Spatially Modulated Interaction Induced Bound States and Resonances

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Ran Qi and Hui Zhai, PRL. 106, 163201 (2011)

DAMOP
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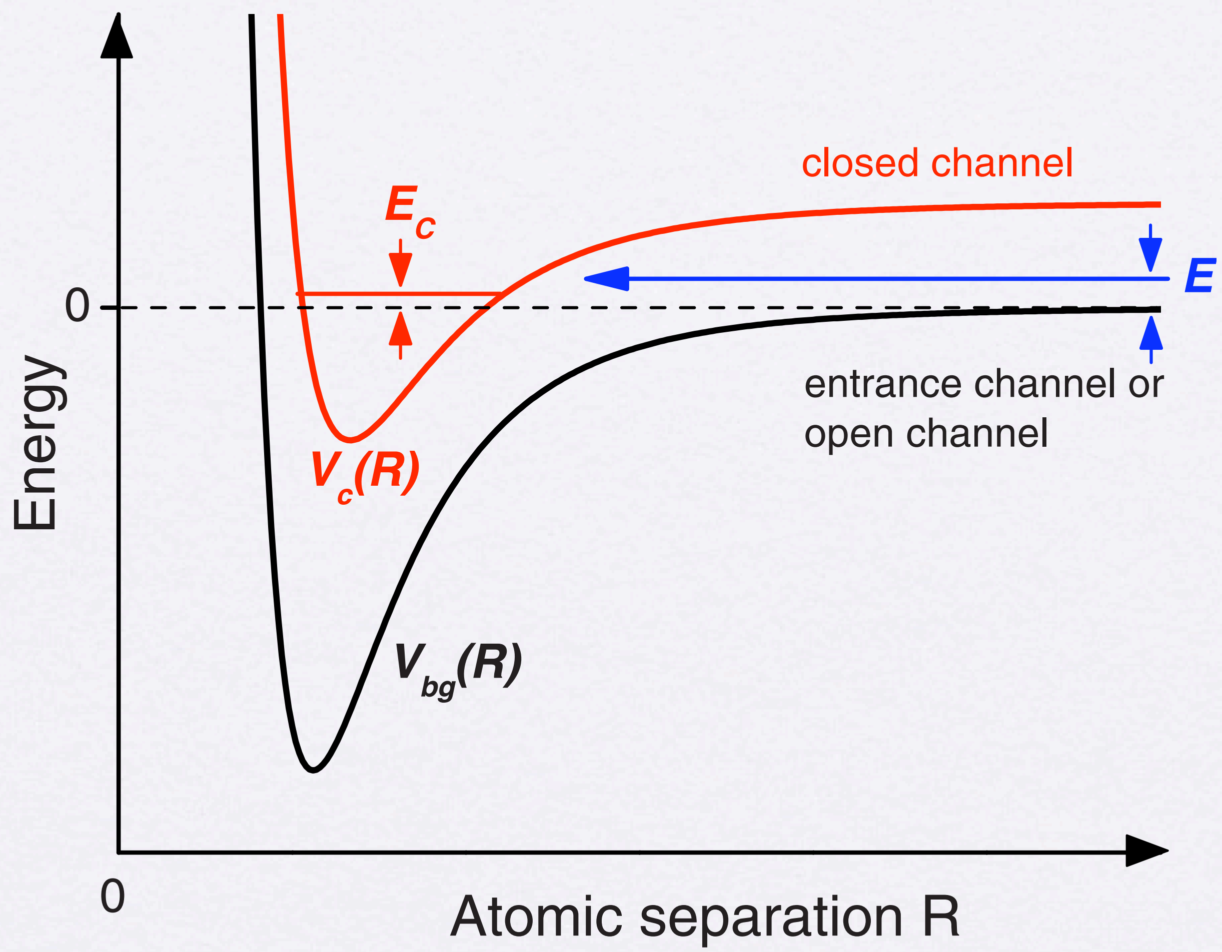
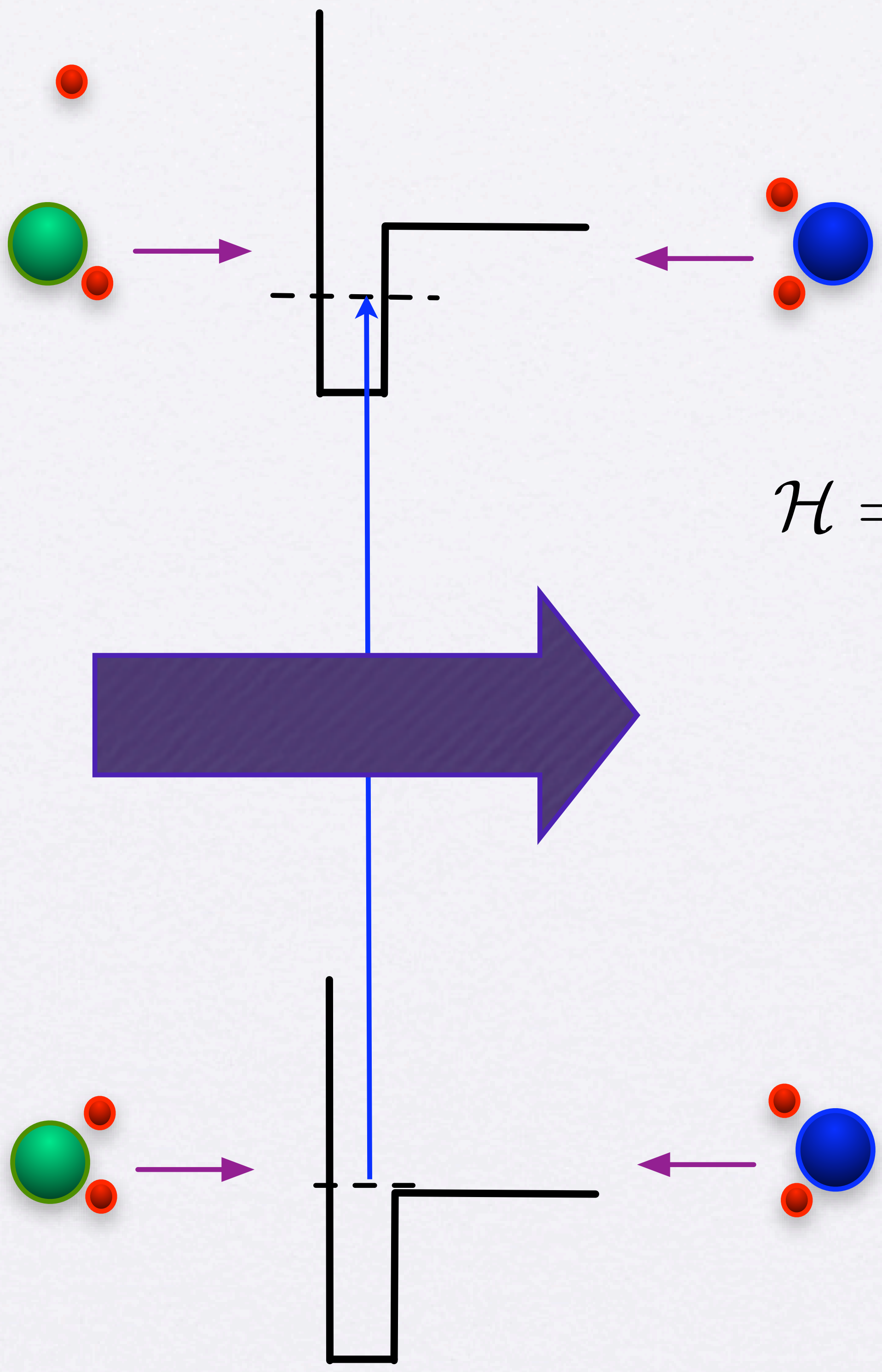
Feshbach resonance is an important tool to achieve strong interactions in ultracold Fermi gases



