2D and Quasi-2D Fermi Gases with Rashba Spin-Orbit Coupling

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Outline

- Introduction
  - BCS-BEC crossover in 2D
  - 2D Fermi gas with Rashba SOC
- Two-body physics in 2D and Q2D
  - Scattering state
  - Bound state
- Many-body physics in Q2D
  - Effective 2D Hamiltonian
  - Q2D Fermi gas with Rashba SOC
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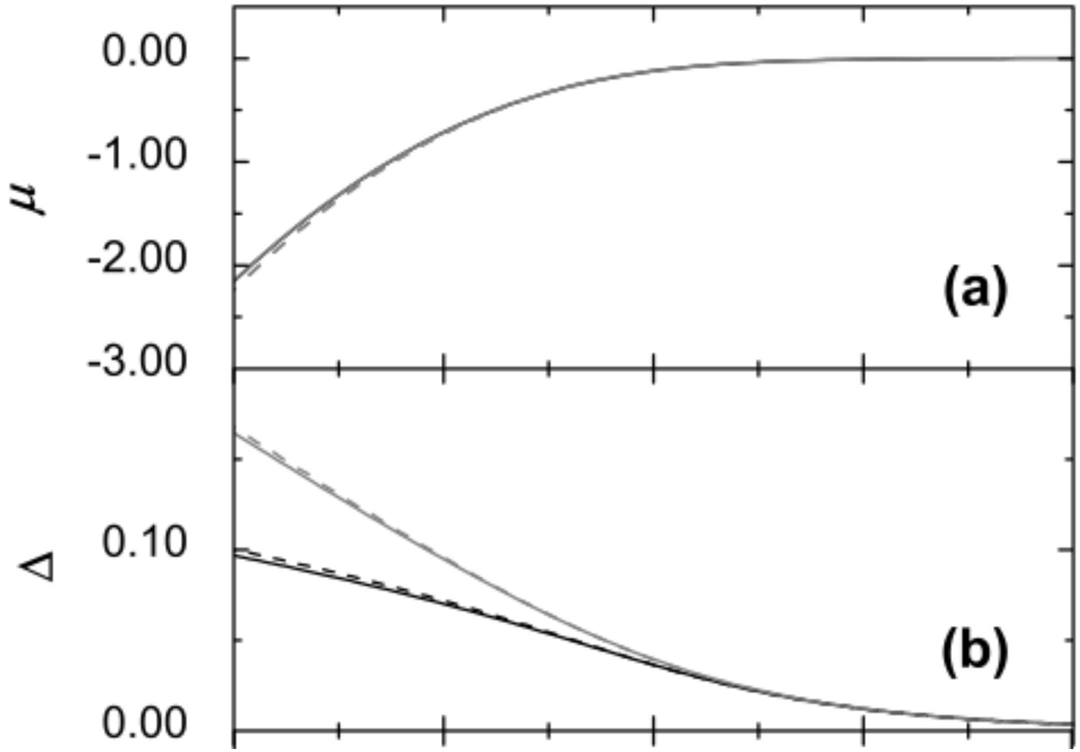
BCS-BEC crossover in 2D

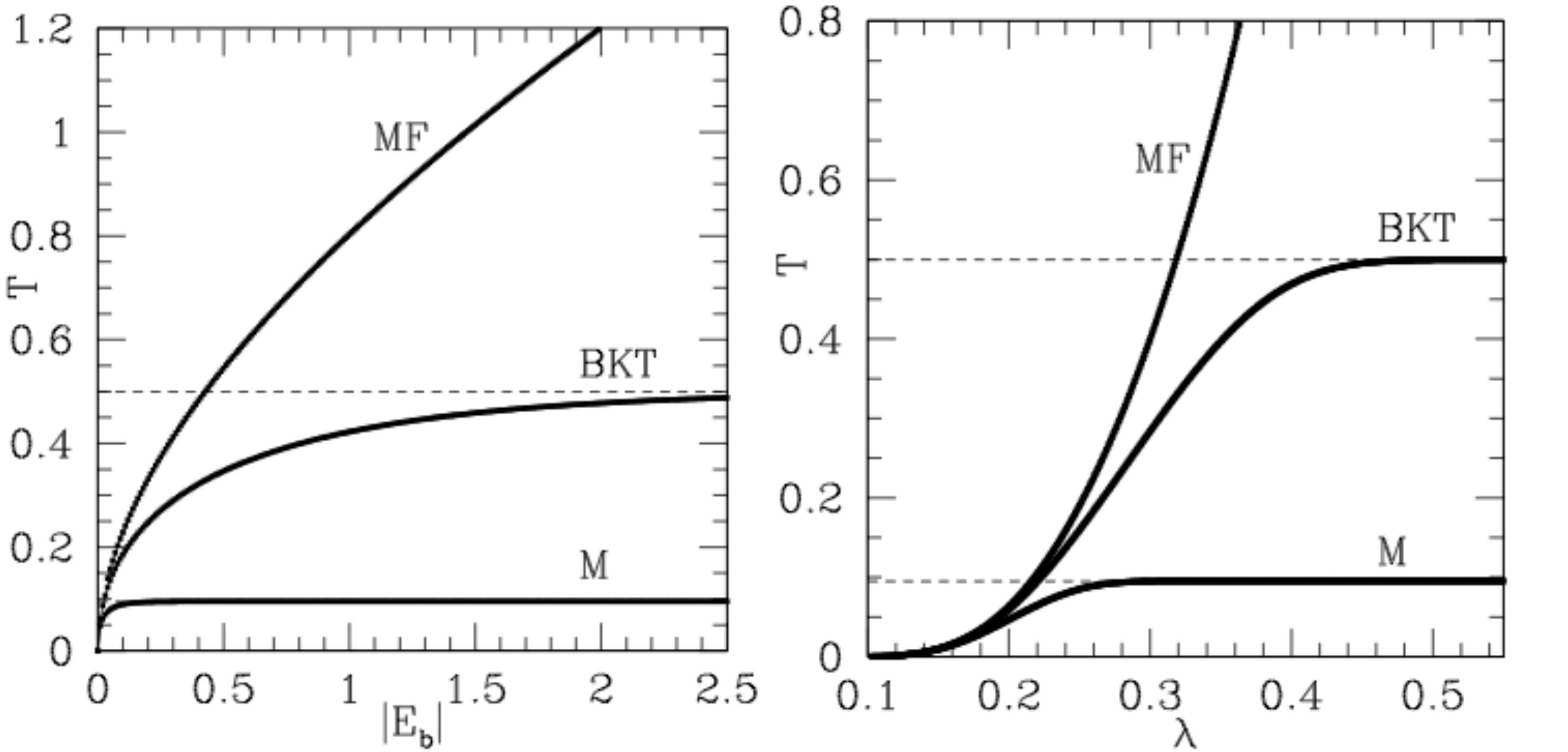
- Zero T



Randeria, Duan, Shieh
PRB 41, 327 (1990)

BCS-BEC crossover in 2D

- Finite T

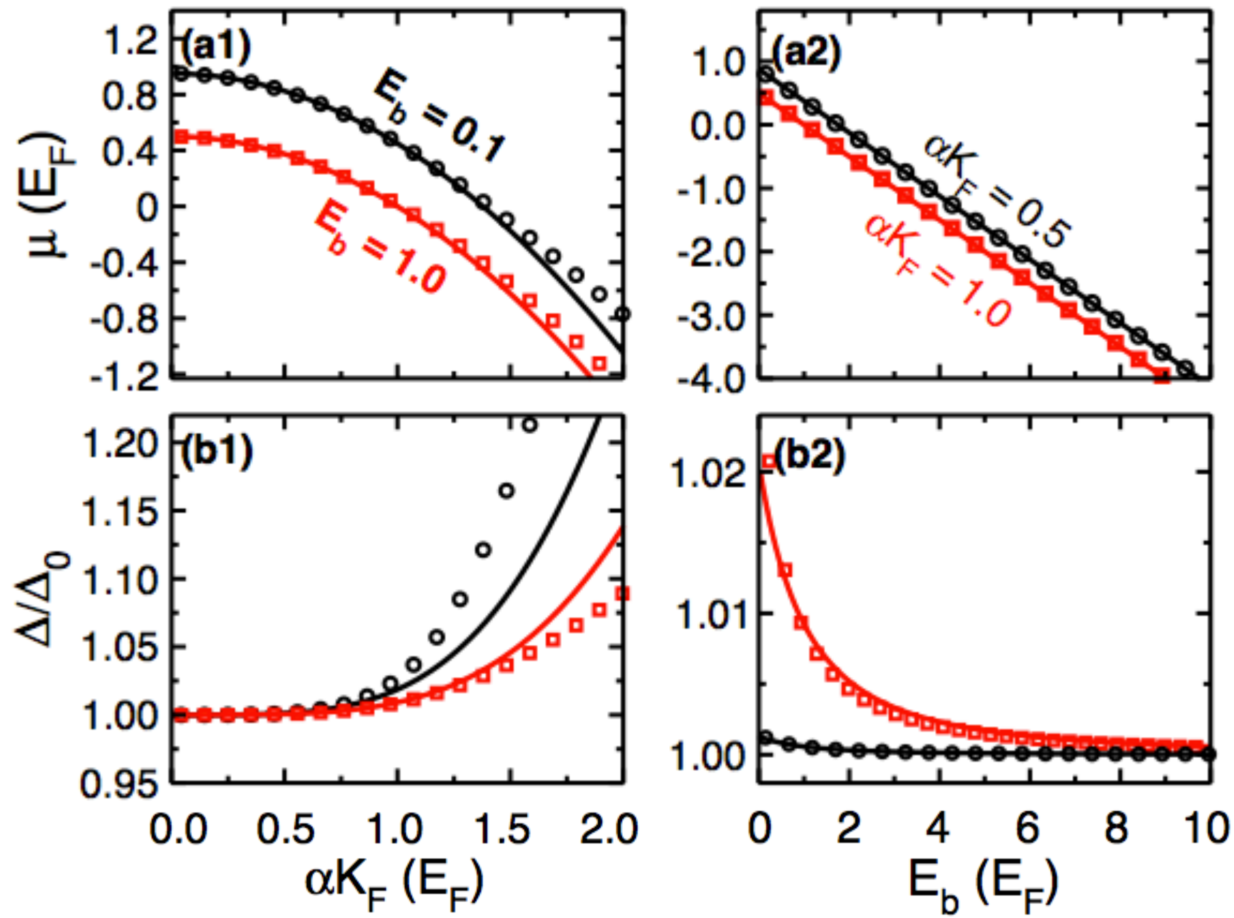


Botelho and Sa de Melo, PRL 96, 040404 (2006)

