Ground State and Dynamics of One Dimensional Hard Core Anyons

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Acknowledge

- Yunbo Zhang(张云波) (SXU)
- Shu Chen(陈澍) (IOP, CAS)
- Qiang Gu(顾强) (USTB),开放系统中的BEC

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Outline

1. Trapped in optical lattice combined with a weak harmonic trap
   - Model and Method
   - Ground State
   - closing harmonic trap
   - harmonic trap becoming weaker
2. Hard core anyons in a harmonic trap
   - method for ground state
   - dynamics–similar to the case of ground state
3. Conclusion

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Bose-Hubbard Model

$$H = -t\sum_{l=1}^{L}\left(b_l^\dagger b_{l+1} + H.C.\right) + \sum_{l=1}^{L} V_l b_l^\dagger b_l + \frac{U}{2}\sum_{l=1}^{L} n_l(n_l - 1) \quad (1)$$

satisfy the Bose commutation relations

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$$a_j a_l^\dagger = \delta_{jl} - e^{-i\theta\epsilon(j-l)} a_l^\dagger a_j,$$
$$a_j a_l = -e^{i\theta\epsilon(j-l)} a_l a_j \quad (3)$$

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$$a_j = \exp\left(i\theta \sum_{1\le s<j} b_s^\dagger b_s\right) b_j,$$
$$a_j^\dagger = b_j^\dagger \exp\left(-i\theta \sum_{1\le s<j} b_s^\dagger b_s\right)$$

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Bose Hamiltonian

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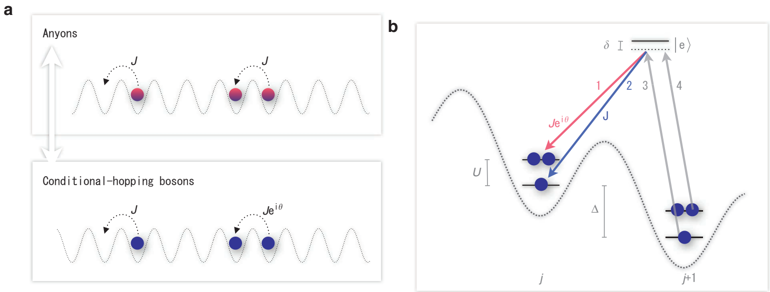

Figure: nature commu 2, 361 (2011): Anyon–Boson mapping and schematic of the proposed experiment.

$J = t$

Hamiltonian

$$H = -t\sum_{l=1}^{L}\left(a_{l+1}^{\dagger}a_l + H.C.\right) + \sum_{l=1}^{L} V_l a_l^{\dagger} a_l \qquad (6)$$

with $V_l = V_0(l-(L+1)/2)^2$. Quench V_0!!!

generalized commutation relations

$$\begin{aligned} a_j a_l^{\dagger} &= \delta_{jl} - e^{-i\chi\pi\epsilon(j-l)} a_l^{\dagger} a_j, \\ a_j a_l &= -e^{i\chi\pi\epsilon(j-l)} a_l a_j \end{aligned} \qquad (7)$$

χ:statistical parameter $\in [0,1]$.

The hard core condition:

$a_l^2 = a_l^{\dagger 2} = 0$ and $\left\{a_l, a_l^{\dagger}\right\} = 1$.

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using the generalized Jordan-Wigner transformation

$$a_j = \exp\left(i\chi\pi \sum_{1\le s<j} f_s^\dagger f_s\right) f_j, \tag{8}$$

$$a_j^\dagger = f_j^\dagger \exp\left(-i\chi\pi \sum_{1\le s<j} f_s^\dagger f_s\right), \tag{9}$$

$$H_F = -t\sum_{l=1}^{L}\left(f_{l+1}^\dagger f_l + H.C.\right) + \sum_{l=1}^{L} V_l f_l^\dagger f_l, \tag{10}$$

Based on the single particle wavefunction, the exact many body wavefunction of anyons can be constructed! Therefore......

