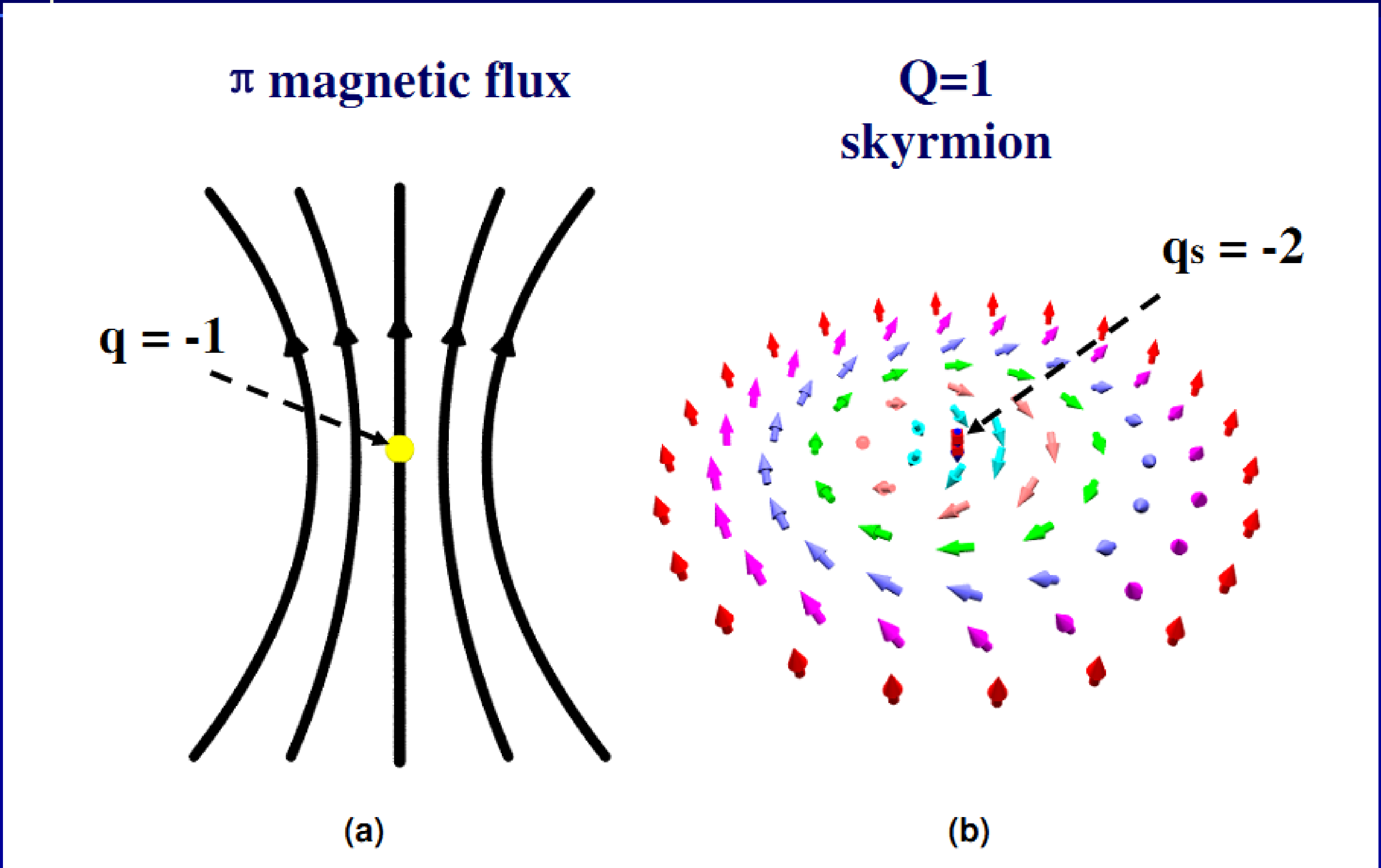
K-matrix formulation

$$\mathcal{L}_{CSH} = -i \sum_{I,J} \frac{\mathcal{K}_{IJ}}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu^I \partial_\nu a_\lambda^J$$