magnetic Feshbach resonance; optical Feshbach resonance; confinement induced resonance

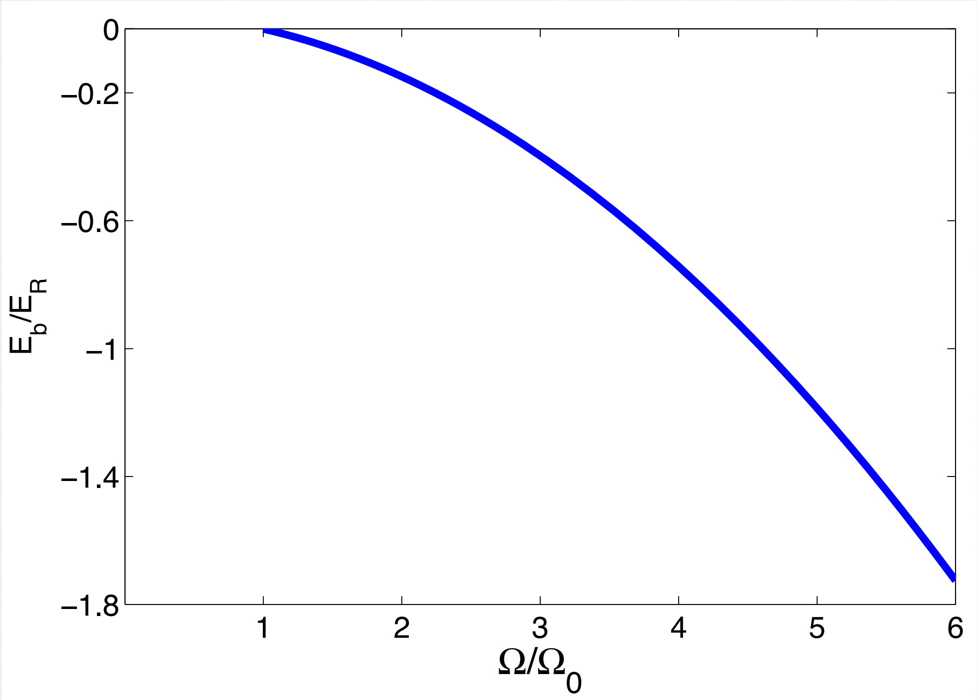
Idea of Optical Feshbach resonance



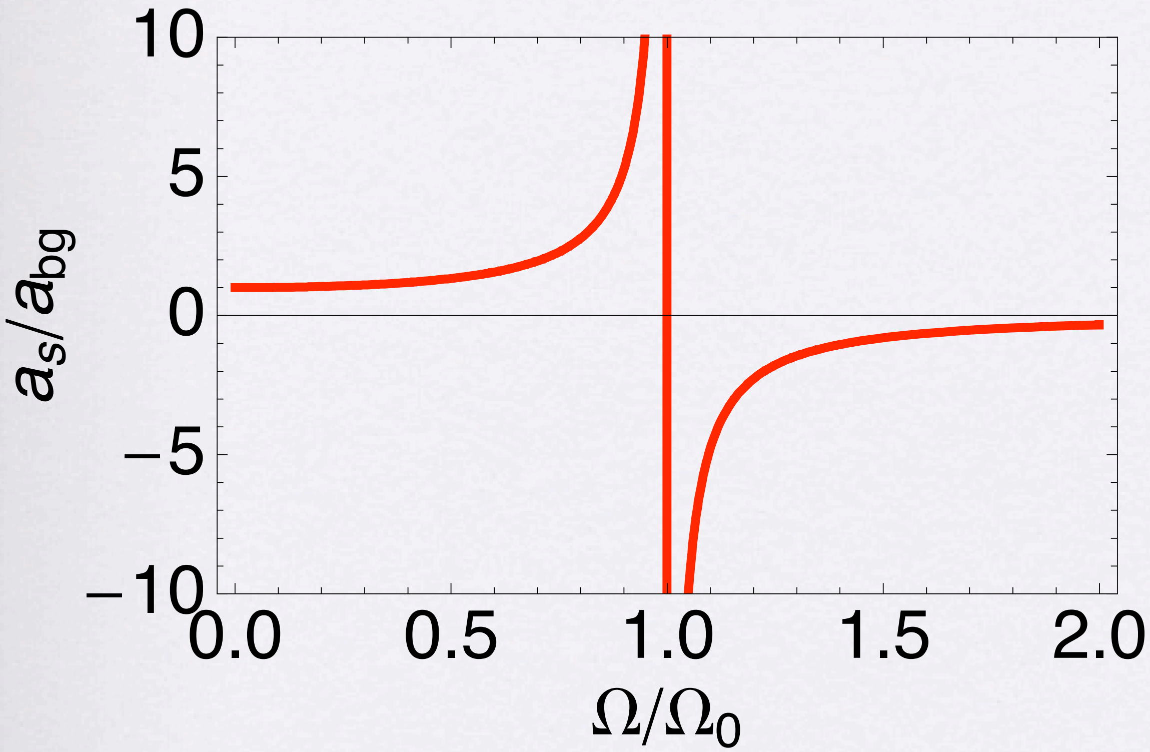
$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega \\ \hbar\Omega & -V_c \end{pmatrix}$$

