Antiferromagnetism and superfluidity of a dipolar Fermi gas in a 2D optical lattice

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(cond-mat/1106.2194)

Outline

(1) Background introduction

(2) Effective lattice model in a resonant AC field

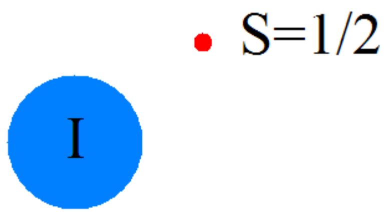
(3) Antiferromagnetic state at half filling

(4) Superfluid states with d-wave and extended s-wave symmetries

(5) Conclusion

(1) Background Introduction

Alkali atom



Hyperfine spin **F=S+I**

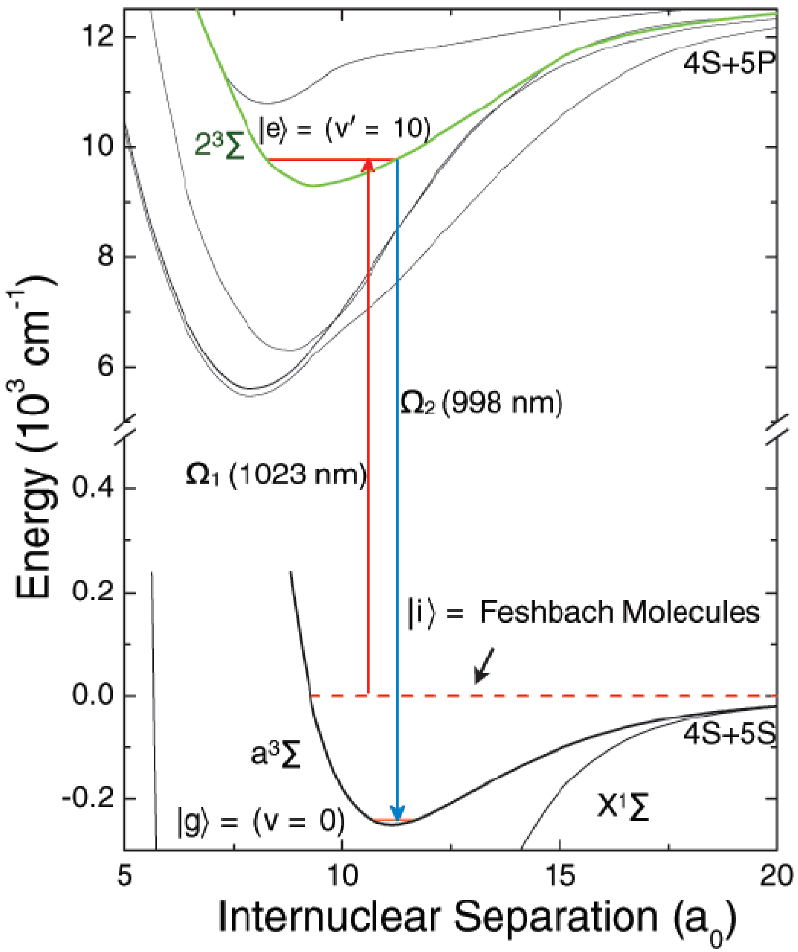
Magnetic moment $\mu_a \approx \mu_B$

Bose atoms: ^{7}Li, ^{23}Na, ^{87}Rb…

Fermi atoms: ^{6}Li, ^{40}K …

Hetero-nuclear molecule: ^{87}Rb^{40}K …

Creating $^{87}Rb^{40}K$ polar molecules (JILA)



Electric dipole:

0.052(2) Debye
(Triplet rovibrational ground state)

0.566(17) Debye (Singlet)

Density~10^{12} cm^{-3}

Temperature~$1.5T_F$

Stimulated Raman adiabatic passage

Dipole-dipole interaction

(Long-range and anisotropic)

$$V_{\mathrm{dd}}(\mathbf{r}) = \left(\frac{1}{r^3}\right)\{\mathbf{D}_{\mathrm{A}} \cdot \mathbf{D}_{\mathrm{B}} - 3(\mathbf{D}_{\mathrm{A}} \cdot \hat{e}_r)(\hat{e}_{\mathrm{r}} \cdot \mathbf{D}_{\mathrm{B}})\}$$

