2D and Quasi-2D Fermi Gases with Rashba Spin-Orbit Coupling

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Outline

- Introduction
  - BCS-BEC crossover in 2D
  - 2D Fermi gas with Rashba SOC
- Two-body physics in 2D and Q2D
  - Scattering state
  - Bound state
- Many-body physics in Q2D
  - Effective 2D Hamiltonian
  - Q2D Fermi gas with Rashba SOC
- Summary
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• Summary
BCS-BEC crossover in 2D

- Zero $T$

Randeria, Duan, Shieh
PRB 41, 327 (1990)
BCS-BEC crossover in 2D

- Finite T

Botelho and Sa de Melo, PRL 96, 040404 (2006)
2D Fermi gas with Rashba SOC

- Without polarization

Chen, Gong, Zhang, PRA 85, 013601 (2012)
2D Fermi gas with Rashba SOC

- With polarization: Homogeneous case

Zhou, WZ, Yi
PRA 84, 063603 (2011)

Tewari et al.
NJP 13, 065004 (2011)
\[ p_x + ip_y \quad (p_x - ip_y) \]
Yang and Wan
PRA 85, 023633 (2012)
2D Fermi gas with Rashba SOC

- In trap

Zhou, WZ, Yi
PRA 84, 063603 (2011)

\[ E_b/E_F = 0.5, \; \alpha k_F/E_F = 0.6, \; h/E_F = 1.45, \; P = 0.662. \]
2D Fermi gas with Rashba SOC

- Finite-$T$

$h = 0$

$\alpha K_F = 1.0 \bar{E}_F$

He and Huang, PRL 108, 145302 (2012)
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Two-body scattering state (2D)

\[ H^{(2D)} = H_0^{(2D)} + V_{2D}(\rho), \]

\[ \downarrow \langle \rho | \psi_c^{(0)} \rangle = \frac{e^{ik \cdot \rho}}{2^{3/2} \pi} |\alpha(q,k)\rangle_s - \frac{e^{-ik \cdot \rho}}{2^{3/2} \pi} |\bar{\alpha}(q,-k)\rangle_s. \]

\[ \Downarrow \langle \rho | \psi_c^{(+)} \rangle \approx \downarrow \langle \rho | \psi_c^{(0)} \rangle + A(c)_{\Downarrow} \langle \rho | g(\varepsilon_c) |0\rangle_{\Downarrow} |0,0\rangle_s, \]

\[ f^{(2D)}(c' \leftarrow c) = -2\pi^2 \langle \psi_{c'}^{(0)} |0\rangle_{\Downarrow} |00\rangle_s A(c). \]

\[ A(c) = \frac{(2\pi)_S \langle 00 | \Downarrow \langle 0 | \psi_c^{(+)} \rangle}{i\pi/2 - C - \ln(d\sqrt{\varepsilon_c/2}) - (2\pi)\lambda(\varepsilon_c, q)}. \]
Two-body scattering state (2D)

\[ A(c) = \frac{(2\pi)S\langle 00|\hat{\nabla}|0|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln \left(d\sqrt{\varepsilon_c/2}\right) - (2\pi)\lambda(\varepsilon_c, q)}. \]

- Scattering amplitude is q-dependent
- Qualitative change of behavior at low-energy limit

\[
\lim_{\varepsilon \to 0} f_0^{(2D)} \propto \frac{1}{\ln \varepsilon_c}.
\]

\[
\lim_{\varepsilon_c \to \varepsilon_{\text{thre}}(q)} f^{(2D)} \propto \sqrt{\varepsilon_c - \varepsilon_{\text{thre}}(q)}.
\]
Two-body scattering state (2D)

\[ F \equiv \frac{f^{(2D)} (c' \leftarrow c)}{\langle \psi^{(0)}_{c'} | 0 \rangle_\perp \langle 0 | \psi^{(+)}_c \rangle} \]
Two-body scattering state (Q2D)

\[ A_{\text{eff}}(c) = \frac{(2\pi)_S \langle 00 \parallel \langle 0 | \psi_c^{(+)} \rangle}{i\pi/2 - C - \ln \left\{ d_{\text{eff}}(\varepsilon_c, q) \sqrt{\varepsilon_c/2} \right\} - (2\pi) \lambda(\varepsilon_c, q)}. \]

\[ \ln d_{\text{eff}}(\varepsilon_c, q) = -\sqrt{2\pi w} \left( \frac{\varepsilon_c}{2\omega} \right) - \ln \left( -\frac{i\sqrt{\varepsilon_c}}{2} \right) \]

\[ -C - \frac{\pi l_0}{a} + (2\pi)^2 l_0 \sum_{n_z=1}^{\infty} |\varphi_{n_z}(0)|^2 \lambda(\varepsilon_c - n_z \omega; q). \]
Two-body scattering state (Q2D)