2D Fermi gas with Rashba SOC

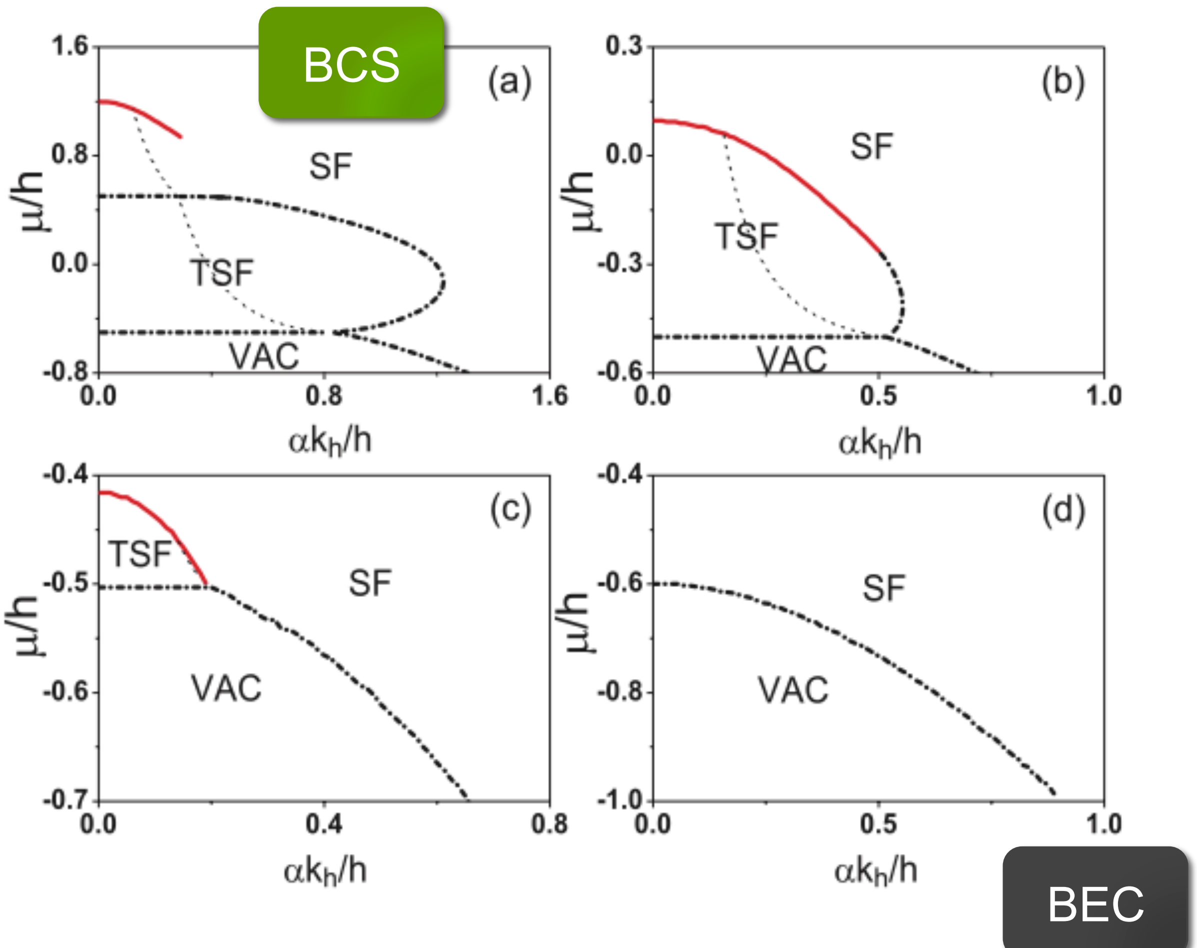
- Without polarization

Chen, Gong, Zhang, PRA 85, 013601 (2012)

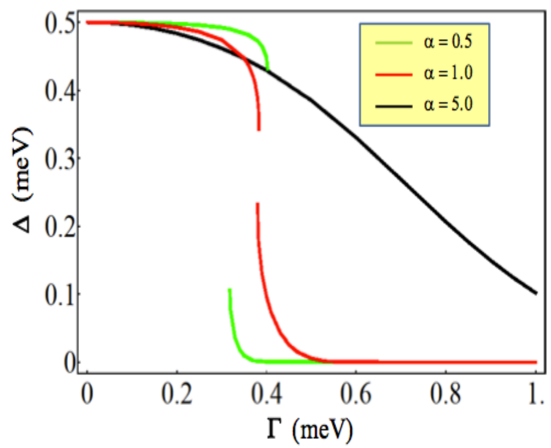


2D Fermi gas with Rashba SOC

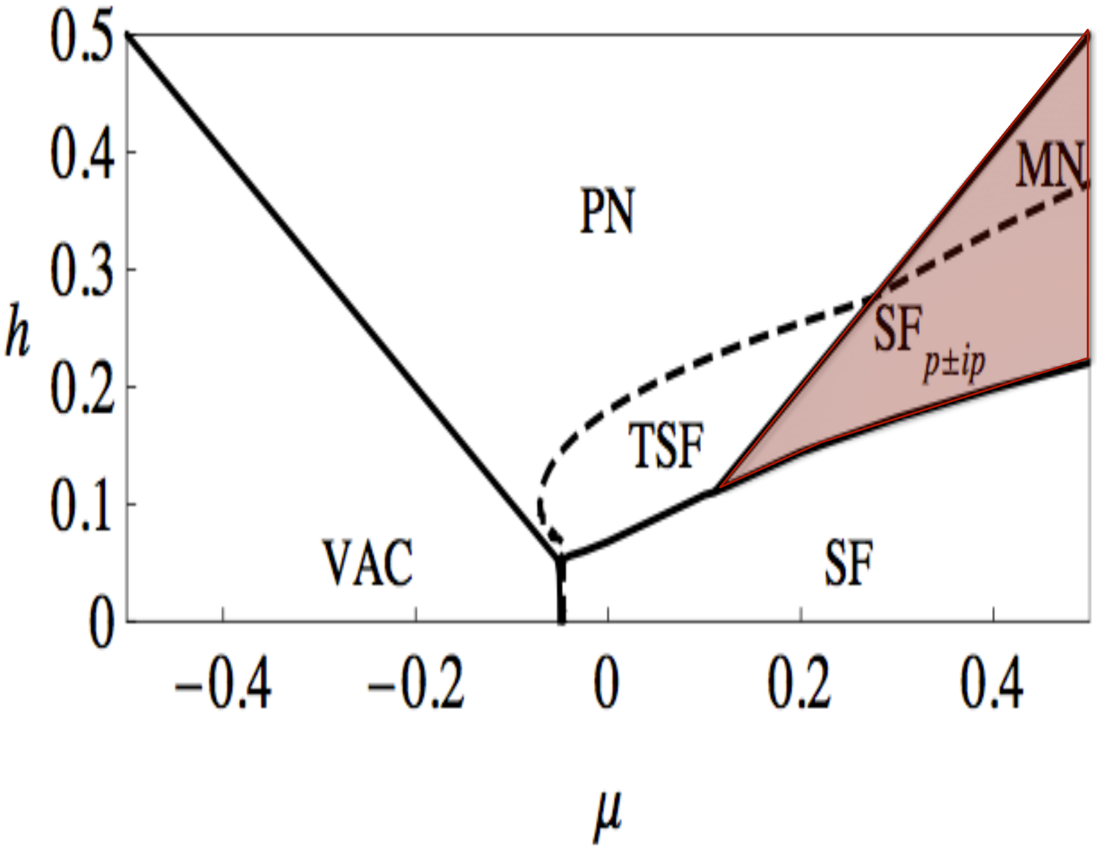
- With polarization: Homogeneous case



Zhou, WZ, Yi
PRA 84, 063603 (2011)



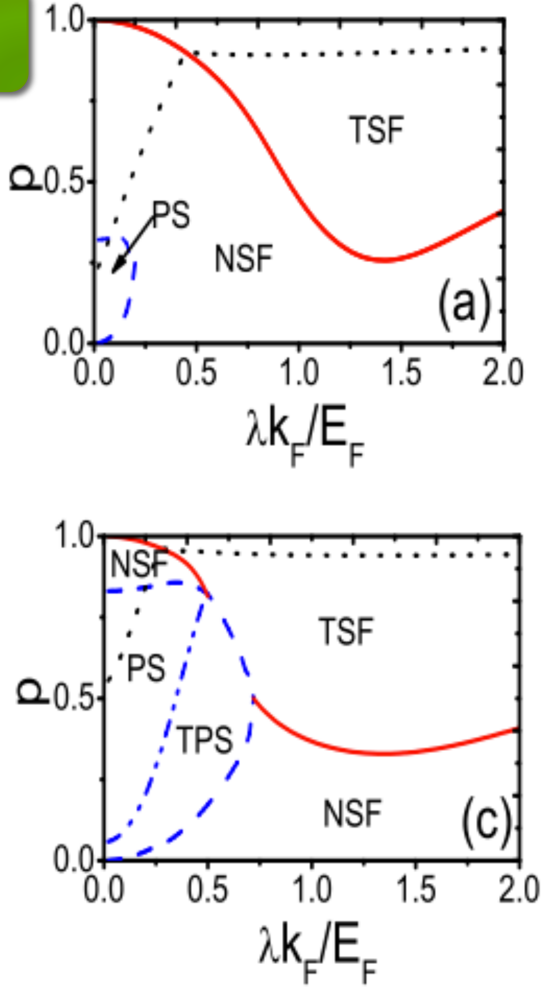
Tewari et al.
NJP 13, 065004 (2011)



$p_x + ip_y \ (p_x - ip_y)$

Zhang, Chan, Duan, arXiv:1110:2241

BCS



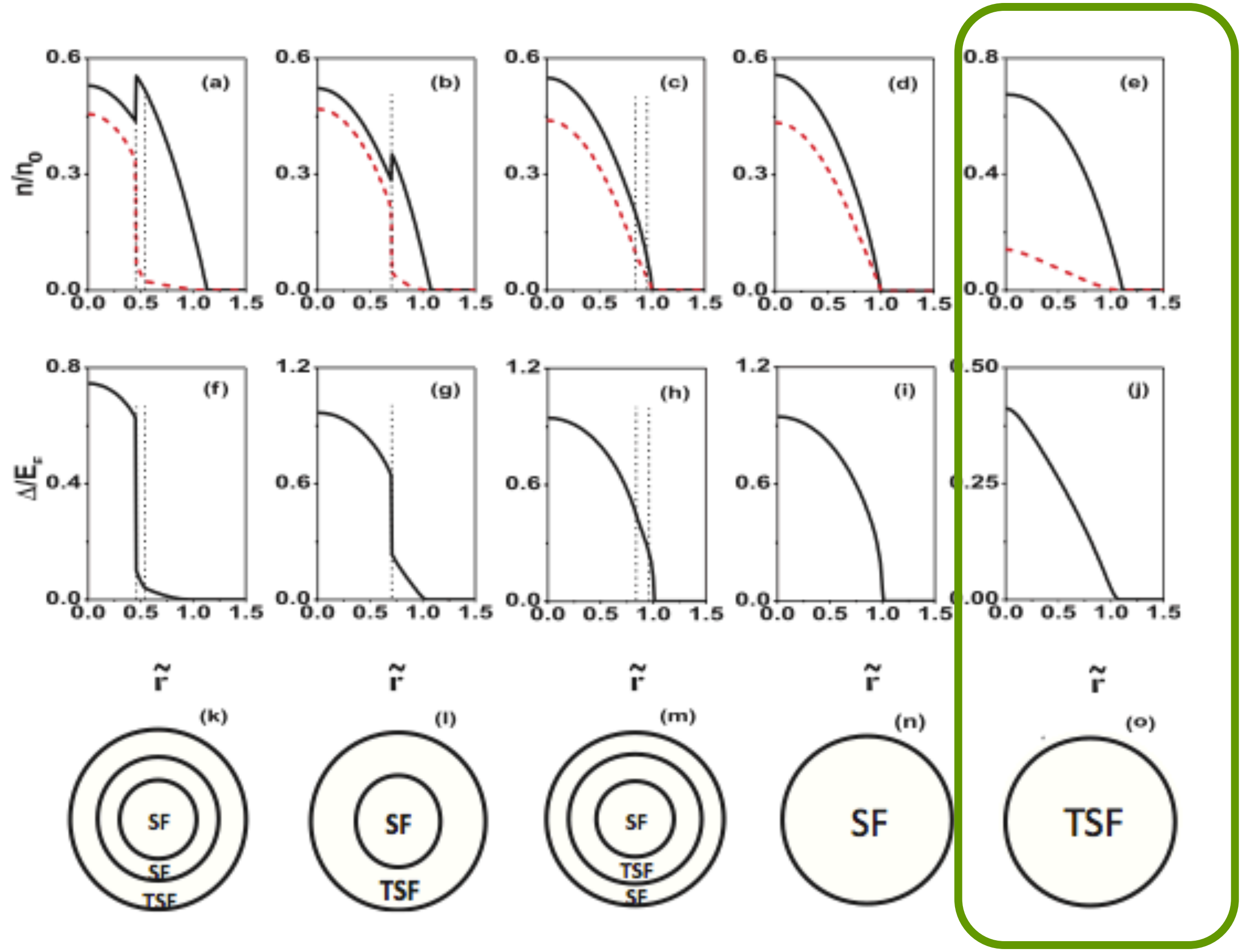
Yang and Wan
PRA 85, 023633
(2012)