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Fermion's wavefunction

The eigenstates of single-particle

$$|\alpha\rangle = c_\alpha^\dagger |0\rangle = \sum_l \varphi_\alpha(l) f_l^\dagger |0\rangle .$$

The many body state of N_f free spinless Fermions

$$\begin{aligned}
|\Psi_F\rangle &= c_1^\dagger c_2^\dagger \cdots c_{N_f}^\dagger |0\rangle \\
&= \sum_{l_1 l_2 \cdots l_{N_f}} \varphi_1(l_1)\varphi_2(l_2)\cdots\varphi_{N_f}(l_{N_f}) f_{l_1}^\dagger f_{l_2}^\dagger \cdots f_{l_{N_f}}^\dagger |0\rangle \\
&= \prod_{n=1}^{N_f}\sum_{l=1}^{L} P_{ln} f_l^\dagger |0\rangle
\end{aligned}$$

with $P_{ln} = \varphi_n(l)$.

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with $P_{ln} = \varphi_n(l)$.

The equal-time Green's function for the hard core anyons at time τ should be expressed as

$$G_{jl}(\tau) = \left\langle \Psi_{HCA}(\tau) \left| a_j a_l^\dagger \right| \Psi_{HCA}(\tau) \right\rangle \tag{11}$$
$$= \langle \Psi_F(\tau) | \exp\left(i\chi\pi \sum_{\beta}^{j-1} f_\beta^\dagger f_\beta \right) f_j f_l^\dagger \exp\left(-i\pi \sum_{\gamma}^{l-1} f_\gamma^\dagger f_\gamma \right) |\Psi_F(\tau)\rangle$$
$$= \left\langle \Psi_F^A(\tau) | \Psi_F^B(\tau) \right\rangle$$

with

$$\left\langle \Psi_F^A(\tau) \right| = \left(f_j^\dagger \exp\left(-i\chi\pi \sum_\beta^{j-1} f_\beta^\dagger f_\beta \right) |\Psi_F(\tau)\rangle \right)^\dagger$$
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$$\left|\Psi_F^A\right\rangle = \prod_{n=1}^{N_f+1} \sum_{l=1}^{L} P_{ln}^{\prime A} f_l^{\dagger} \left|0\right\rangle$$

$$G_{jl} = \left\langle \Psi_F^A | \Psi_F^B \right\rangle = \det\left[\left(\mathbf{P}^{\prime A}\right)^T \mathbf{P}^{\prime B}\right] . \tag{14}$$

The Green's function can be obtained by evaluate the determinant of $(N+1) \times (N+1)$ matrix.

The reduced one body density matrix (ROBDM) can be evaluated by Green's function

$$\rho_{jl}(\tau) = \left\langle a_j^{\dagger} a_l \right\rangle = \delta_{jl}\left(1 - G_{jl}(\tau)\right) - (1-\delta_{jl})e^{-i\chi\pi} G_{jl}(\tau).$$

momentum distribution

$$n(k) = \frac{1}{2\pi} \sum_{j,l=1}^{L} e^{-ik(j-l)} \rho_{jl}(\tau). \tag{15}$$

natural orbitals ϕ^{η}

$$\sum_{j=1}^{L} \rho_{jl}\phi^{\eta} = \lambda_{\eta}\phi^{\eta}, j = 1, 2, ...L, \tag{16}$$

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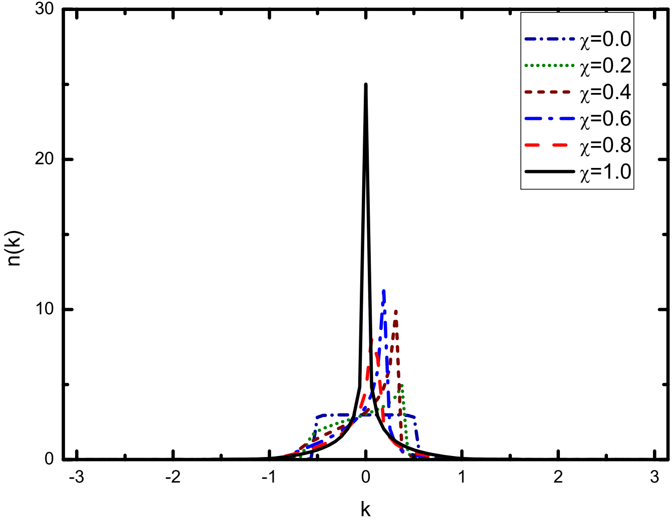

Figure: The momentum distributions for 50 hard core anyons in a lattice of 300 sites. $V_0 = 0.0$.