In DC field, $V_{dd}(\mathbf{r}) = (d^2/r^3)[1 - 3(z^2/r^2)]$

Interesting properties of a dipolar Fermi gas:

(1) Anisotropic Fermi surface

(2) Mechanical collapse at high density

(3) P-wave superfluid and other novel states

…

(2) Effective model in a resonant AC field

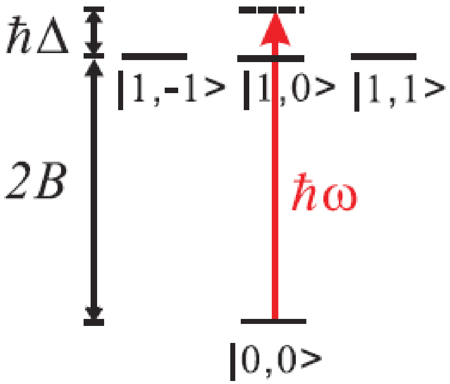
Single molecule Hamiltonian

$$H_0 = \frac{p^2}{2m} + H_{opt} + H_{rot},$$

rotational energy $H_{rot} = BJ^2 - \mathbf{d} \cdot \mathbf{E}_{AC}(t),$

AC field $\mathbf{E}_{AC}(t) = E_{AC} \cos(\omega t)\hat{z}$

Rotational states |0,0> and |1,0> can be coupled by an AC field.



Effective rotational energy

$$H_{rot} = \begin{pmatrix} 0 & \Omega \\ \Omega & 2B - \hbar\omega \end{pmatrix},$$

$$\Omega = dE_{AC}/\sqrt{3}$$

In resonance limit, |0,0> and |1,0> states are degenerate.

Dipole matrix

$$\langle 0,0|\mathbf{d}|0,0\rangle = 0,\ \langle 1,0|\mathbf{d}|1,0\rangle = 0,$$

$$\langle 0,0|\mathbf{d}_{x,y}|1,0\rangle = 0,$$

$$\langle 0,0|d_z|1,0\rangle = d/\sqrt{3}.$$

Pseudospin states

$$|\uparrow\rangle \equiv (|0,0\rangle + |1,0\rangle)/\sqrt{2} \text{ and } |\downarrow\rangle \equiv (|0,0\rangle - |1,0\rangle)/\sqrt{2},$$

$$\langle\uparrow|\mathbf{d}|\downarrow\rangle = 0,\ \langle\uparrow|\mathbf{d}|\uparrow\rangle = d\hat{z}/\sqrt{3},\ \text{and}\ \langle\downarrow|\mathbf{d}|\downarrow\rangle = -d\hat{z}/\sqrt{3}.$$

Dipolar Ising interaction between pseudospins

$$V_{dd}(\mathbf{r}_i - \mathbf{r}_j) = 4d^2 \frac{1 - 3\cos^2\theta_{ij}}{3\mid\mathbf{r_i} - \mathbf{r_j}\mid^{\mathbf{3}}} s_{iz} s_{jz}$$

Effective model in a 2D optical lattice

$$H = -\sum_{\langle i,j \rangle,\sigma} t(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + \frac{1}{2}\sum_{i\neq j} J_{ij} s_{iz} s_{jz} + U \sum_i n_{i\uparrow} n_{i\downarrow},$$

$$J_{ij} = 4d^2/(3|\mathbf{r}_j - \mathbf{r}_i|^3)$$

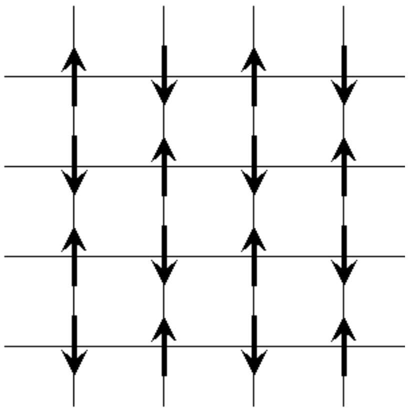
Remarks:

(1) Onsite interaction *U* may lead to molecule loss through chemical reaction, and may be tuned by Feshbach resonace. It can make the phase diagram much more complicated. We consider only *U*=0.

(2) Dipolar Ising interaction leads to antiferromagnetic spin order or spin-singlet pairing between nearest-neighbor fermions.