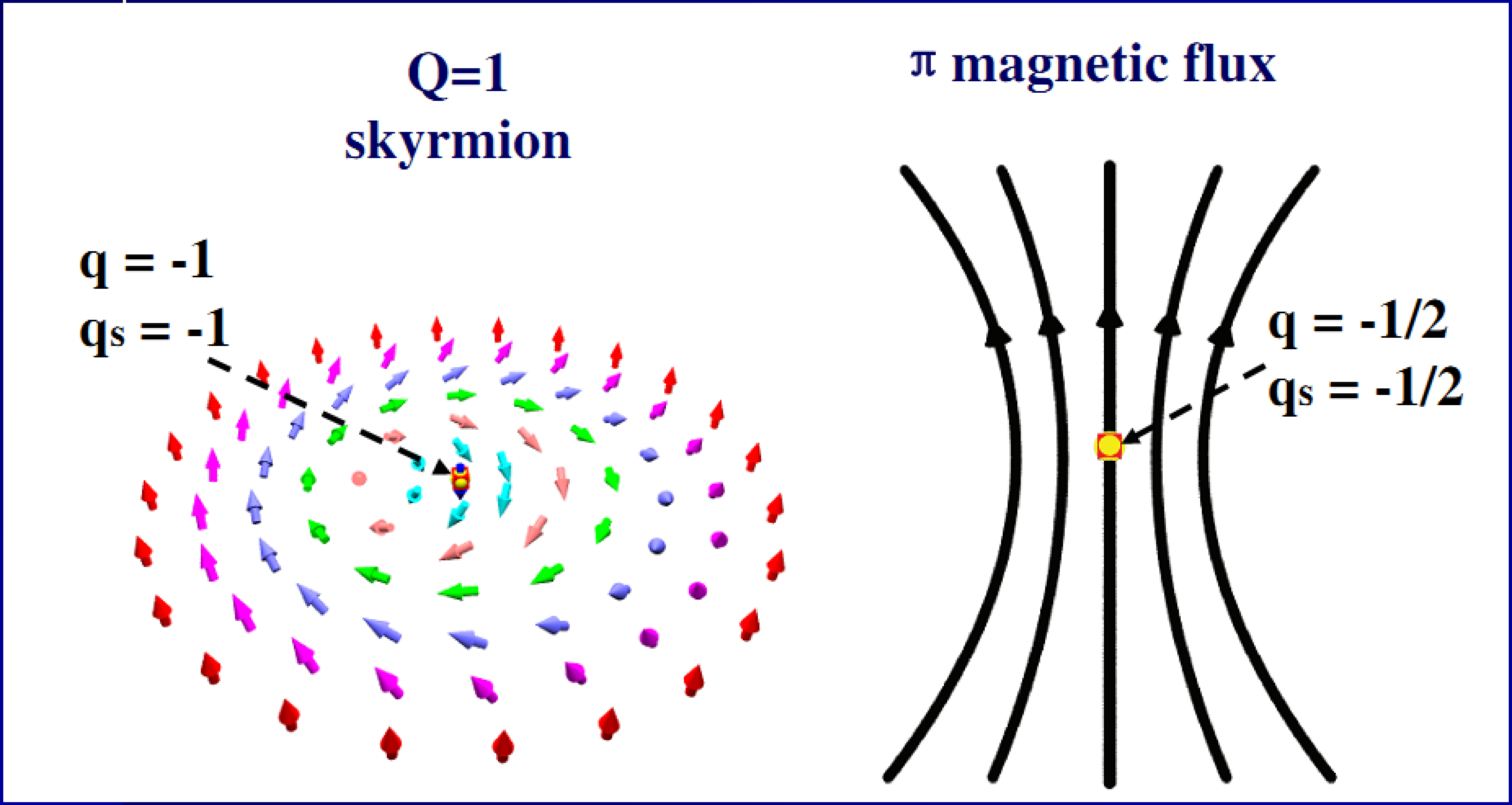
for A-TSDW order with $m_1, m_2 > \Delta_M$, $\mathcal{K} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