BEC

2D Fermi gas with Rashba SOC

- In trap

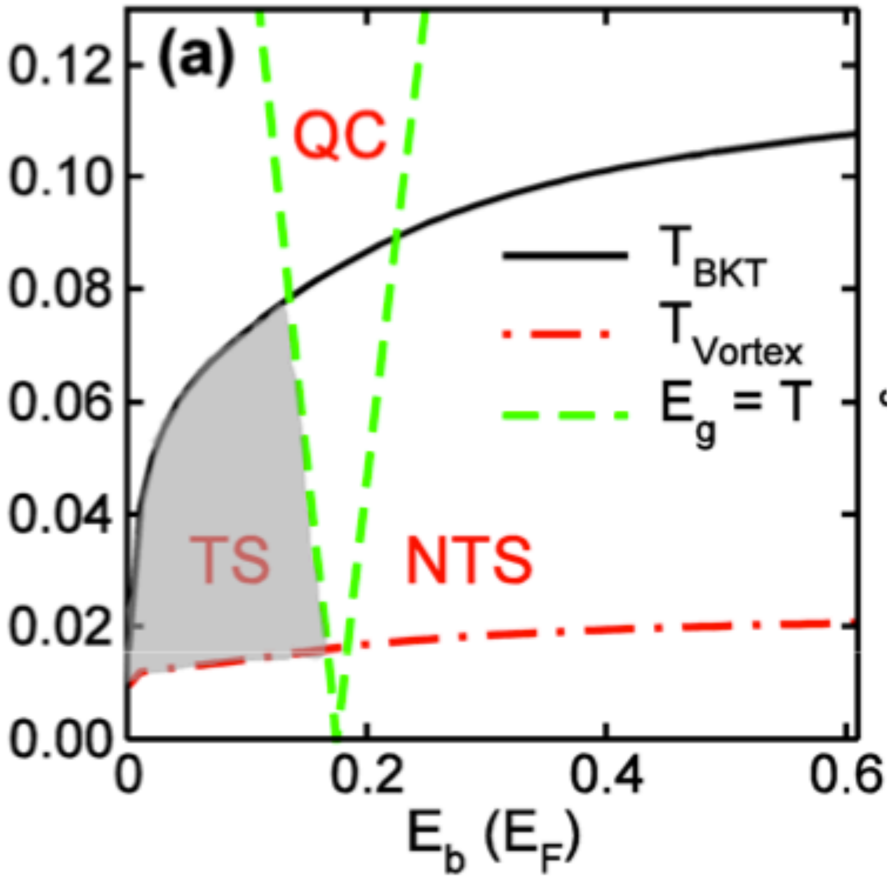
Zhou, WZ, Yi
PRA 84, 063603
(2011)



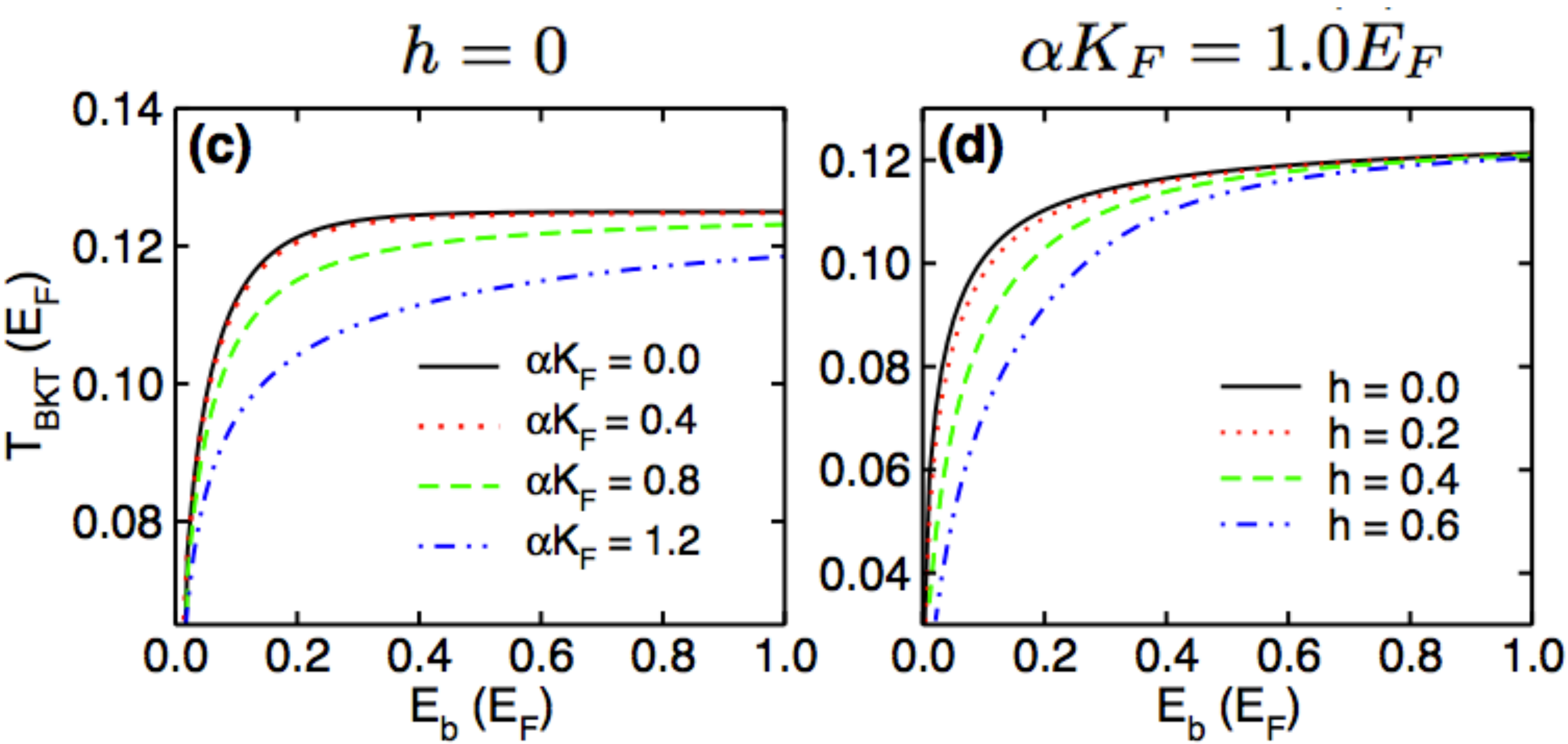
$$E_b/E_F = 0.5,\ \alpha k_F/E_F = 0.6, h/E_F = 1.45,\ P = 0.662.$$

2D Fermi gas with Rashba SOC

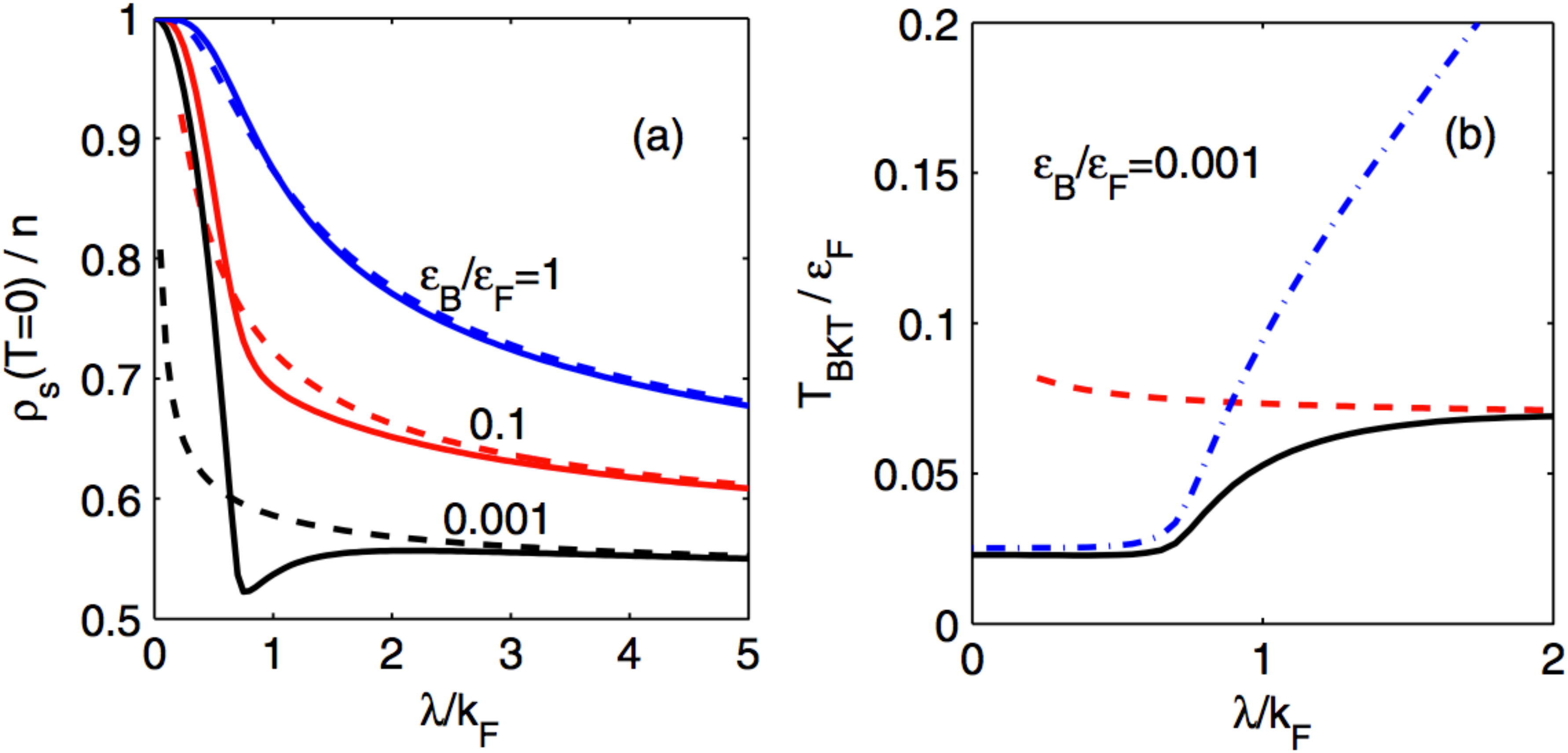
- Finite-T



Gong, Chen, Jia, Zhang, arXiv:1201.2238



Gong, Chen, Jia, Zhang, arXiv:1201:2238



He and Huang, PRL **108**,145302 (2012)