(3) Antiferromagnetic state at half filling

Antiferromagnetic order



$$\langle s_{iz} \rangle = m \text{ for sublattice A}$$

$$\langle s_{jz} \rangle = -m \text{ for sublattice B}$$

Mean-field approximation

$$s_{iz} s_{jz} \approx \langle s_{iz} \rangle s_{jz} + \langle s_{iz} \rangle s_{jz} - \langle s_{iz} \rangle \langle s_{jz} \rangle$$

Mean-field Hamiltonian

$$H_{AF} = -\sum_{\langle i,j \rangle,\sigma} t(c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma}) + hm(\sum_{i \in A} s_{iz} - \sum_{j \in B} s_{jz}) - \frac{hm^2 N}{2},$$

Mean-field Hamiltonian in k-space

$$H_{AF} = \frac{hm}{2} \sum_{\mathbf{k}} (a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{k}\uparrow} + b^\dagger_{\mathbf{k}\downarrow} b_{\mathbf{k}\downarrow} - a^\dagger_{\mathbf{k}\downarrow} a_{\mathbf{k}\downarrow} - b^\dagger_{\mathbf{k}\uparrow} b_{\mathbf{k}\uparrow})$$

$$+ \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} (a^\dagger_{\mathbf{k}\sigma} b_{\mathbf{k}\sigma} + b^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}) - \frac{hm^2 N}{2},$$

Bare band

$$\varepsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_y a),$$

Effective spin-coupling constant

$$h \equiv \sum_{i \neq 0} (-1)^i J_{0i} = -2.646J$$

$$J \equiv 4d^2/(3a^3)$$

Canonical transformation

$$\alpha_{\mathbf{k}\uparrow} = u_{\mathbf{k}} a_{\mathbf{k}\uparrow} + v_{\mathbf{k}} b_{\mathbf{k}\uparrow},\ \alpha_{\mathbf{k}\downarrow} = u_{\mathbf{k}} b_{\mathbf{k}\downarrow} + v_{\mathbf{k}} a_{\mathbf{k}\downarrow},$$

$$\beta_{\mathbf{k}\uparrow} = u_{\mathbf{k}} b_{\mathbf{k}\uparrow} - v_{\mathbf{k}} a_{\mathbf{k}\uparrow},\ \beta_{\mathbf{k}\downarrow} = u_{\mathbf{k}} a_{\mathbf{k}\downarrow} - v_{\mathbf{k}} b_{\mathbf{k}\downarrow},$$

Diagonalized mean-field Hamiltonian

$$H_{AF} = \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}}^{-} \alpha_{\mathbf{k}\sigma}^{\dagger} \alpha_{\mathbf{k}\sigma} + \varepsilon_{\mathbf{k}}^{+} \beta_{\mathbf{k}\sigma}^{\dagger} \beta_{\mathbf{k}\sigma}) - \frac{hm^2 N}{2},$$