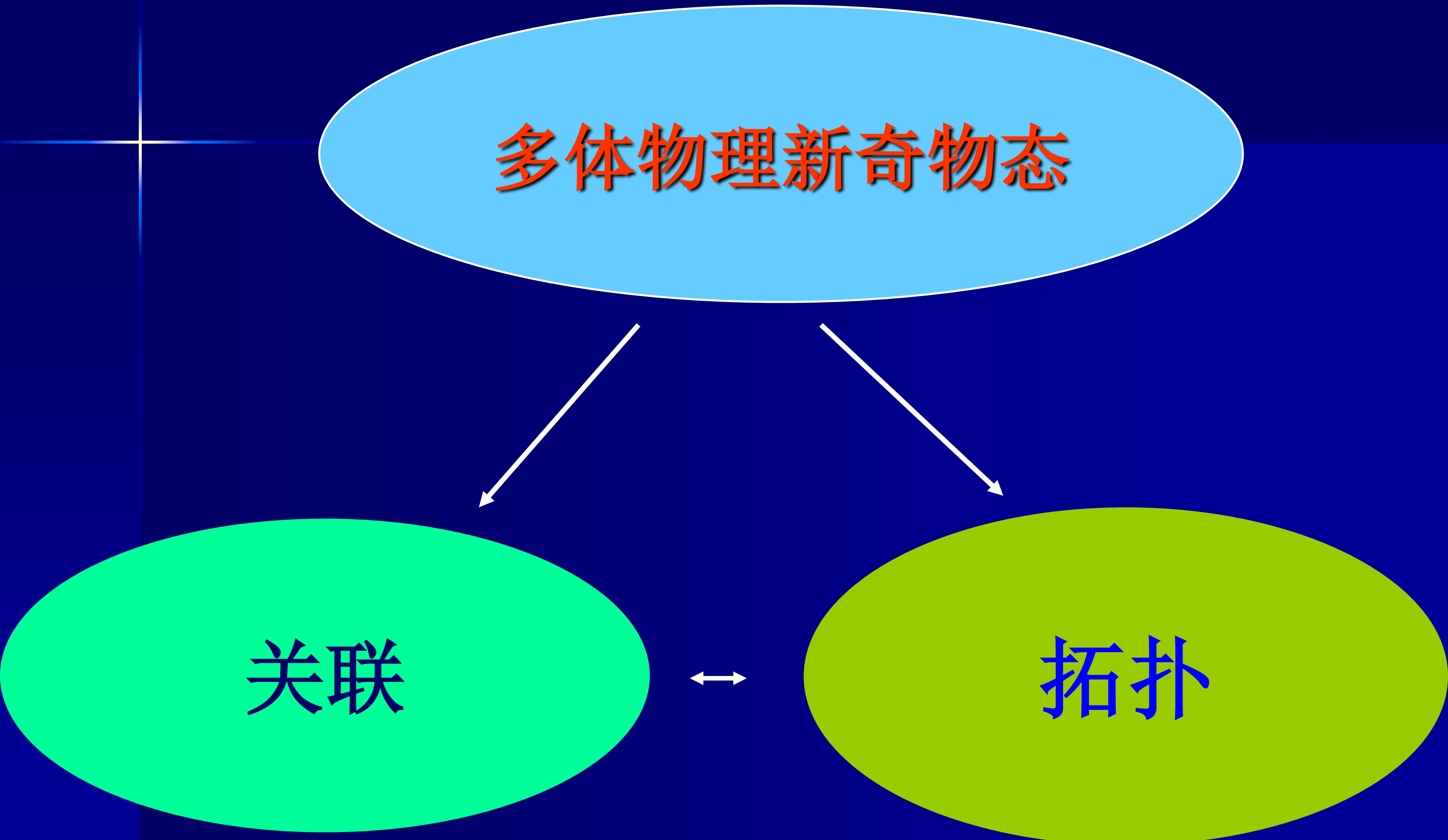
for B-TSDW order with $m_2 > \Delta_M > m_1$, $\mathcal{K} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

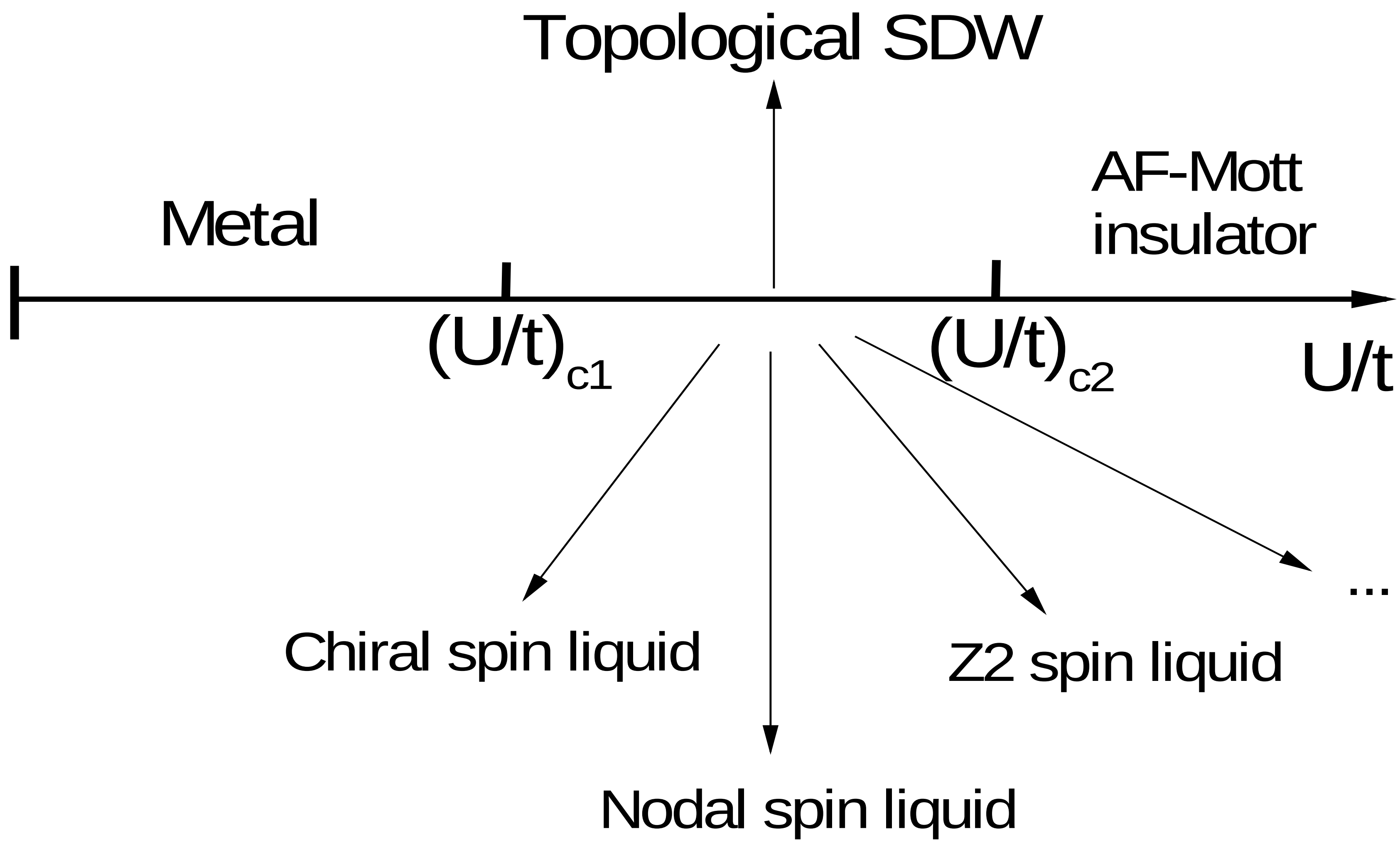
Spin-charge separated charge-flux binding effect in A-TSDW