Zhang, Zhang, WZ
arXiv:1203:0623
Two-body bound state (2D)

\[-\ln d = C + \ln \left(-\frac{i\sqrt{\varepsilon_b}}{2}\right) + (2\pi)\lambda(\varepsilon_b, q)\,.

Takei, et al,
PRA 85, 023626 (2012)

Zhang, Zhang, WZ, arXiv:1203:0623
Two-body bound state (Q2D)

- Two-channel model

\[
H = H_0 + H_{soc} + H_{bf} + H_{int}.
\]

\[
|\Psi\rangle_{\ell,q} = \left( \beta_{\ell,q} b_{\ell,q}^{\dagger} \right)
+ \sum_{m,n,k} \sum_{\sigma,\sigma'} \eta_{m,n,k,q}^{\sigma\sigma'} c_{m,k+q/2,\sigma}^{\dagger} c_{n,-k+q/2,\sigma'}^{\dagger} |0\rangle
\]
Two-body bound state (Q2D)

- $q=0$

![Graph showing $-|E_b(q=0)|$ vs. $a_t/a_s$ for different values of $\lambda$. The graph includes curves for $\lambda = 0$, $\lambda = 1$, $\lambda = 2$, and $\lambda = 4$. The y-axis ranges from -20 to 0, and the x-axis ranges from -2 to 4.](image)
Two-body bound state (Q2D)

- $q=0$
Two-body bound state (Q2D)

- General $q$
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Effective 2D Hamiltonian (w/o SOC)

Kestner and Duan, PRA 74, 053606 (2006)
Effective 2D Hamiltonian (w/o SOC)

WZ, Lin, Duan, PRA 77, 063613 (2008)
Effective 2D Hamiltonian (w/SOC)

- Q2D model:
  - fermions in ground state \( n=0 \)
  - fermions in excited states \( n=1,2,3\ldots \)
  - Feshbach molecules

- Effective 2D Hamiltonian (2-channel model)
  - 2D Fermions
  - dressed molecules (structureless)

- Matching conditions
  - open channel threshold
  - background scattering
  - two-body binding energy
  - fermions in ground state

\[
\Delta \varepsilon = O \left( \frac{\mu - E_b / 2}{\hbar \omega_z} \right)^2
\]
Effective 2D Hamiltonian

\[ H_{\text{eff}} = \sum_{k,\sigma} \varepsilon_k a_{k,\sigma}^\dagger a_{k,\sigma} + \delta_b d_0^\dagger d_0 + \frac{\alpha_b}{L} \sum_k \left( d_0^\dagger a_{k,\uparrow} a_{-k,\downarrow} + \text{H.C.} \right) \]

\[ + \frac{V_b}{L^2} \sum_{k,k'} a_{k,\uparrow}^\dagger a_{-k,\downarrow}^\dagger a_{-k',\downarrow} a_{k',\uparrow} + \gamma' \sum_k \left[ (k_x - i k_y) a_{k,\uparrow}^\dagger a_{k,\downarrow} + (k_x + i k_y) a_{k,\downarrow}^\dagger a_{k,\uparrow} \right] \]

\[ \gamma' = \gamma; \]

\[ V_p^{-1} = \sqrt{2\pi} \left( U_p^{-1} - C_p \right) \]
Effective 2D Hamiltonian

Zhang et al., in preparation
Q2D Fermi gas with Rashba SOC
Q2D Fermi gas with Rashba SOC
unitarity

P=0.1

\[ \frac{n}{P} \]

P=0.42

\[ \frac{n}{P} \]

P=0.82

\[ \frac{n}{P} \]
Summary

• SOC changes the qualitative behavior of 2D scattering state in the low-energy limit (logarithmic -> polynomial)
• SOC enhances the two-body binding energy in Q2D
• The axial excited states become more important
• An effective 2D model incorporating these DOF is required
• In-trap phase diagrams in Q2D Fermi gas can be qualitatively different from the 2D case
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