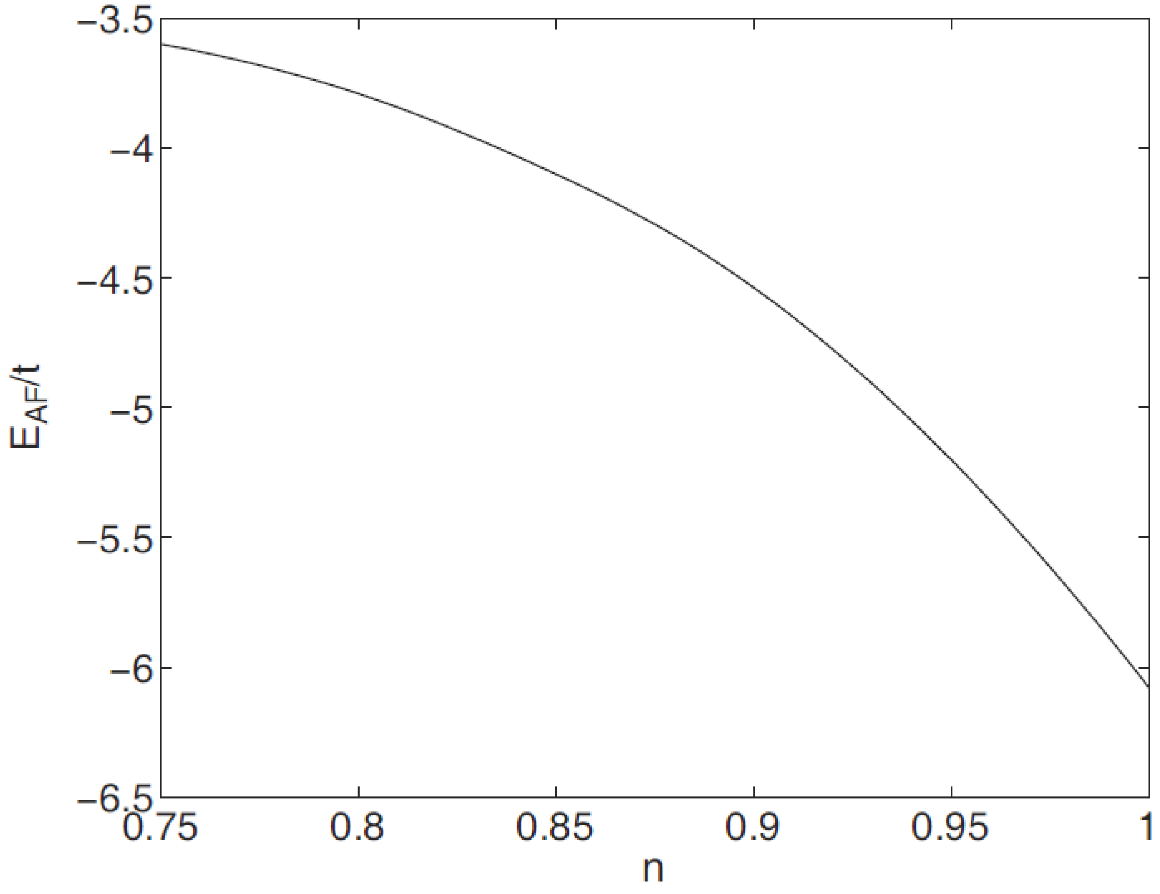
quasi-particle energy $\varepsilon_{\mathbf{k}}^{\pm} = \pm\sqrt{\varepsilon_{\mathbf{k}}^{2} + (\frac{hm}{2})^2}.$

Self-consistency equation

$$\frac{1}{h} = \frac{1}{N} \sum_{\mathbf{k}} \frac{1}{2\varepsilon_{\mathbf{k}}^{+}} [\theta(\mu - \varepsilon_{\mathbf{k}}^{-}) - \theta(\mu - \varepsilon_{\mathbf{k}}^{+})],$$

At half filling, the antiferromagnetic state always exists.

Antiferromagnetic-state energy vs. filling factor



$J/t = 3.$

Beyond half filling, the negative compressibility shows that the antiferromagnetic state is subject to mechanical collapse.

(4) Superfluid states with d-wave and extended s-wave symmetries

Antiferromagnetic Dipolar Ising interaction leads to attraction between opposite spins.

Effective pairing interaction

$$-\frac{1}{4}\sum_{i\neq j} J_{ij} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{j\downarrow} c_{j\downarrow},$$

in mean-field approximation

$$-\frac{1}{4}\sum_{i\neq j} J_{ij}[c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow}\langle c_{j\downarrow} c_{i\uparrow}\rangle + \langle c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow}\rangle c_{j\downarrow} c_{i\uparrow} - \langle c^{\dagger}_{i\uparrow} c^{\dagger}_{j\downarrow}\rangle\langle c_{j\downarrow} c_{i\uparrow}\rangle].$$

Mean-field Hamiltonian

$$H_{SF} = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\Delta^{*}_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} - \Delta_{\mathbf{k}} g^{*}_{\mathbf{k}}),$$

bare band $\varepsilon_{\mathbf{k}} = -2t(\cos k_x a + \cos k_y a),$

gap $\Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} f_{\mathbf{k}\mathbf{k}'} g_{\mathbf{k}'}$

pair susceptibility $g_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$

pairing interaction $f_{\mathbf{k}\mathbf{k}'} = -\frac{1}{4N^2} \sum_{i \neq j} J_{ij} \exp[i(\mathbf{k} - \mathbf{k}') \cdot (\mathbf{r}_i - \mathbf{r}_j)].$