spin-charge synchronization charge-flux binding effect in B-TSDW




多体物理新奇物态
关联
拓扑

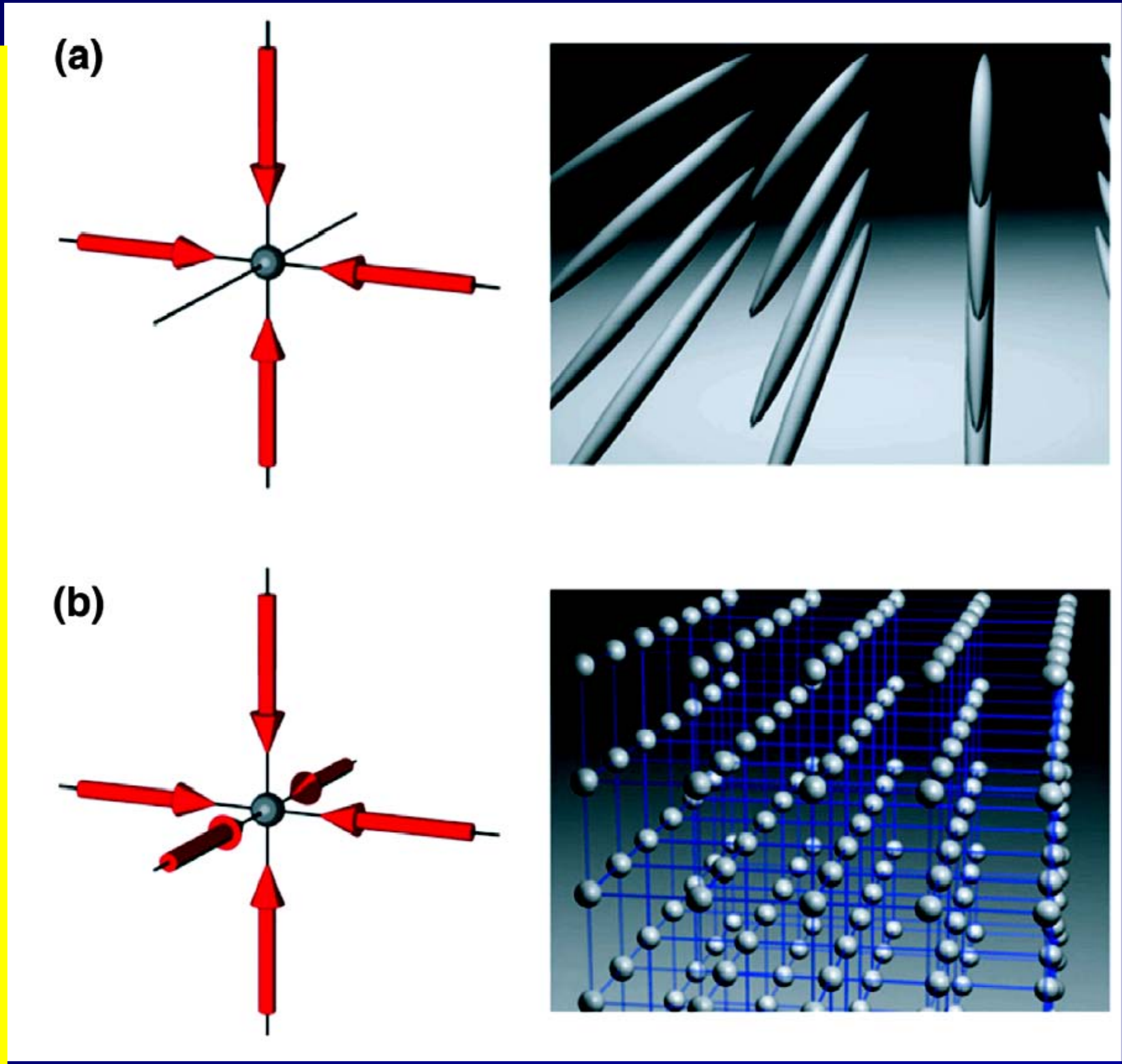

Topological SDW
AF-Mott
insulator
Metal
$(U/t)_{c1}$
$(U/t)_{c2}$
U/t
...
Chiral spin liquid
Z2 spin liquid
Nodal spin liquid