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Two-body scattering state (2D)

$$H^{(2D)} = H_0^{(2D)} + V_{2D}(\rho),$$

$$_\perp\langle \boldsymbol{\rho} | \psi_c^{(0)} \rangle = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{\rho}}}{2^{3/2}\pi} |\boldsymbol{\alpha}(\boldsymbol{q},\boldsymbol{k})\rangle_S - \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{\rho}}}{2^{3/2}\pi} |\bar{\boldsymbol{\alpha}}(\boldsymbol{q},-\boldsymbol{k})\rangle_S.$$

$$_\perp\langle \boldsymbol{\rho} | \psi_c^{(+)} \rangle \approx {}_\perp\langle \boldsymbol{\rho} | \psi_c^{(0)} \rangle + A(c)\,{}_\perp\langle \boldsymbol{\rho} | g(\varepsilon_c) | \mathbf{0} \rangle_\perp |0,0\rangle_S,$$

$$\boxed{f^{(2D)}(c' \leftarrow c) = -2\pi^2 \langle \psi_{c'}^{(0)} | \mathbf{0} \rangle_\perp |00\rangle_S A(c).}$$

$$\boxed{A(c) = \frac{(2\pi)\,{}_S\langle 00|\,{}_\perp\langle \mathbf{0} | \psi_c^{(+)} \rangle}{i\pi/2 - C - \ln\left(d\sqrt{\varepsilon_c}/2\right) - (2\pi)\lambda(\varepsilon_c, \boldsymbol{q})}.}$$

Two-body scattering state (2D)

$$A(c) = \frac{(2\pi)_S\langle 00|_\perp \langle \mathbf{0}|\psi_c^{(+)}\rangle}{i\pi/2 - C - \ln\left(d\sqrt{\varepsilon_c}/2\right) - (2\pi)\lambda\left(\varepsilon_c, \boldsymbol{q}\right)}.$$

- Scattering amplitude is q-dependent
- Qualitative change of behavior at low-energy limit

w/o SOC

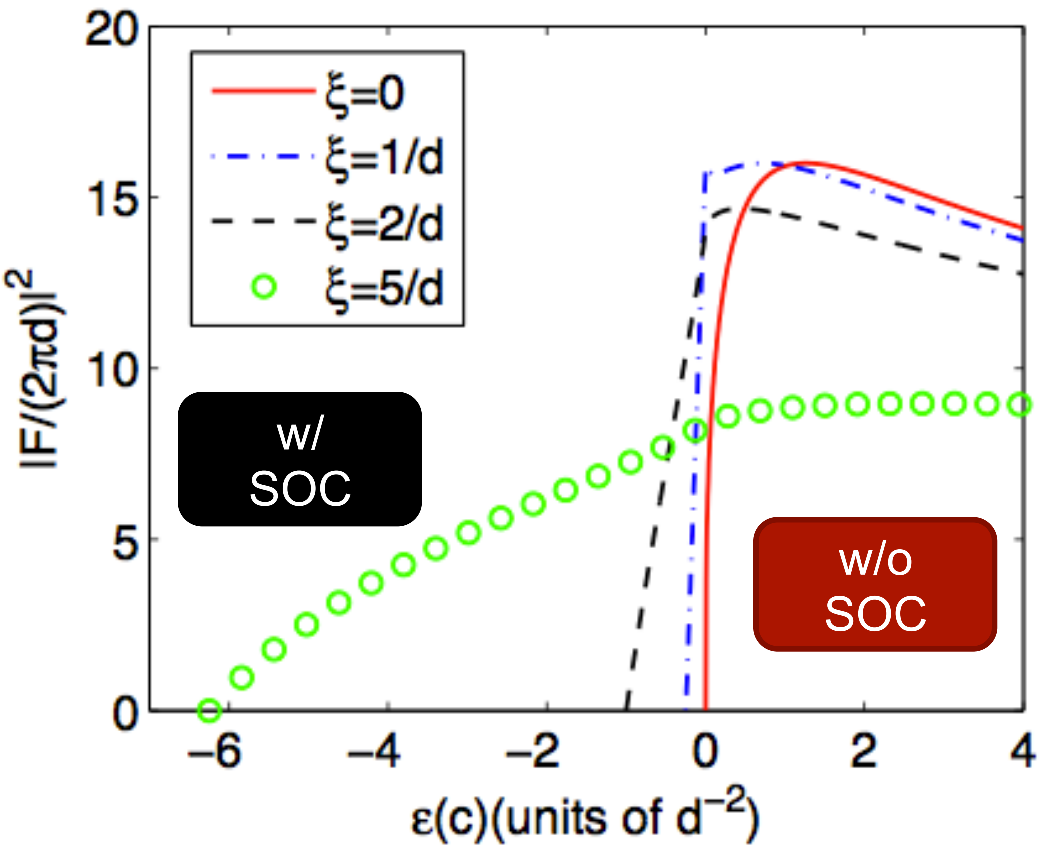
$$\lim_{\varepsilon\to 0} f_0^{(2\mathrm{D})} \propto \frac{1}{\ln \varepsilon_c}.$$

w/ SOC

$$\lim_{\varepsilon_c\to\varepsilon_{\mathrm{thre}}(q)} f^{(2\mathrm{D})} \propto \sqrt{\varepsilon_c - \varepsilon_{\mathrm{thre}}(q)}.$$

Two-body scattering state (2D)

$$F \equiv \frac{f^{(2\mathrm{D})}(c' \leftarrow c)}{\langle \psi_{c'}^{(0)} | \mathbf{0} \rangle_{\perp} | 00 \rangle_{S} \langle 00 |_{\perp} \langle \mathbf{0} | \psi_{c}^{(+)} \rangle}$$



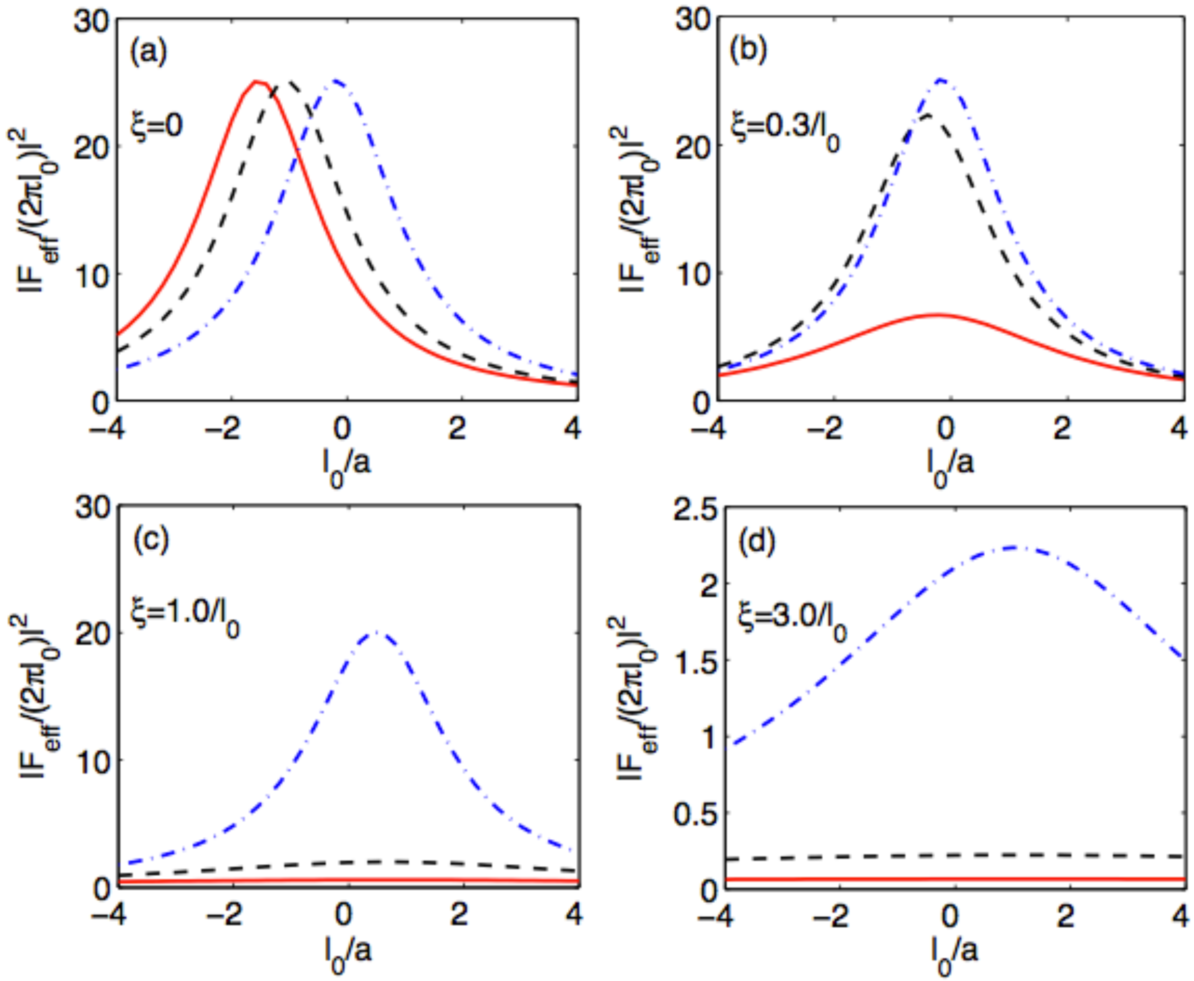
Zhang, Zhang, WZ
arXiv:1203:0623

Two-body scattering state (Q2D)

$$A_{\text{eff}}(c) = \frac{(2\pi)\,{}_S\langle 00|_\perp \langle \mathbf{0}|\psi_c^{(+)}\rangle}{i\pi/2 - C - \underline{\ln\left\{d_{\text{eff}}(\varepsilon_c, \boldsymbol{q})\sqrt{\varepsilon_c}/2\right\}} - (2\pi)\lambda(\varepsilon_c, \boldsymbol{q})}.$$

$$\ln d_{\text{eff}}(\varepsilon_c, \boldsymbol{q}) = -\frac{\sqrt{2\pi}w\left(\frac{\varepsilon_c}{2\omega}\right)}{2} - \ln\left(-\frac{i\sqrt{\varepsilon_c}}{2}\right)$$
$$-C - \frac{\pi l_0}{a} + (2\pi)^2 l_0 \sum_{n_z=1}^{\infty} |\varphi_{n_z}(0)|^2 \lambda(\varepsilon_c - n_z\omega; \boldsymbol{q}).$$