Optical Feshbach resonance



$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

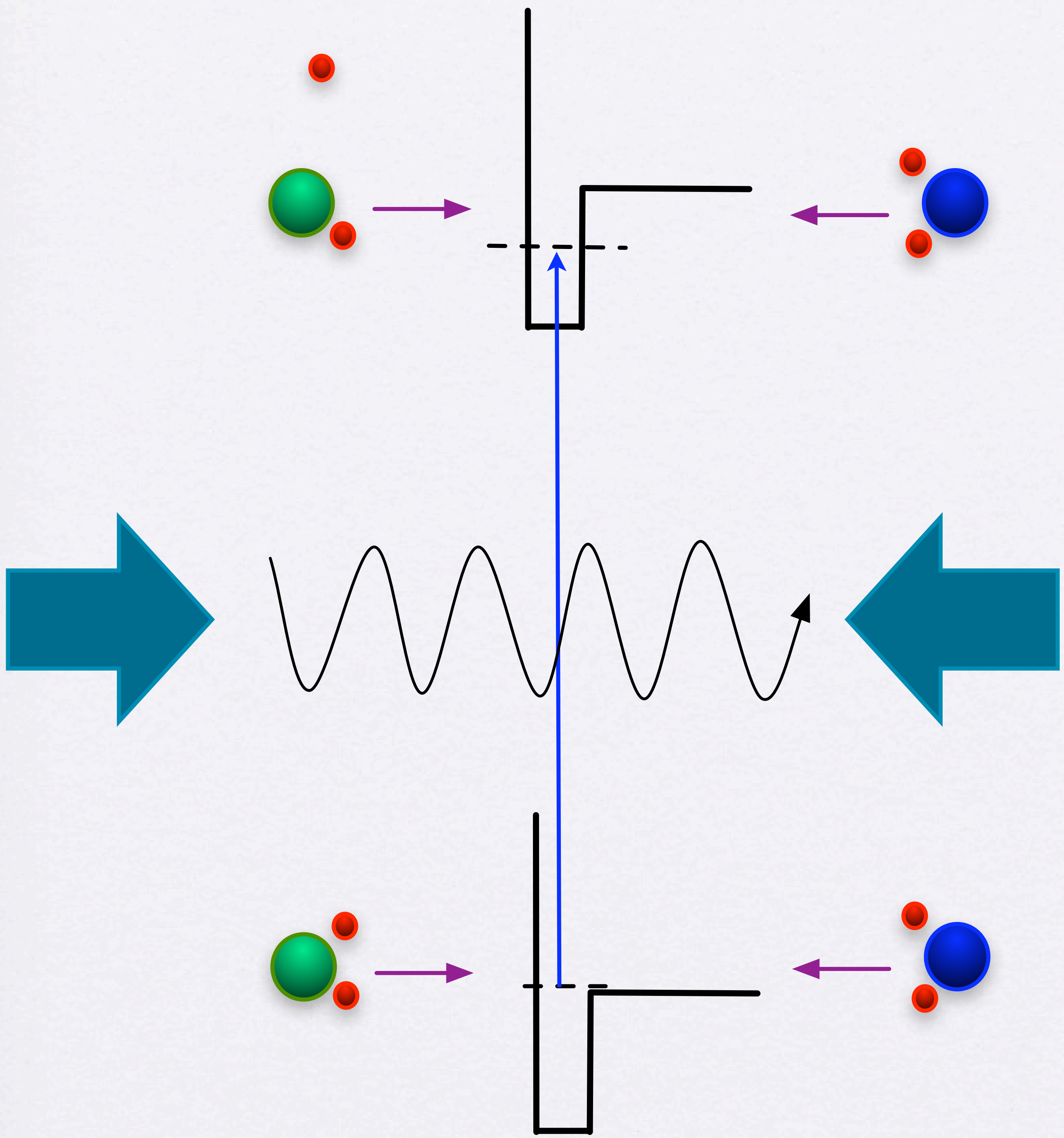


$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega \\ \hbar\Omega & -V_c \end{pmatrix}$$

$$a_{\mathrm{s}} = a_{\mathrm{bg}}\left(1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2}\right)$$

C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).

Optical Feshbach resonance with Standing wave

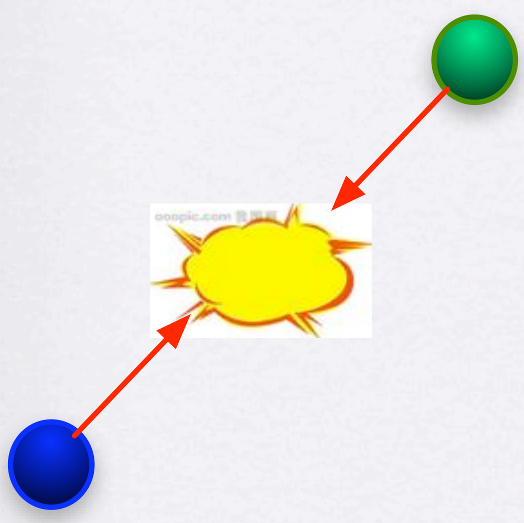


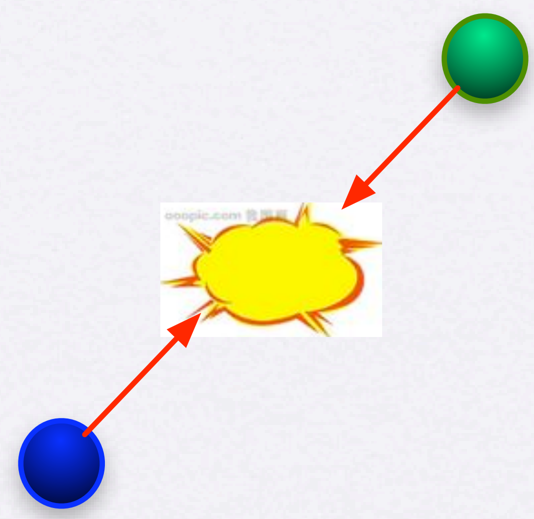
Spatial dependent interaction

Two-body interaction potential:

$$V(\mathbf{r_1}, \mathbf{r_2}) = V(\mathbf{r_1} - \mathbf{r_2})$$

Spatial independent





Spatial dependent interaction

Two-body interaction potential:

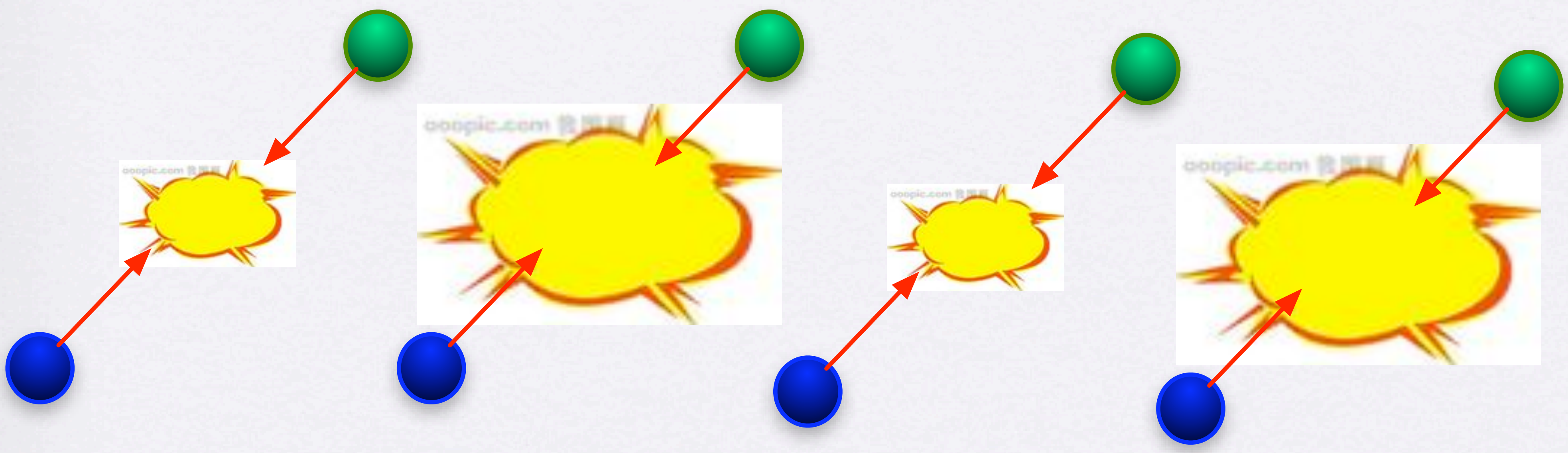
$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla_{\mathbf{R}}^2 - \frac{\hbar^2}{m}\nabla_{\mathbf{r}}^2 + v(\mathbf{r}, \mathbf{R})$$

$$V(\mathbf{r_1}, \mathbf{r_2}) = V\left(\mathbf{r_1} - \mathbf{r_2}, \frac{\mathbf{r_1} + \mathbf{r_2}}{2}\right)$$

$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega(\mathbf{R}) \\ \hbar\Omega(\mathbf{R}) & -V_c \end{pmatrix}$$