III. Realization in optical lattice

Possible experimental realizations of the generalized Hubbard model in optical lattices :

1. ***The Hubbard model***
2. ***The π-Hubbard model***
3. ***The Hubbard model on honeycomb lattice***
4. ***The interacting spinful Haldane model***

Optical lattices in cold atoms – many-body systems in different dimensions with tunable parameters

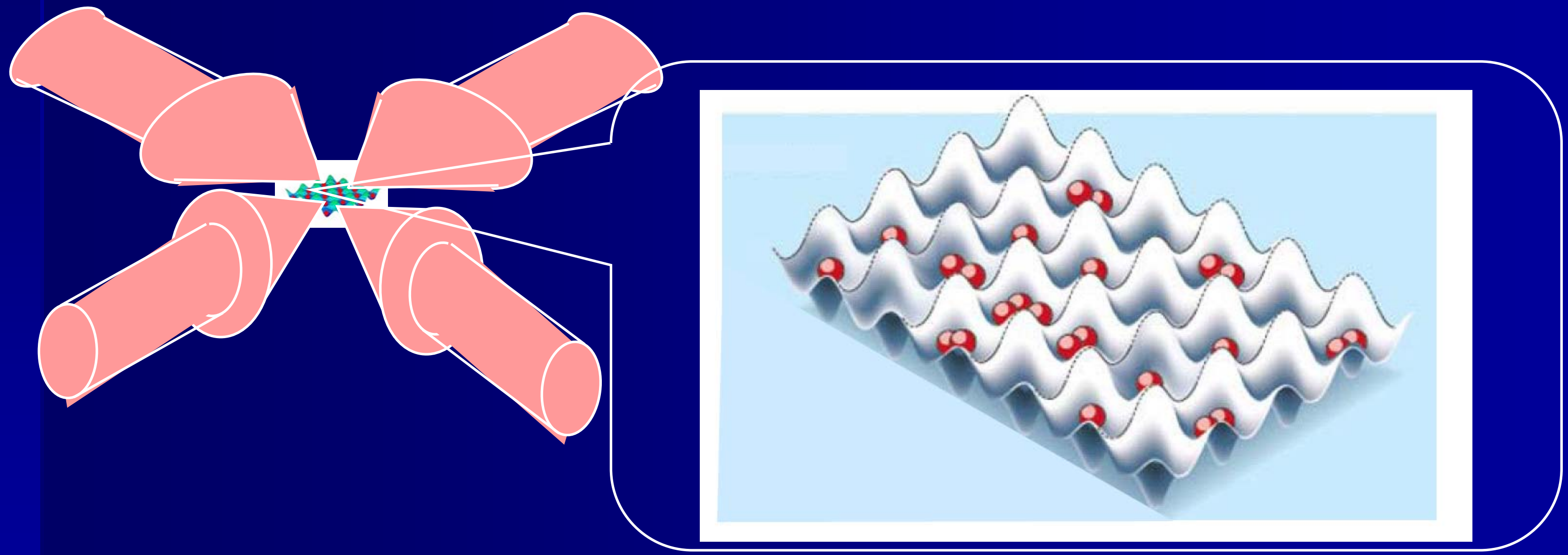


Bloch, I., Dalibard, J. & Zwerger, W..
Rev. Mod. Phys. 80, 885 (2008)

The Hubbard model on square lattice

Greiner et al., Nature (2001)

Jaksch et al. PRL (1998)



The π-Hubbard model on square lattice

- A U(1) adiabatic phase is created by two laser beams for the tunneling of atoms between neighbor lattice sites

$$U = \begin{pmatrix} \cos\theta & -\sin\theta e^{iqz} & 0 & 0 \\ \frac{\sqrt{2}}{2}\sin\theta e^{-iqz} & \frac{\sqrt{2}}{2}\cos\theta & -\frac{\sqrt{2}}{2}e^{-iq_2z} & 0 \\ \frac{\sqrt{2}}{2}\sin\theta e^{-iqz} & \frac{\sqrt{2}}{2}\cos\theta & \frac{\sqrt{2}}{2}e^{-iq_2z} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{\mathbf{A}} = i\hbar U \nabla U^{\dagger}$$