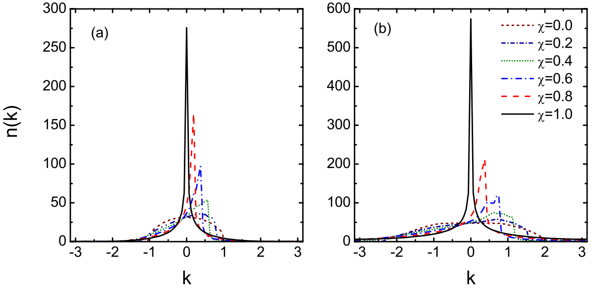

Figure: The momentum distributions for 50 (a) and 150 (b) hard core anyons in lattice of 300 sites combined with a harmonic trap. $V_0^I = 1.0 \times 10^{-4} t$.

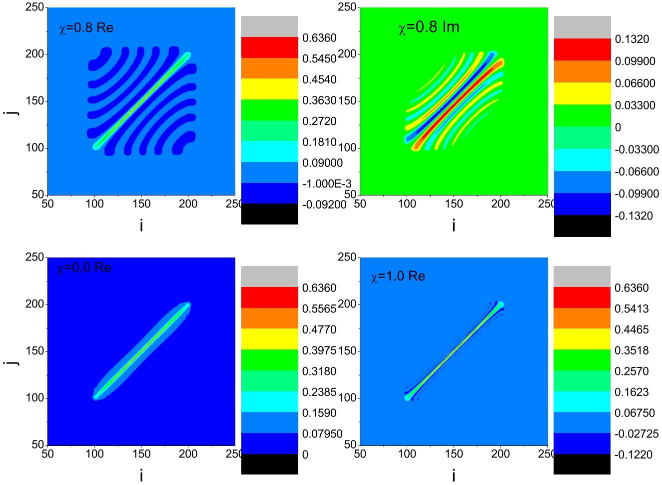

Figure: The reduced one body density matrix for 50 hard core anyons in lattice of 300 sites. $V_0^I = 1.0 \times 10^{-4} t$.

Outline

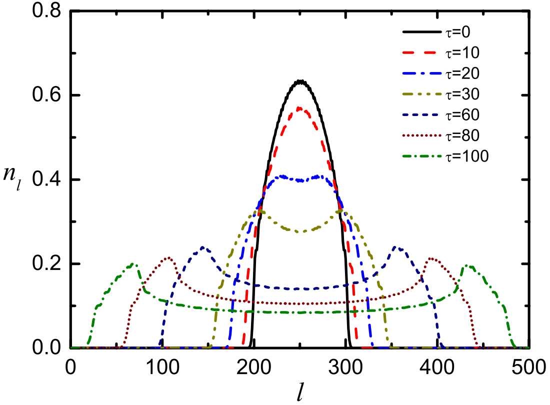

Figure: The evolving of density distribution for 50 hard core anyons in optical lattice of 500 sites. $V_0^I = 1.0 \times 10^{-4} t$ and $V_0 = 1.0 \times 10^{-8} t$.

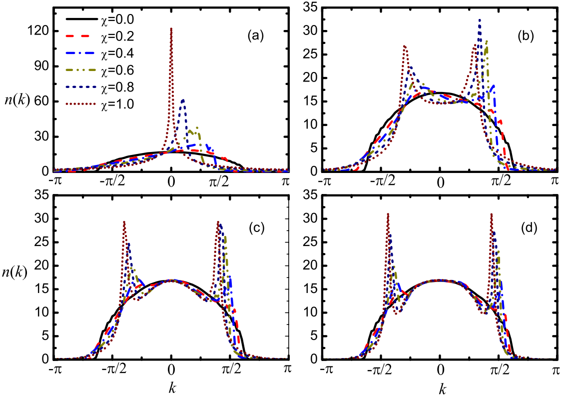

Figure: The momentum distributions for 50 hard core anyons in lattice of 500 sites, $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$. (a) $\tau = 0$, (b) $\tau = 20$, (c) $\tau = 50$,(d) $\tau = 100$.