Bogoliubov transformation

$$c_{\mathbf{k}\uparrow} = u_{\mathbf{k}} d_{\mathbf{k}\uparrow} + v_{\mathbf{k}} d^{\dagger}_{-\mathbf{k}\downarrow},\ c_{-\mathbf{k}\downarrow} = u_{\mathbf{k}} d_{-\mathbf{k}\downarrow} - v_{\mathbf{k}} d^{\dagger}_{\mathbf{k}\uparrow},$$

Diagonalized mean-field Hamiltonian

$$H_{SF} = N(E_{SF} - \mu n) + \sum_{\mathbf{k}\sigma} E_{\mathbf{k}} d^{\dagger}_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma},$$

Quasi-particle energy

$$E_{\mathbf{k}} = \sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2}$$

Gap equation

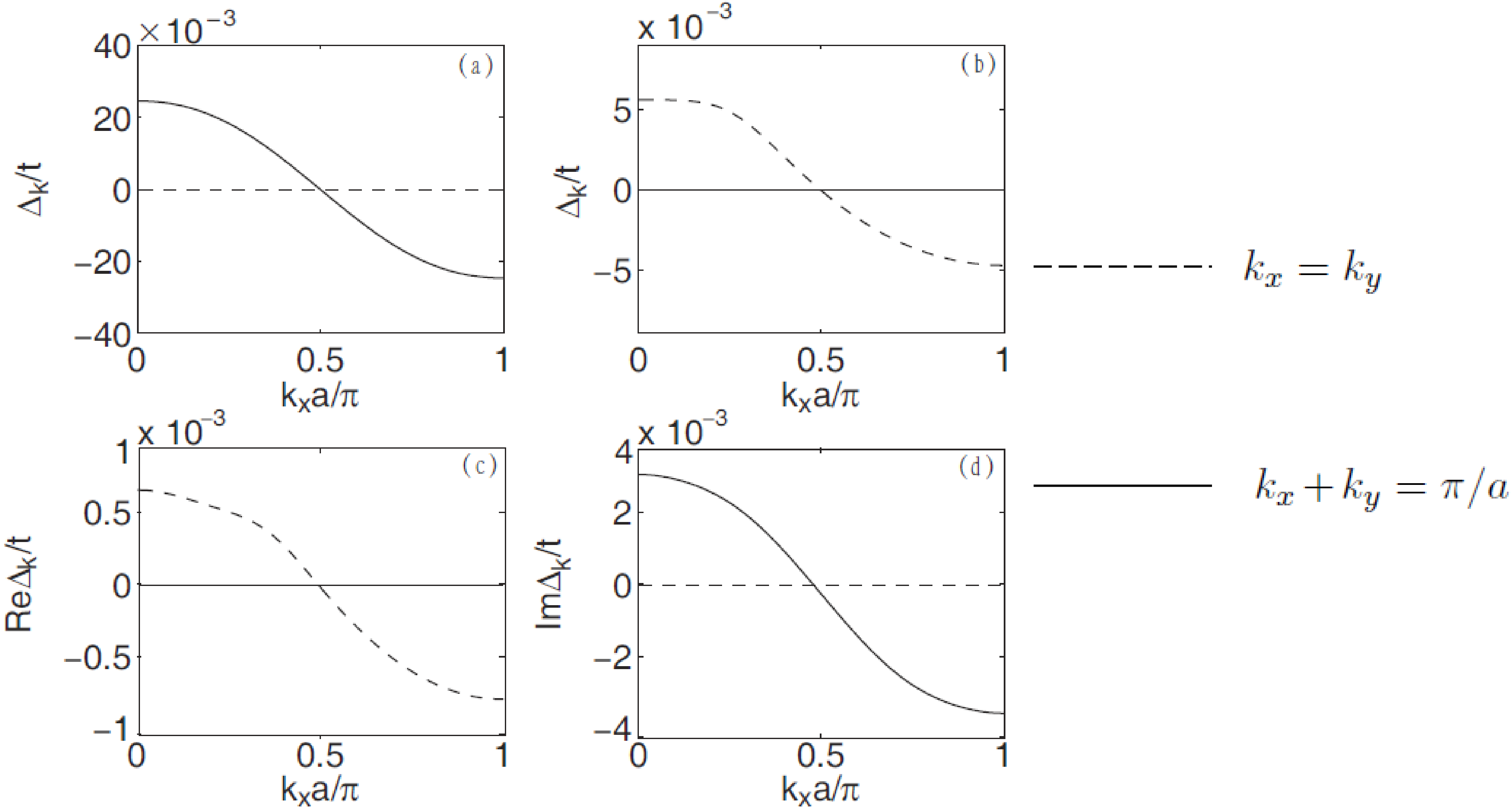
$$\Delta_{\mathbf{k}} = -\frac{1}{N} \sum_{\mathbf{k}'} f_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}}$$