Two-body scattering state (Q2D)



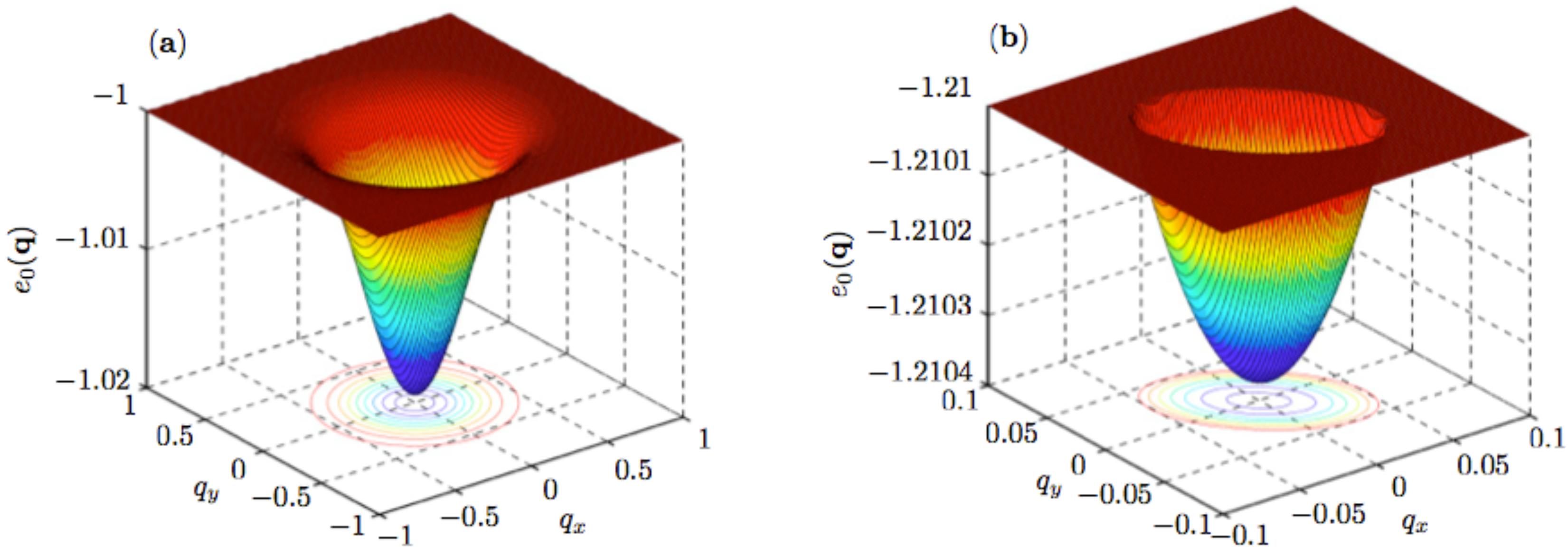
Zhang, Zhang, WZ
arXiv:1203:0623

Two-body bound state (2D)

$$\boxed{-\ln d = C + \ln\left(-\frac{i\sqrt{\varepsilon_b}}{2}\right) + (2\pi)\,\lambda(\varepsilon_b, \boldsymbol{q}).}$$

Zhang, Zhang, WZ, arXiv:1203:0623

Takei, et al,
PRA 85, 023626 (2012)



Two-body bound state (Q2D)

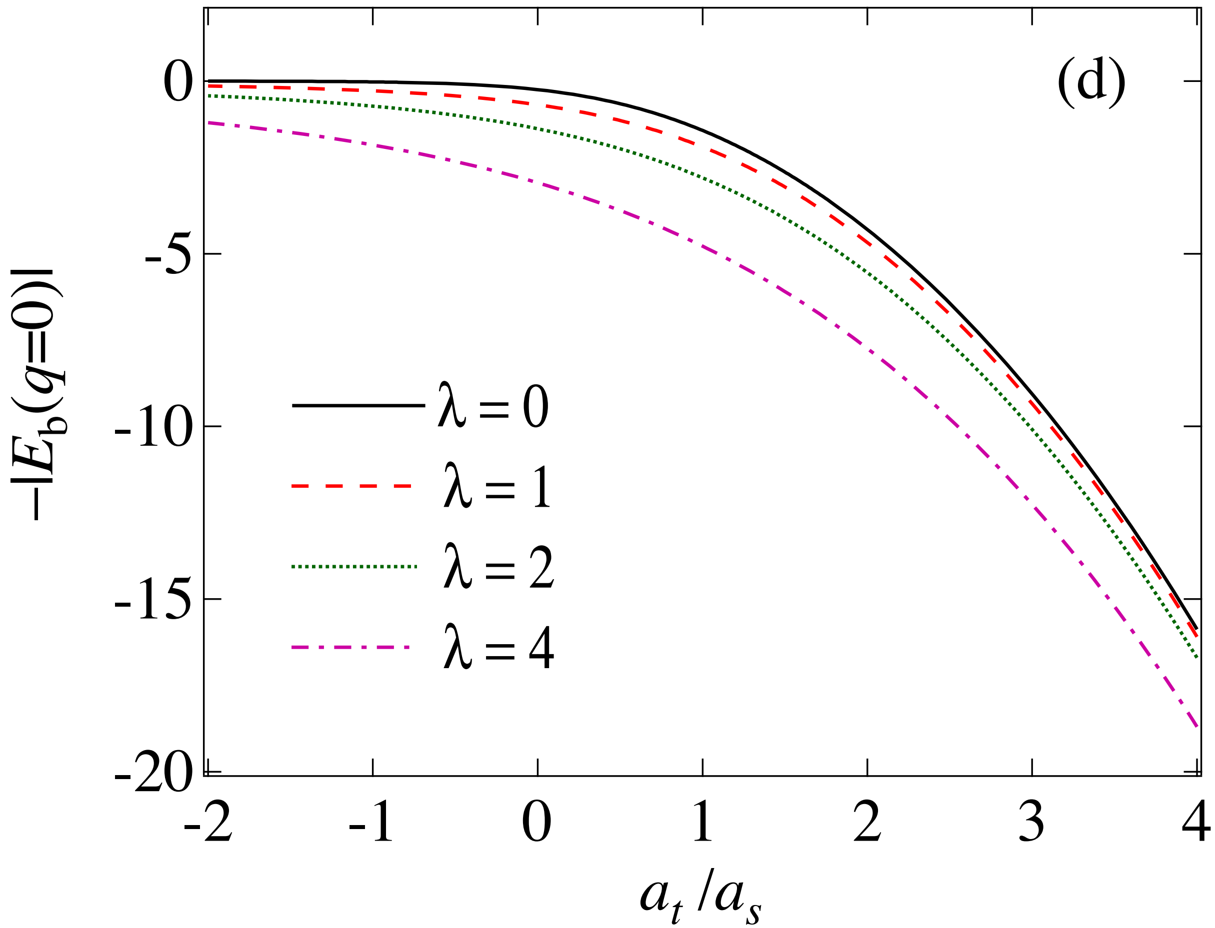
- Two-channel model

$$H = H_0 + H_{\mathrm{soc}} + H_{\mathrm{bf}} + H_{\mathrm{int}}.$$

$$|\Psi\rangle_{\ell,\boldsymbol{q}} = \Bigg(\beta_{\ell,\boldsymbol{q}} b^{\dagger}_{\ell,\boldsymbol{q}} + \sum_{m,n,\boldsymbol{k}}{}' \sum_{\sigma,\sigma'} \eta^{\sigma\sigma'}_{m,n,\boldsymbol{k},\boldsymbol{q}} c^{\dagger}_{m,\boldsymbol{k}+\boldsymbol{q}/2,\sigma} c^{\dagger}_{n,-\boldsymbol{k}+\boldsymbol{q}/2,\sigma'}\Bigg)|0\rangle$$

Two-body bound state (Q2D)