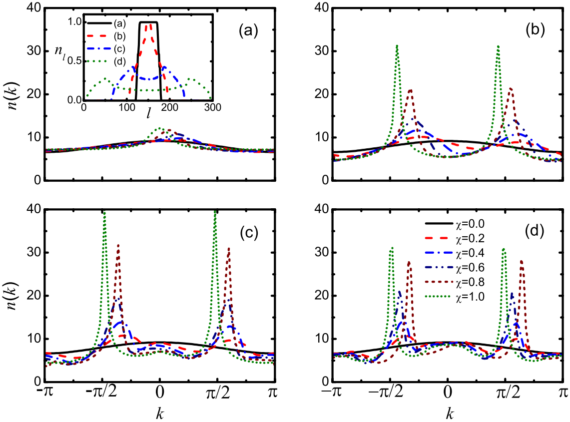

Figure: The momentum distribution of 50 anyons in lattice of 300 sites. $V_0^I = 1.0 \times 10^{-2} t$ and $V_0 = 1.0 \times 10^{-8} t$. (a) $\tau = 0$, (b) $\tau = 10$, (c) $\tau = 30$,(d) $\tau = 60$. Inset: Density distributions.

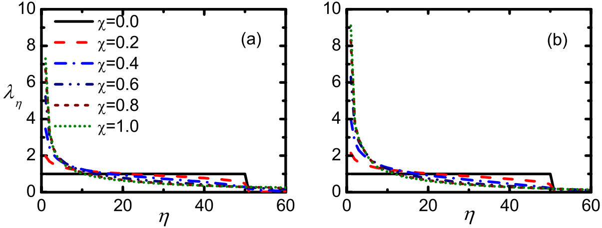

Figure: The occupation distributions for 50 hard core anyons in lattice of 500 sites, $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$. (a) $\tau = 0$, (b) $\tau = 100$.

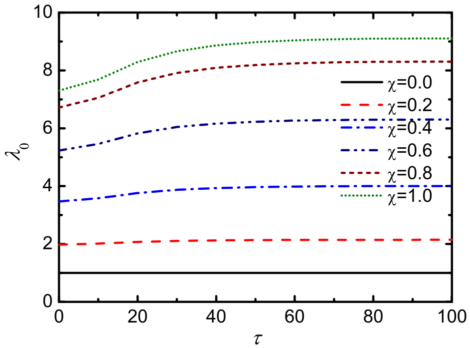

Figure: The occupations of the lowest natural orbital for 50 hard core anyons in lattice of 500 sites. $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 1.0 \times 10^{-8}t$.

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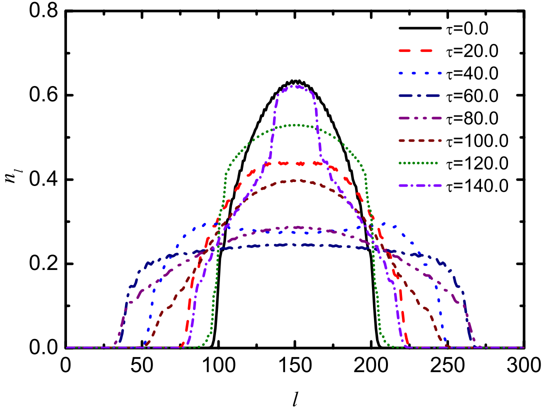


Figure: The density distribution of 50 hard core anyons in optical lattice of 300 sites. $V_0^I = 1.0 \times 10^{-3}t$ and $V_0 = 2.0 \times 10^{-4}t$.

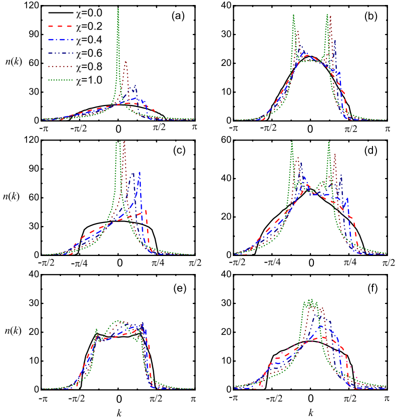
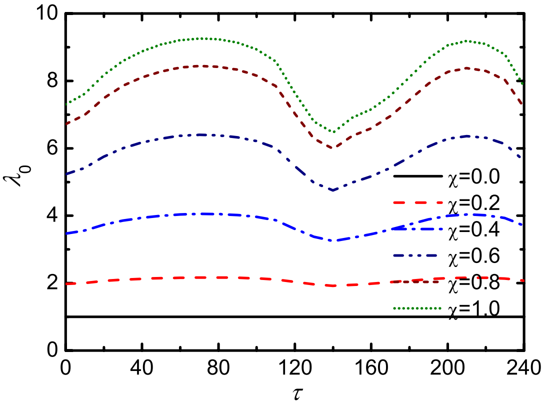


Figure: (The momentum distribution of 50 anyons in optical lattice of 300 sites. $V_0^I = 1.0 \times 10^{-3} t$ and $V_0 = 2.0 \times 10^{-4} t$. (a) $\tau = 0$, (b) $\tau = 30$, (c) $\tau = 60$, (d) $\tau = 80$, (e) $\tau = 100$, (f) $\tau = 120$.