Spatially periodically modulated

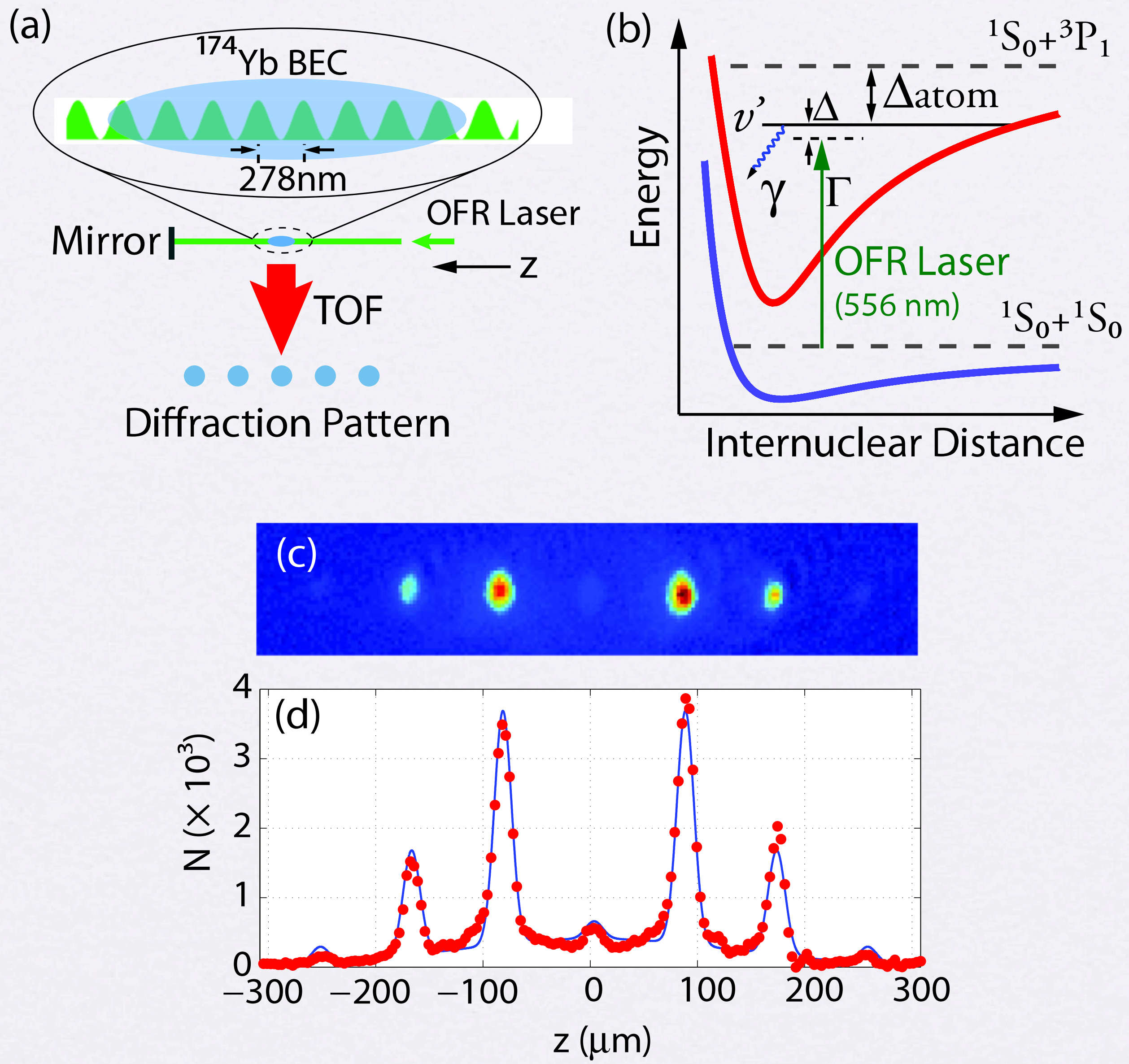
$$\Omega(\mathbf{R}) = \Omega\cos(Kx)$$



$a_{\mathrm{s}}(x)$ **is spatially dependent and modulates periodically in space**

Experimental Realization

Submicron spatial modulation of an interatomic interaction in a Bose-Einstein condensate, PRL, 105, 050405 (2010) Kyoto group



How $a_{\rm s}(x)$ modulates in space?

$$a_{\rm s} = a_{\rm bg}\left(1 - \frac{\Omega^2}{\Omega^2 - \Omega_0^2}\right) \qquad \Omega(\mathbf{R}) = \Omega\cos(Kx)$$