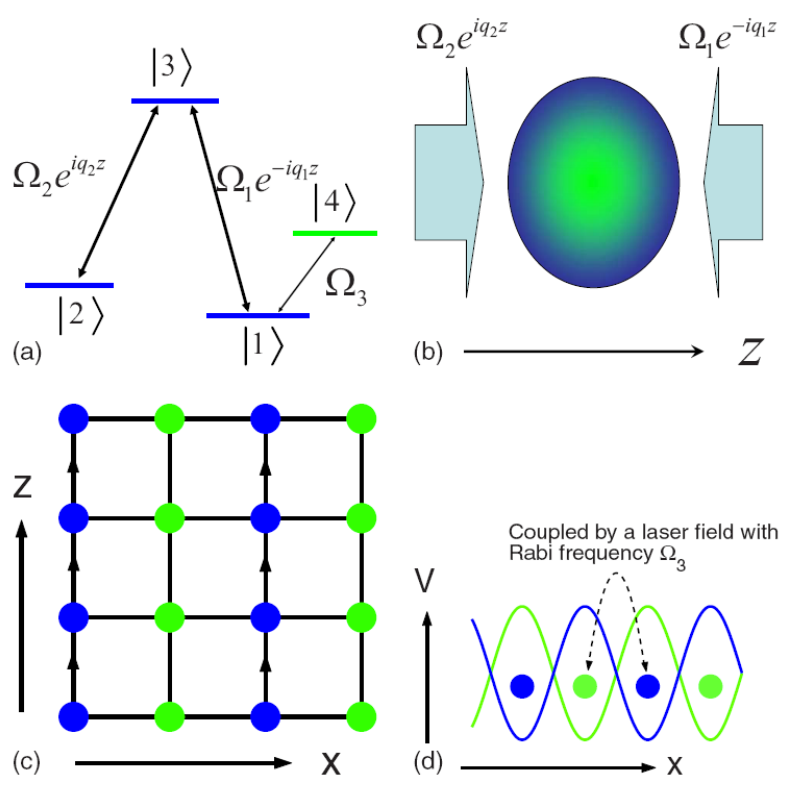
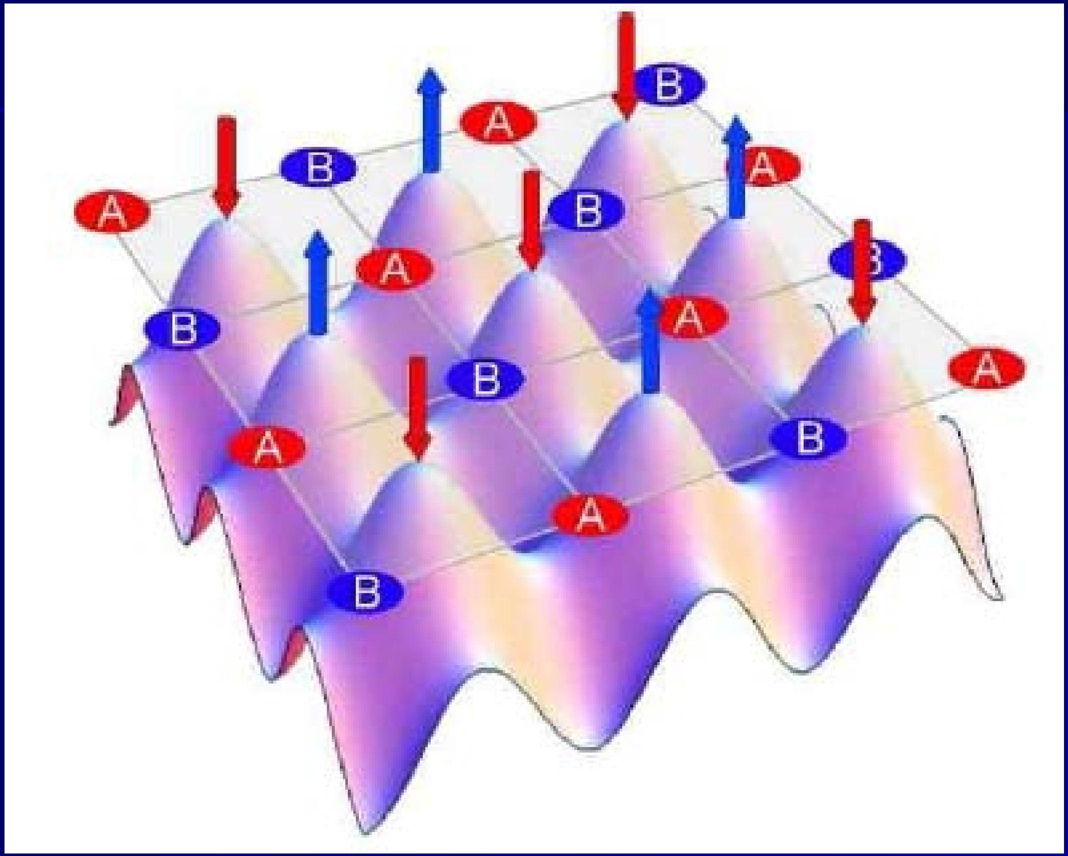


FIG. 1. (Color online). (a) The atomic levels and the interactions between atoms and laser fields. (b) Schematic representation of the experimental setup with the two laser beams incident on the cloud of atoms. (c) Schematic of the square optical lattice and the designed phase factor (denoted by arrows). (d) The scheme of overlapping the two state-selective optical lattices.