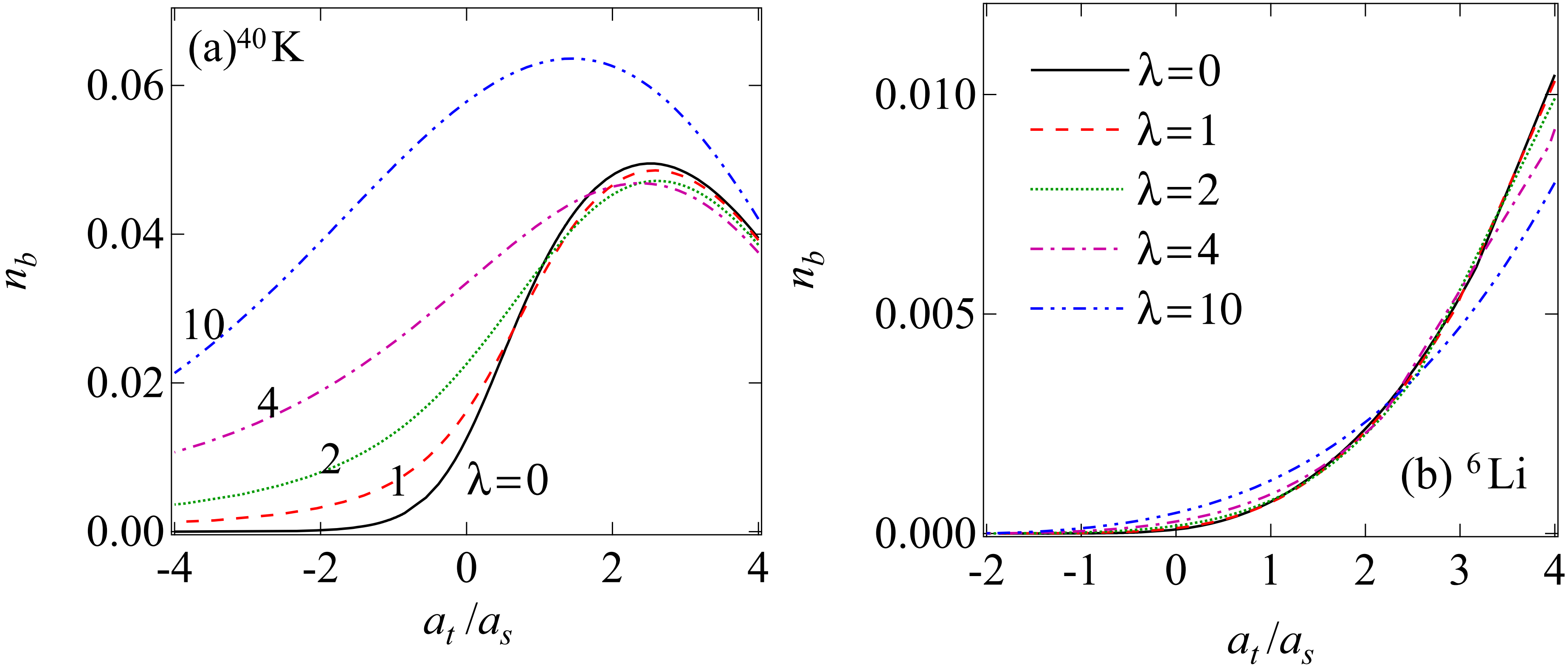
- q=0



Zhang et al., in preparation

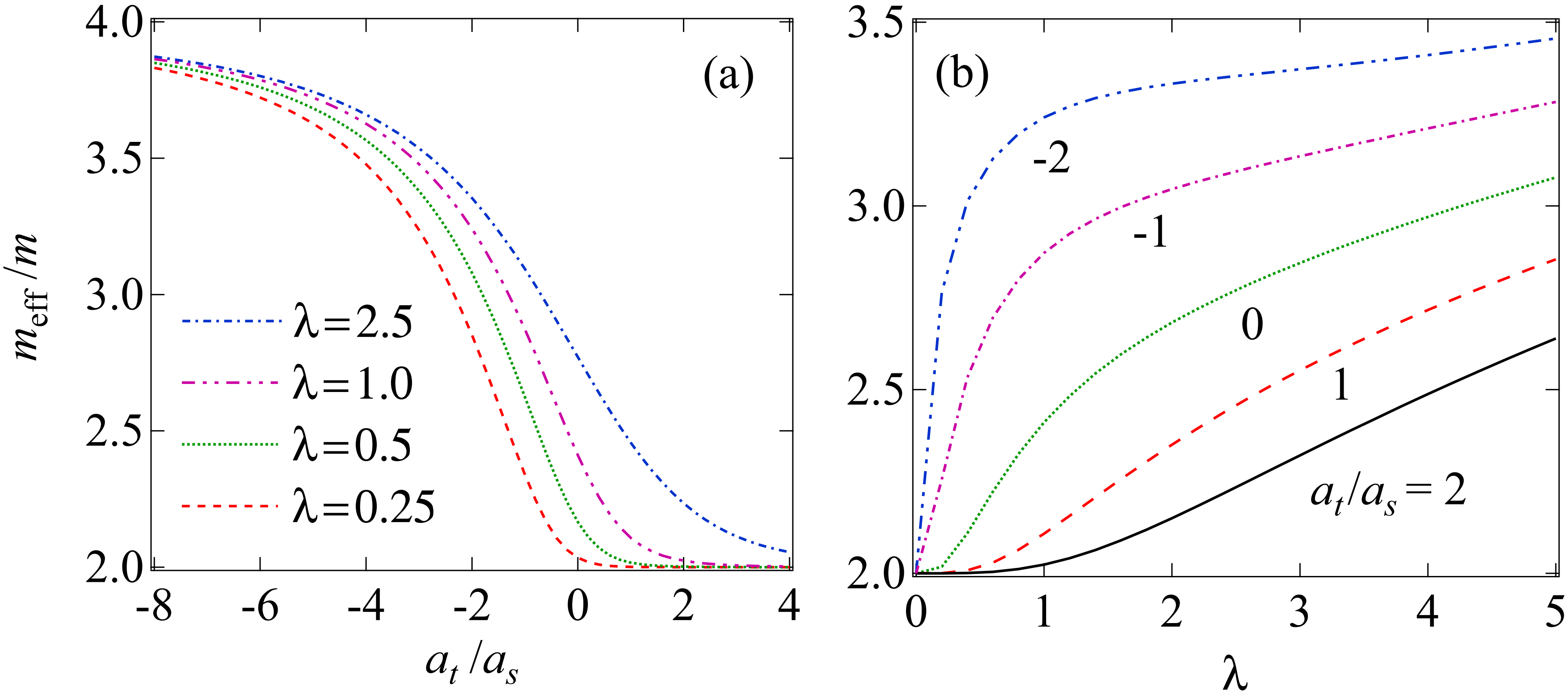
Two-body bound state (Q2D)

- q=0



Two-body bound state (Q2D)

- General q



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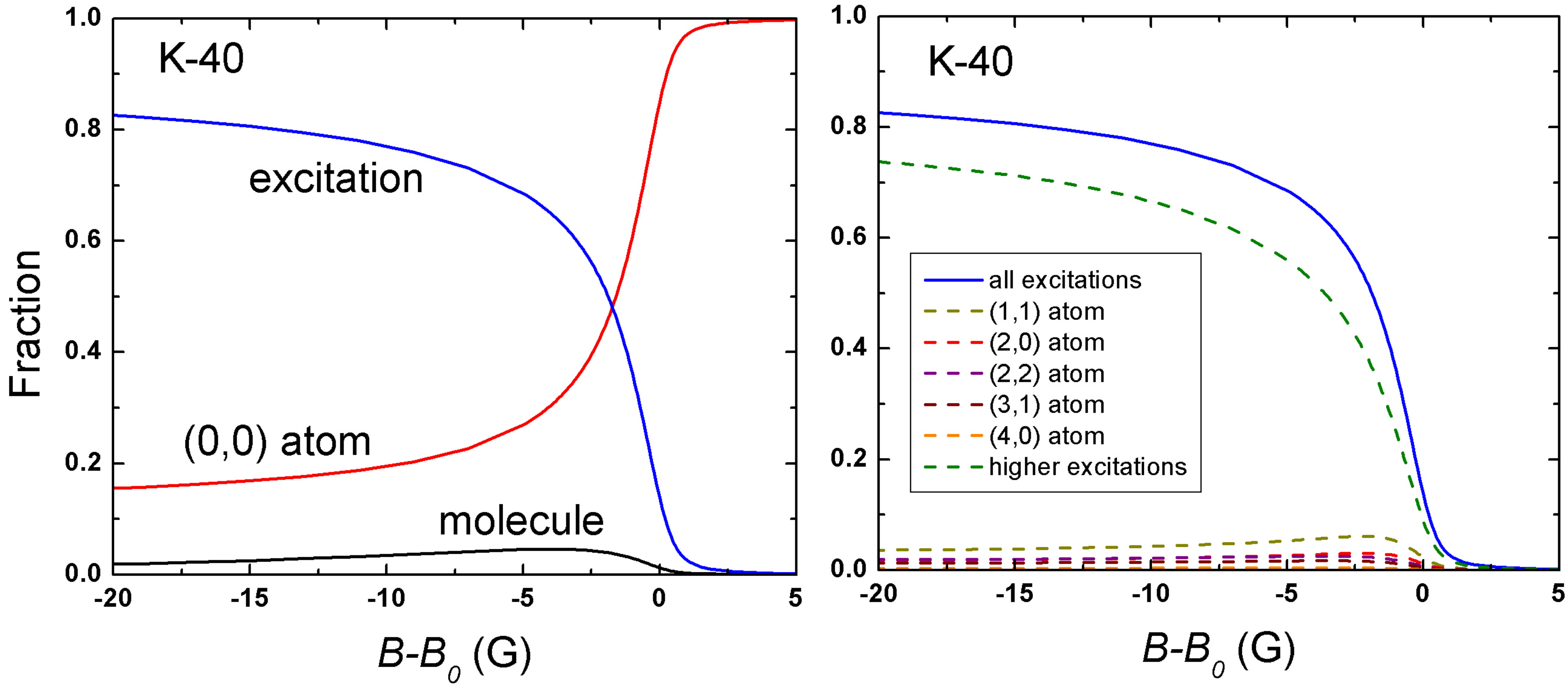
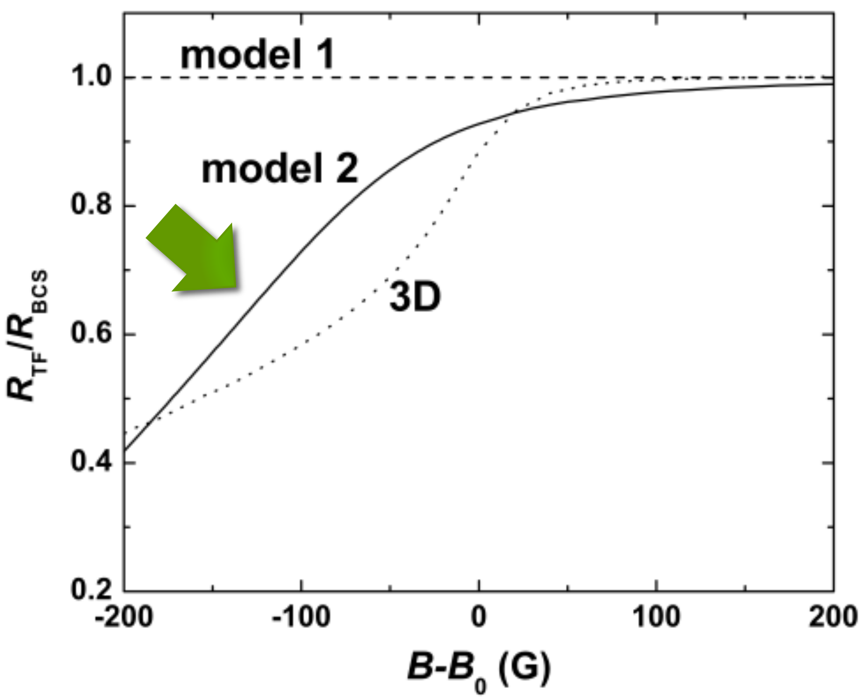
Effective 2D Hamiltonian (w/o SOC)

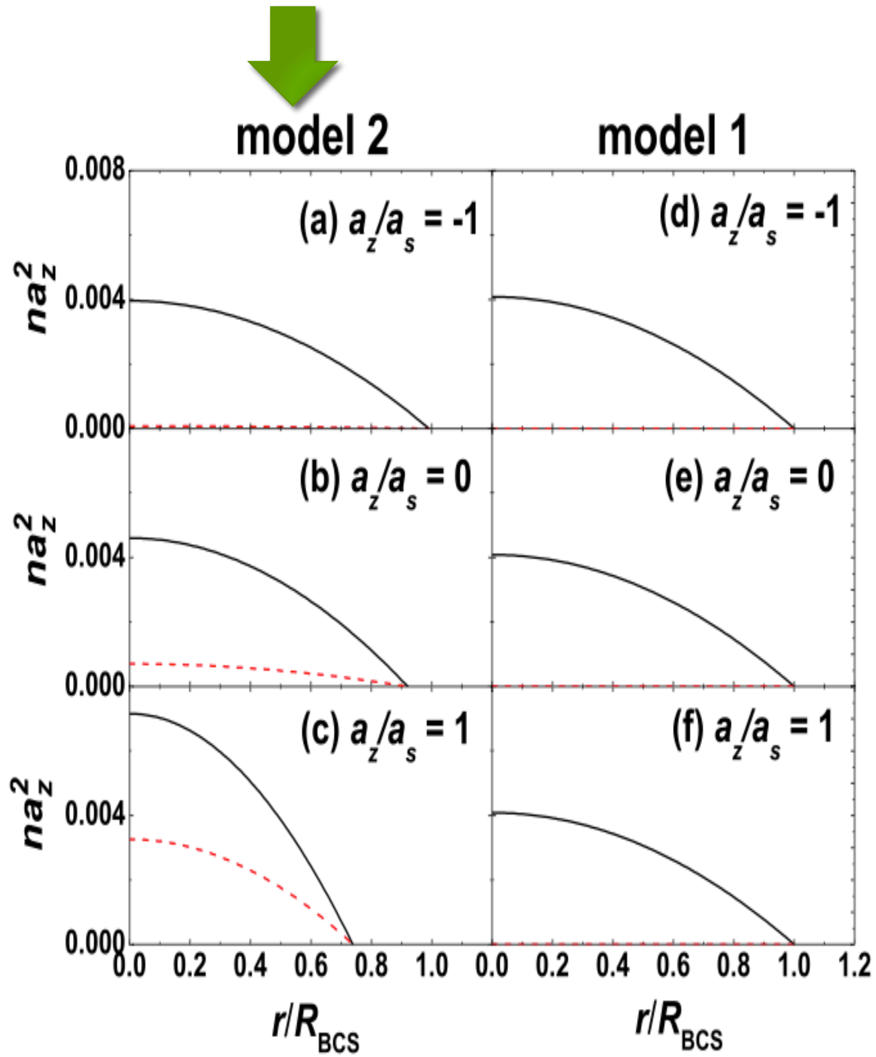


Kestner and Duan, PRA 74, 053606 (2006)