$$a_{\rm s}(x) = a_{\rm bg}\left(1 - \frac{\Omega^2\cos^2(Kx)}{\Omega^2\cos^2(Kx) - \Omega_0^2}\right)$$

?

Any other physics effects?

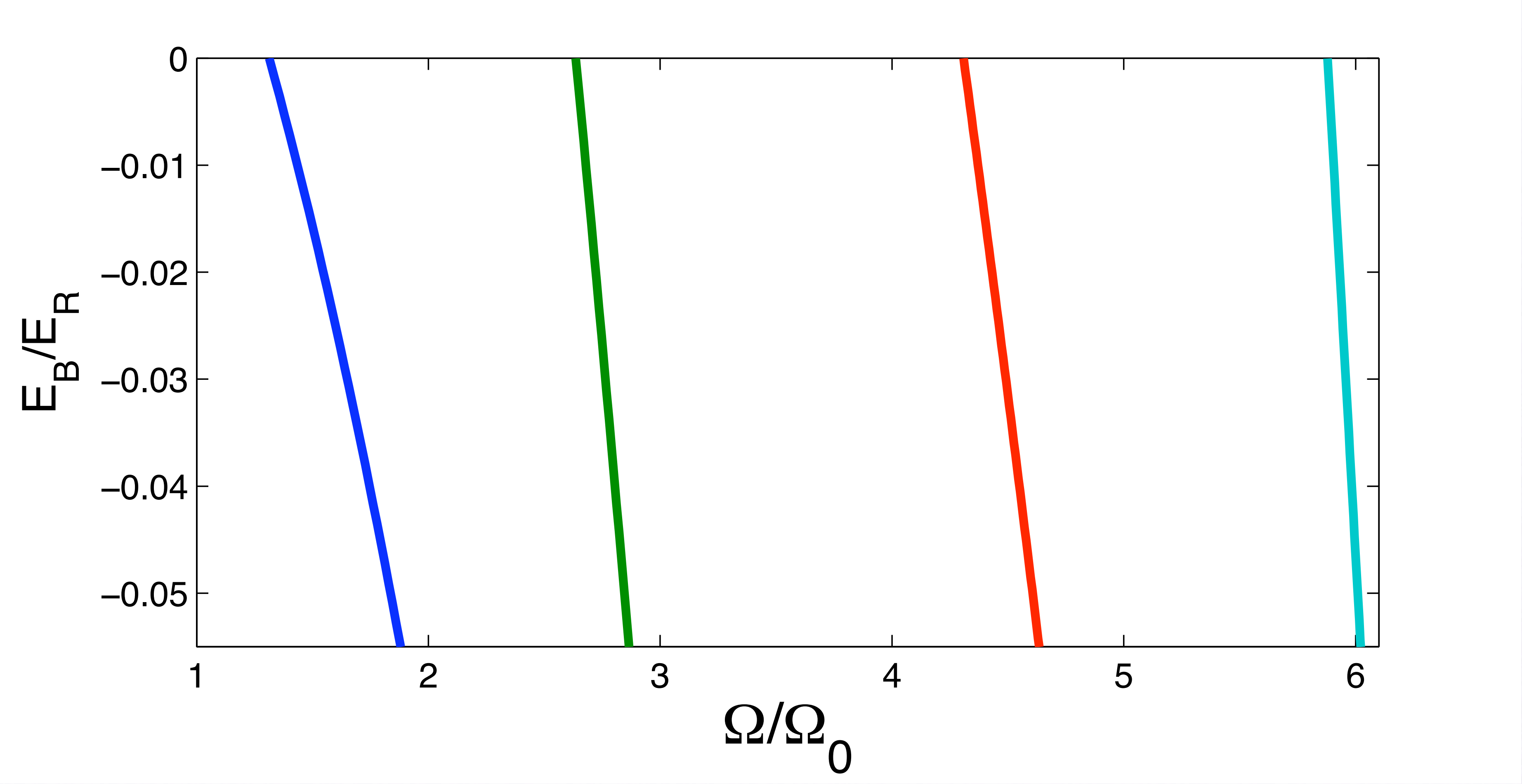
What we have done:
Solve two-body problem of this Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{m}\nabla^2_{\mathbf{r}} + v(\mathbf{r})$$

$$v(\mathbf{r}) = \begin{pmatrix} -V_0 & \hbar\Omega(\mathbf{R}) \\ \hbar\Omega(\mathbf{R}) & -V_c \end{pmatrix}$$