Jing-Min Hou, Wen-Xing Yang, and Xiong-Jun Liu, PRA 79, 043621 (2009)

The staggered flux phase has been proposed to be realized in optical lattice of cold atoms

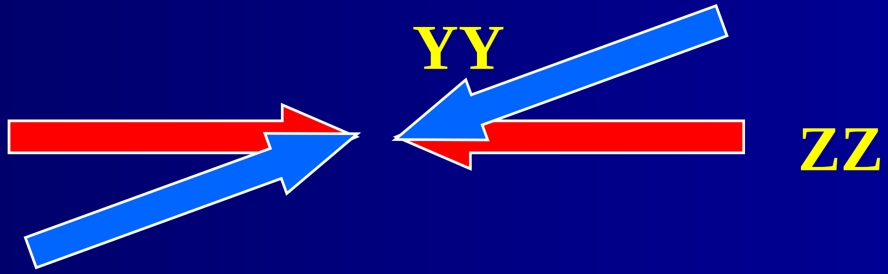


L.-K. Lim, C. Morais Smith, and A. Hemmerich, Phys. Rev. Lett. 100, 130402 (2008).

Lih-King Lim, Achilleas Lazarides, Andreas Hemmerich, C. Morais Smith, arXiv:0905.1281

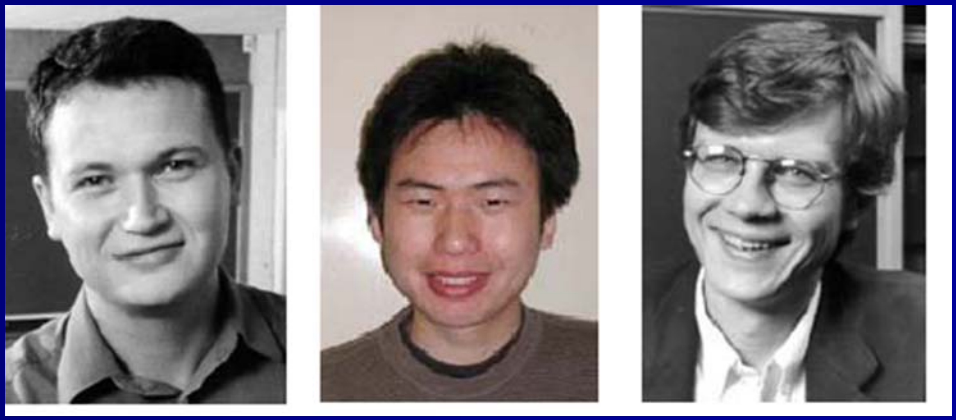
The Hubbard model on the honeycomb lattice in cold atoms

- Optical lattice in 2 dimensions: polarizations & frequencies of standing waves can be different for different directions