Density equation

$$n = 1 - \frac{1}{N} \sum_{\mathbf{k}} \frac{\varepsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}}$$

Gap and density equations can be solved together self-consistently.

Numerical results



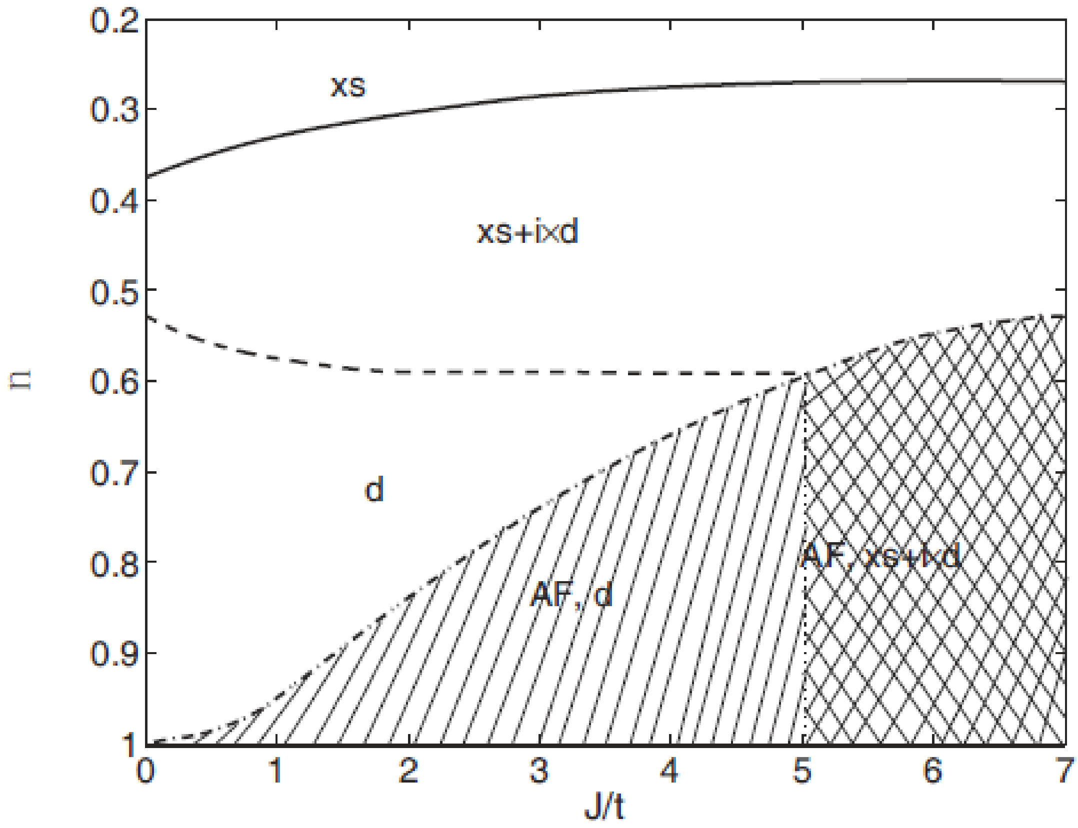
(a) At $J/t = 3$ and $n = 0.656$, d-wave symmetry. $(\cos k_x - \cos k_y)$

(b) At $J/t = 3$ and $n = 0.29$, extended-s-wave symmetry. $(\cos k_x + \cos k_y)$

At $J/t = 3$ and $n = 0.483$, (c) real part shows the extended-s-wave symmetry, (d) the imaginary part of the gap shows d-wave symmetry.

(4) Conclusion

Phase diagram at zero temperature



Remaining issues:

Stability, finite temperature, fluctuations, onsite interaction…