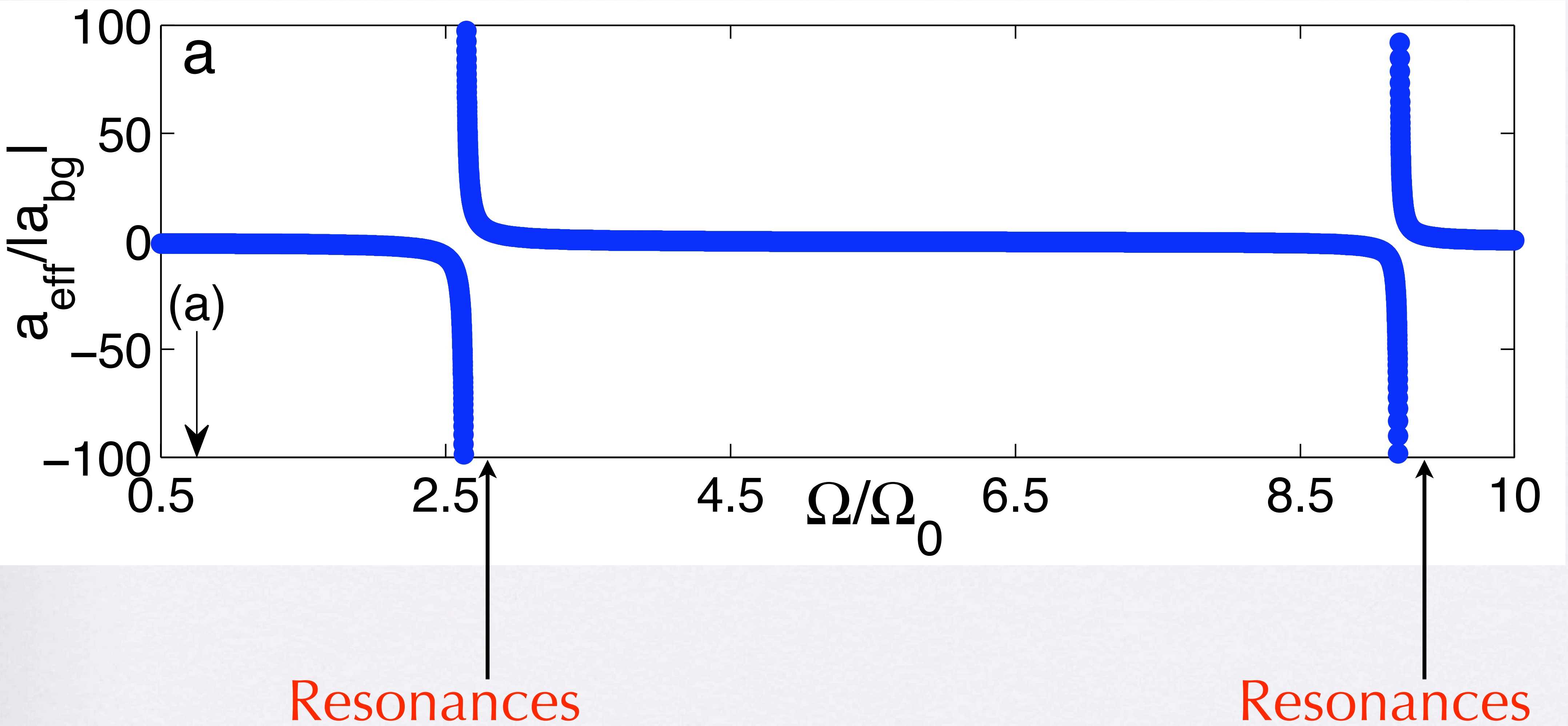
$$\Omega(\mathbf{R}) = \Omega\cos(Kx)$$

Results I: Bound States



Results II: Scattering Resonances

$$a_{\text{eff}} = \lim_{k \to 0} \frac{\tan \delta(k)}{k}$$



Strongly interacting many-body system !!
Universal Behavior ?