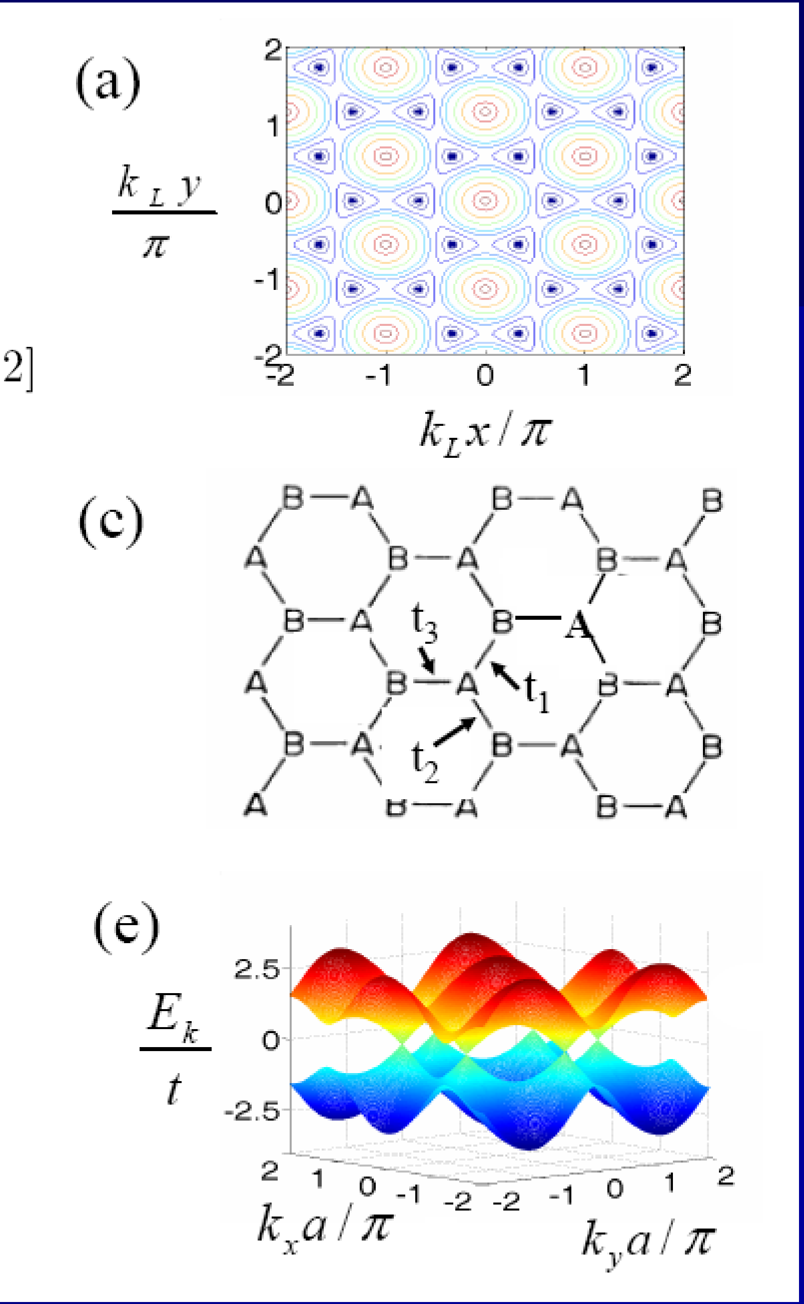
Hubbard model on honeycomb lattice Can be created with 3 sets of standing wave light beams.

L.-M. Duan, E. Demler, and M. D. Lukin,
Phys. Rev. Lett. 91, 090402 (2003).



$$V(x,y) = \sum_{j=1,2,3} V_j \sin^2[k_L(x\cos\theta_j + y\sin\theta_j) + \pi/2]$$

where $\theta_1 = \pi/3$, $\theta_2 = 2\pi/3$, $\theta_3 = 0$



Shi-Liang Zhu, Baigeng Wang, L.-M. Duan, Phys. Rev. Lett 98, 260402 (2007)

The interacting spinful Haldane model

- **For the Hubbard model on honeycomb lattice, one consider next nearest neighbor hopping and artifitional magnetic field.**

L. B. Shao, Shi-Liang Zhu, L. Sheng, D. Y. Xing, Z. D. Wang, Phys. Rev. Lett.101,246810 (2008)

S. L. Zhu et al., Phys. Rev. Lett 97, 240401(2006).

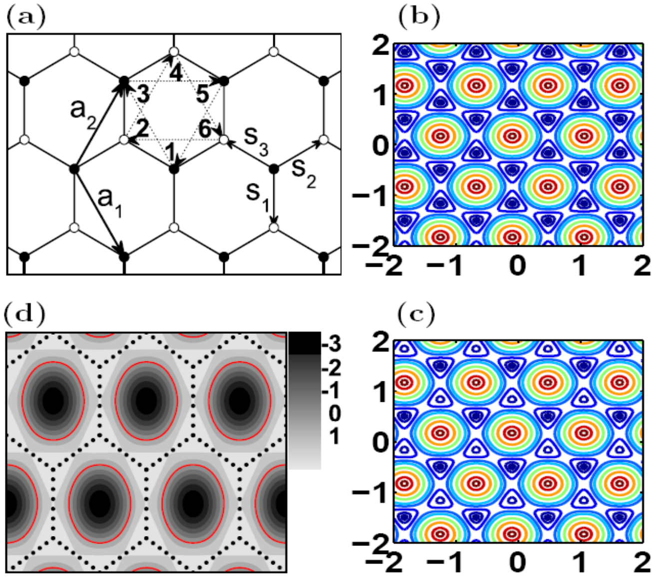


FIG. 1: (Color online) (a) Illustration of the Honeycomb lattice structure of graphene, where open and solid circles represent sites in sublattices $\bar{A}$ and $\bar{B}$. $\mathbf{a}_1 = (\frac{1}{2}a, -\frac{\sqrt{3}}{2}a)$, $\mathbf{a}_2 = (\frac{1}{2}a, \frac{\sqrt{3}}{2}a)$ are the unit vectors of the underlying triangular sublattice. s_1, s_2 and s_3 are three vectors pointing from a $\bar{B}$ site to its three nearest-neighbor sites. (b) and (c) show the contours of the potential V for $\chi = 2\pi/3$ and $\chi = 39\pi/60$, respectively. The vertical (horizontal) axis represents yk_0^L/π (xk_0^L/π). (d) Contours of the magnetic field defined by Eq.(2).

Thanks for your attention!