Effective 2D Hamiltonian (w/o SOC)



WZ, Lin, Duan, PRA 77, 063613 (2008)



Effective 2D Hamiltonian (w/SOC)

- Q2D model:
  - fermions in ground state n=0
  - fermions in excited states n=1,2,3…
  - Feshbach molecules
- Effective 2D Hamiltonian (2-channel model)
  - 2D Fermions
  - dressed molecules (structureless)
- Matching conditions
  - open channel threshold
  - background scattering
  - two-body binding energy
  - fermions in ground state

singular point of T(x)

first derivative of 1/T(x) at singular point

$$\Delta\varepsilon = O\left(\frac{\mu - E_b/2}{\hbar\omega_z}\right)^2$$

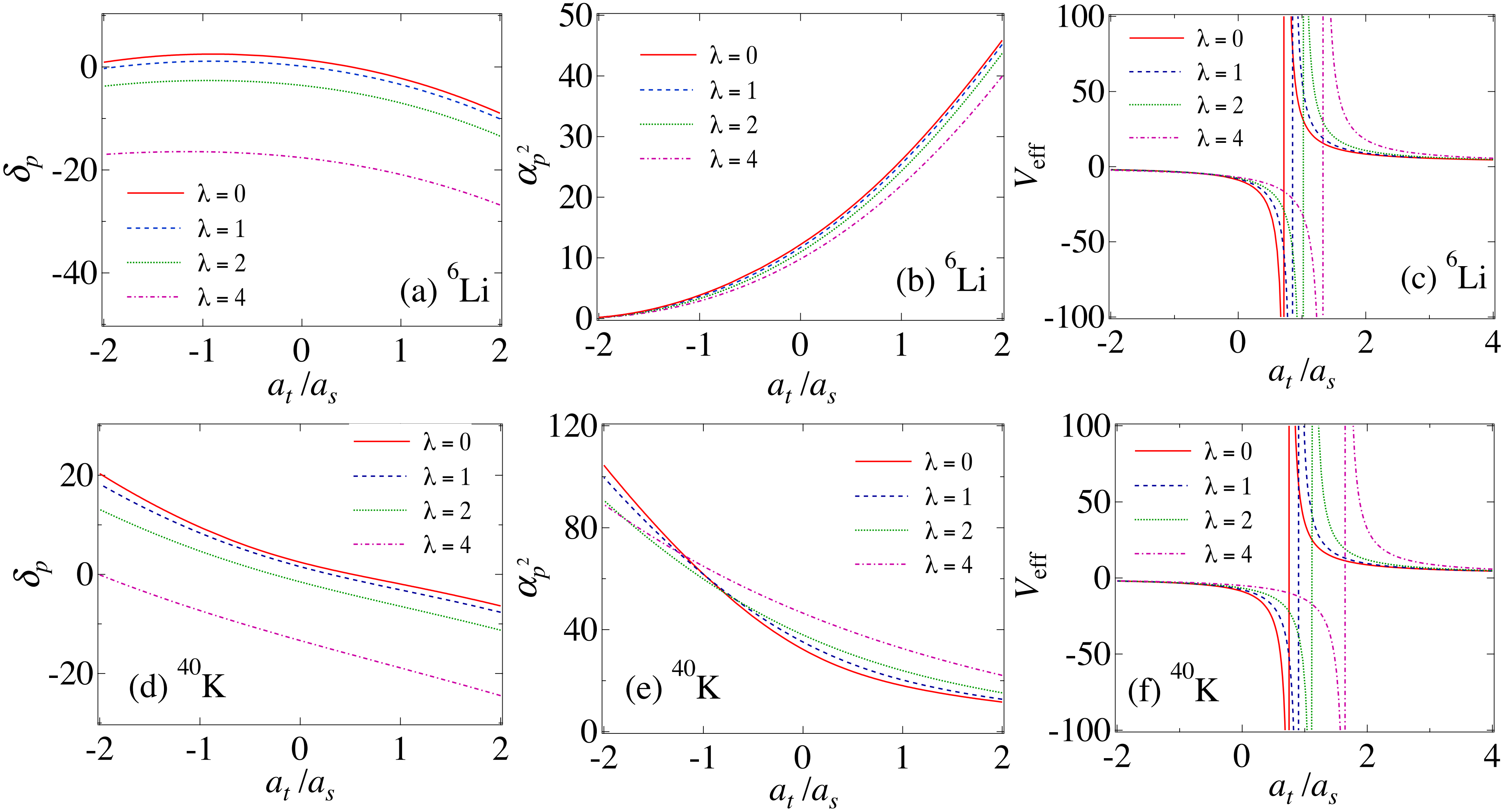
Effective 2D Hamiltonian

$$H_{\text{eff}} = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \delta_b d^{\dagger}_0 d_0 + \frac{\alpha_b}{L} \sum_{\mathbf{k}} \left(d^{\dagger}_0 a_{\mathbf{k},\uparrow} a_{-\mathbf{k},\downarrow} + \text{H.C.} \right)$$
$$+ \frac{V_b}{L^2} \sum_{\mathbf{k},\mathbf{k}'} a^{\dagger}_{\mathbf{k},\uparrow} a^{\dagger}_{-\mathbf{k},\downarrow} a_{-\mathbf{k}',\downarrow} a_{\mathbf{k}',\uparrow} + \gamma' \sum_{\mathbf{k}} \left[(k_x - i k_y) a^{\dagger}_{\mathbf{k},\uparrow} a_{\mathbf{k},\downarrow} + (k_x + i k_y) a^{\dagger}_{\mathbf{k},\downarrow} a_{\mathbf{k},\uparrow} \right]$$

$$\gamma' = \gamma;$$

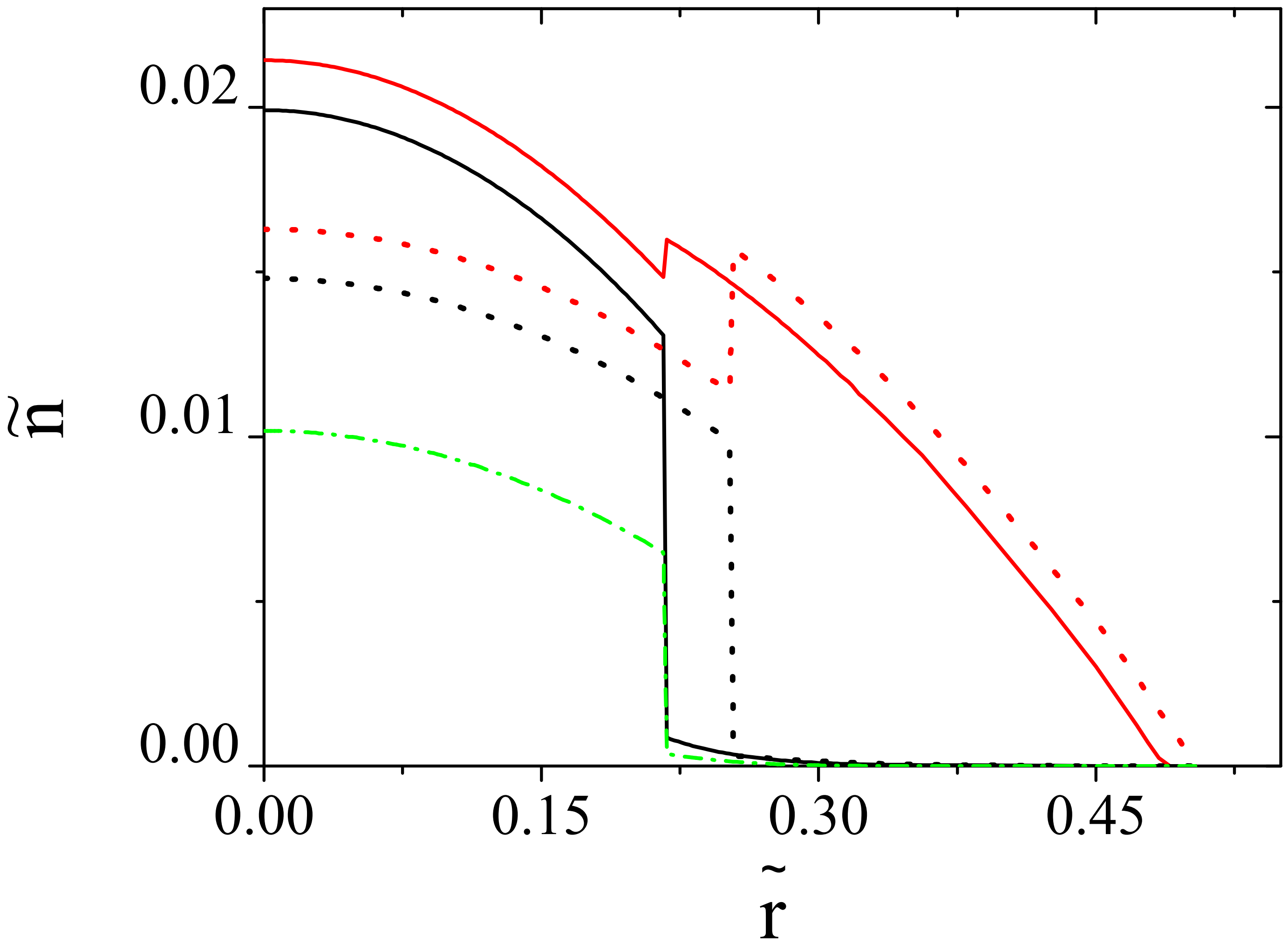
$$V_p^{-1} = \sqrt{2\pi} \left(U_p^{-1} - C_p \right)$$