Results III: Local Scattering Length
--- related to local interaction energy

Bethe-Peierls condition:

$$\lim_{r\to 0}\psi(r,x)=\frac{1}{r}-\frac{1}{a_{loc}(x)}$$

Local scattering length

$$a_{\rm loc}(x)=-\lim_{r\to r_0}\frac{r\psi_{\rm o}(x,r)}{\partial_r(r\psi_{\rm o}(x,r))}$$

The mean-field energy for a BEC:

$$\mathcal{E}=\int dx\left[-\frac{\hbar^2}{2m}\varphi^*\nabla^2\varphi+\frac{4\pi\hbar^2}{m}a_{loc}(x)n^2(x)\right]$$

Results III: Local Scattering Length

Exact formula:

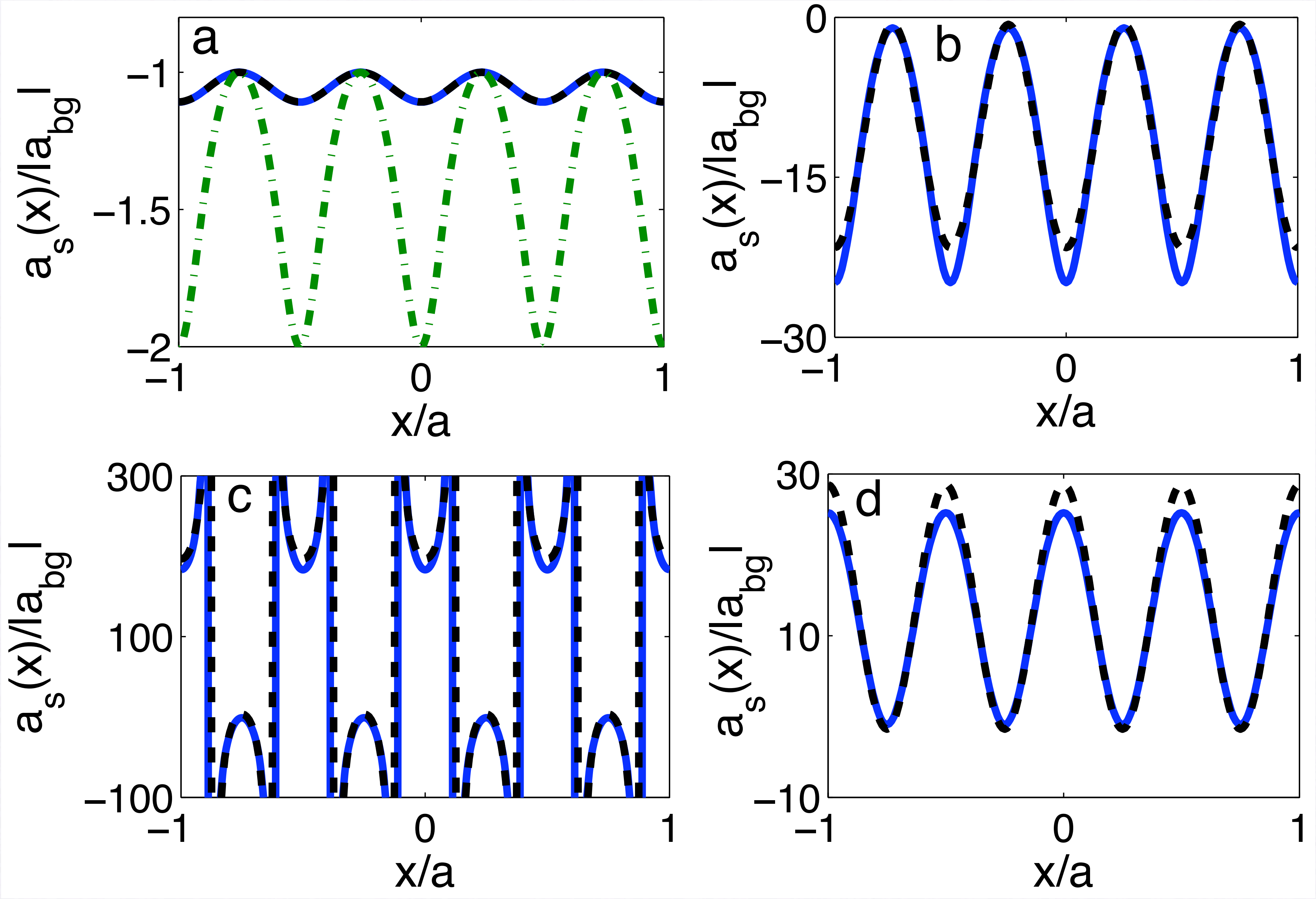
$$a_{\rm loc}(x) = \frac{1 - \sum_{m\neq 0} U_m \cos(mKx)/U_0}{a_{\rm eff}^{-1} - \sum_{m\neq 0} U_m |m| K \cos(mKx)/(2U_0)}$$

Simplified formula

$$K a_{\rm eff} \ll 1 \quad a_{\rm loc}(x) = a_{\rm eff}\left[1 - \frac{2U_2}{U_0}\cos(2Kx)\right]$$

$$K a_{\rm eff} \gg 1 \quad a_{\rm loc}(x) = \frac{1}{K}\left[1 - \frac{U_0}{2U_2\cos(2Kx)}\right]$$

Results III: Local Scattering Length



Summary:
Take Home Message

New Mechanism	New System	New Features
Two-body interaction potential has center-of-mass dependence	Alkali-earth-(like) atomic gases: Sr, Ca, Yb	Spatially dependent local scattering length