Effective 2D Hamiltonian

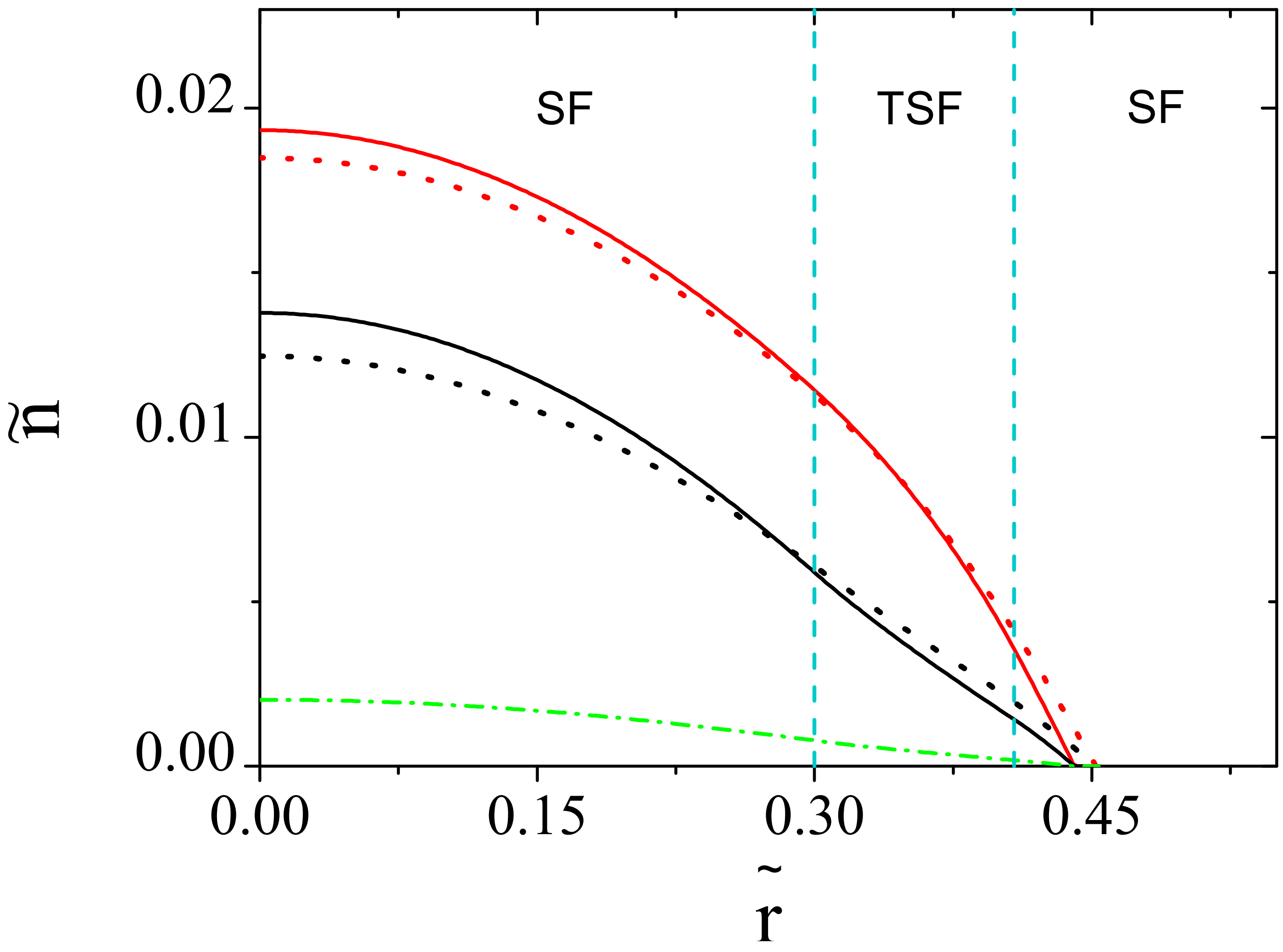


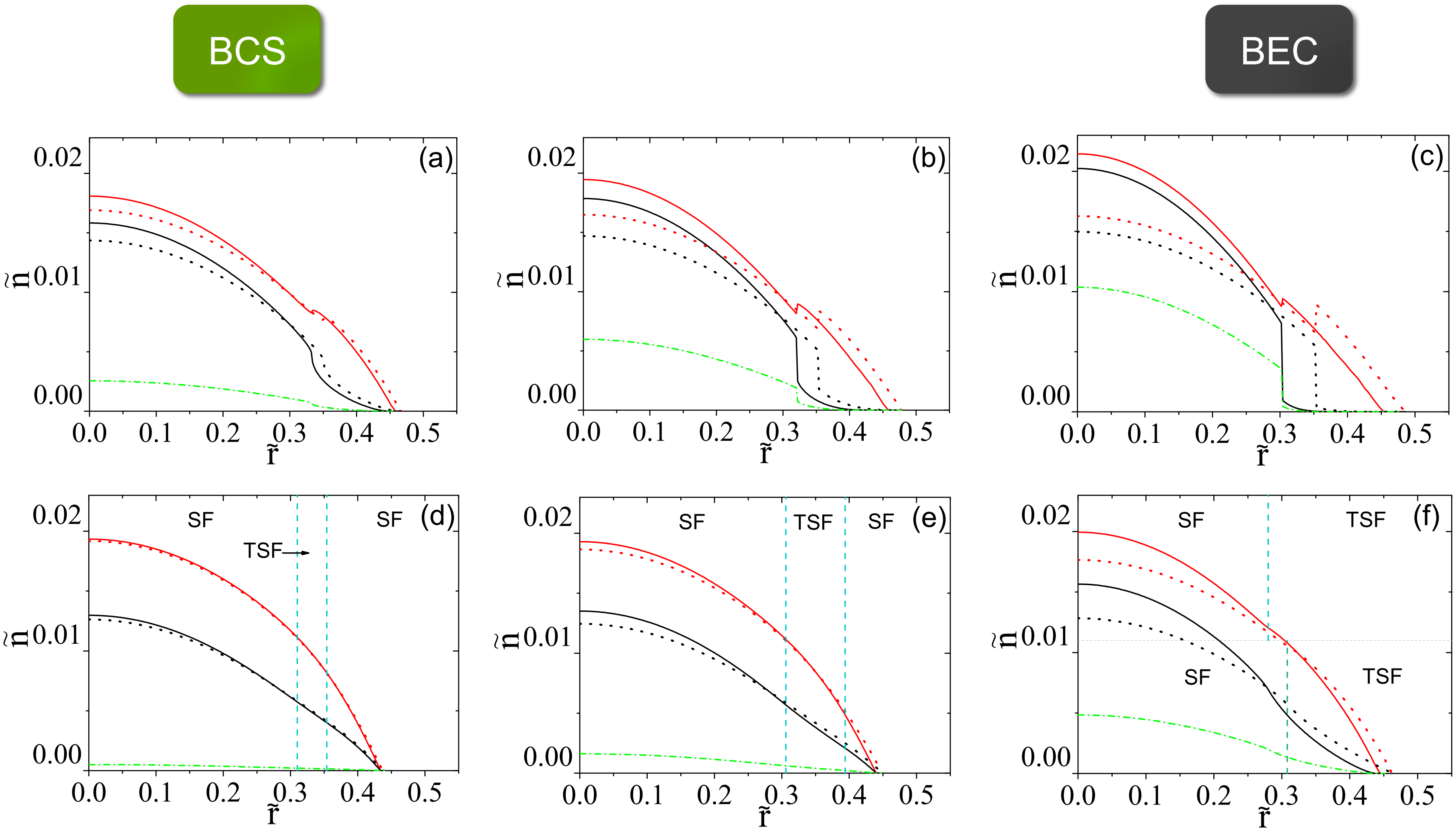
Zhang et al., in preparation

Q2D Fermi gas with Rashba SOC



Q2D Fermi gas with Rashba SOC

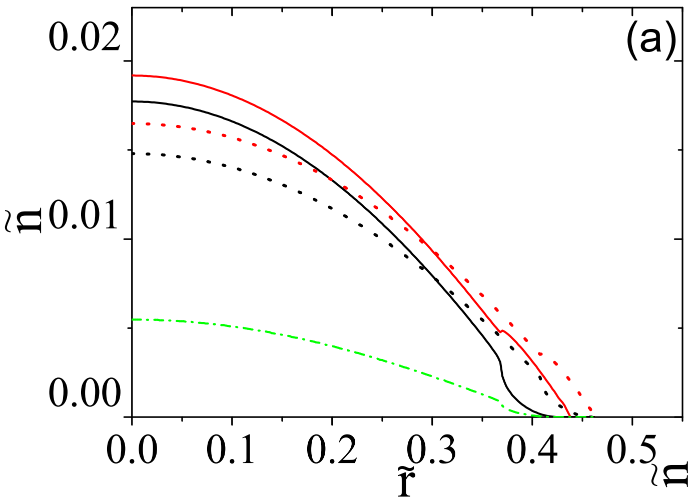




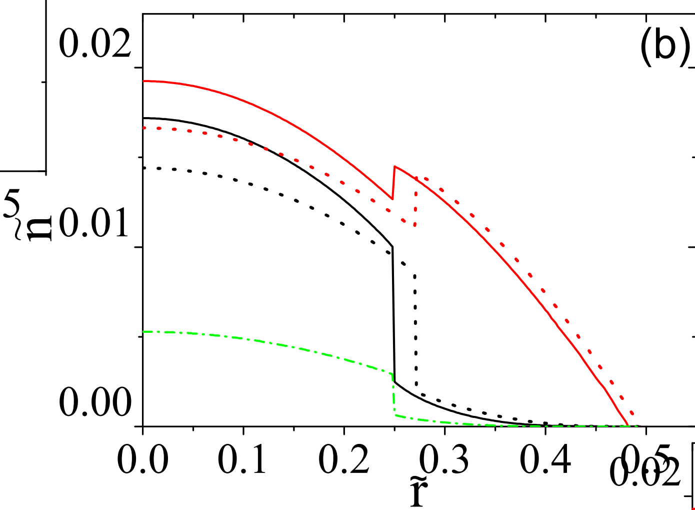
BCS
BEC
(a)
(b)
(c)
(d)
(e)
(f)
SF
TSF
SF
SF
TSF
SF
SF
TSF
SF
TSF
0.02
0.01
0.00
$\tilde{n}$
0.0 0.1 0.2 0.3 0.4 0.5
$\tilde{r}$

unitarity

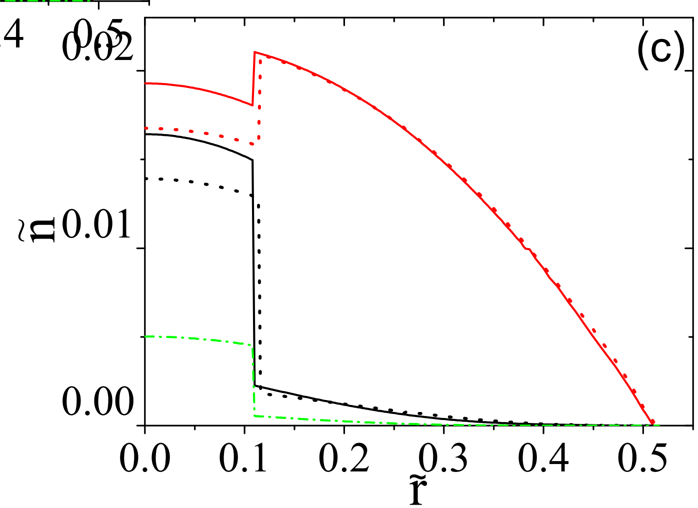
P=0.1



P=0.42



P=0.82



Summary

- SOC changes the qualitative behavior of 2D scattering state in the low-energy limit (logarithmic -> polynomial)
- SOC enhances the two-body binding energy in Q2D
- The axial excited states become more important
- An effective 2D model incorporating these DOF is required
- In-trap phase diagrams in Q2D Fermi gas can be qualitatively different from the 2D case

Acknowledgements

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  - 张芃 (Talk on Sat.)
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