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Outline

Fermi-anyon mapping method

We consider N anyons of mass m with the hard core interaction trapped in a harmonic potential

$$V_{ext} = m\omega^2 x^2/2. \tag{17}$$

anyon Wavefunction

$$\Phi_A(x_1, \cdots, x_N) = \mathcal{A}_\theta(x_1, \cdots, x_N)\Phi_F(x_1, x_2, \cdots, x_N), \tag{18}$$

where the anyonic mapping function is formulated as

$$\mathcal{A}_\theta(x_1, \cdots, x_N) = \prod_{1\le j<k\le N} \exp[-\frac{i\theta}{2}\epsilon(x_{jk})]. \tag{19}$$

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the wavefunction of N polarized fermions

$$\Phi_F\left(x_1, x_2, \cdots, x_N\right) = \left(1/\sqrt{N!}\right) \det_{j,k=1}^{N} \phi_j\left(x_k\right). \tag{20}$$

Here $\phi_j(x) = (\sqrt{\pi}2^j j!)^{-\frac{1}{2}} e^{-x^2/2} H_j(x)$

Using the Vandermonde determinant formula $\det[p_{j-1}(x_k)]_{j,k=1,\cdots,N} = \prod_{1\leq j<k\leq N}(x_j - x_k)$ for $\{p_j(x)\}$=$\{2^{-j}H_j(x)\}$ and $\{p_j(x) = x^j\}$

Fermi wavefunction

$$\Phi_F\left(x_1, x_2, \cdots, x_N\right) = (C_N^H)^{-1} \prod_{j=1}^{N} exp(-x_j^2/2) \prod_{1\leq j<k\leq N} (x_j - x_k) \tag{21}$$

with $(C_N^H)^{-2} = \pi^{-N/2} N!^{-1} \prod_{j=0}^{N-1} 2^j j!^{-1}$

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$\det[p_{j-1}(x_k)]_{j,k=1,\cdots,N} = \prod_{1\le j<k\le N}(x_j - x_k)$ for $\{p_j(x)\}$=$\{2^{-j}H_j(x)\}$
and $\{p_j(x) = x^j\}$

Fermi wavefunction

$$\Phi_F(x_1, x_2, \cdots, x_N) = (C_N^H)^{-1} \prod_{j=1}^{N} exp(-x_j^2/2) \prod_{1\le j<k\le N} (x_j - x_k) \quad (21)$$

with $(C_N^H)^{-2} = \pi^{-N/2} N!^{-1} \prod_{j=0}^{N-1} 2^j j!^{-1}$

the wavefunction of N polarized fermions

$$\Phi_F\left(x_1, x_2, \cdots, x_N\right) = \left(1/\sqrt{N!}\right) \det_{j,k=1}^{N} \phi_j\left(x_k\right). \tag{20}$$

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the wave function of impenetrable anyons

$$\Phi_A(x_1, x_2, \cdots, x_N) = \mathcal{A}_\theta(x_1, x_2, \cdots, x_N) \times \frac{1}{C_N^H} \prod_{j=1}^{N} exp(-x_j^2/2) \prod_{1 \le j < k \le N} (x_j - x_k). \tag{22}$$

ROBDM

the reduced one body density matrix (ROBDM) of anyon gas can be calculated by

$$\rho(x, y) = N \int_{-\infty}^{\infty} dx_1 \cdots \int_{-\infty}^{\infty} dx_{N-1} \times \Psi_A^*(x_1, \cdots, x_{N-1}, x)\Psi_A(x_1, \cdots, x_{N-1}, y). \tag{23}$$

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the ROBDM of impenetrable anyons

By an easy calculation we get the concise expression of ROBDM

$$\rho(x,y) = \frac{2^{N-1}}{\sqrt{\pi}\Gamma(N)} exp(-x^2/2 - y^2/2) \times \det\left[\frac{2^{(j+k)/2}}{2\sqrt{\pi}\sqrt{\Gamma(j)\Gamma(k)}} b_{j,k}(x,y)\right]_{j,k=1,\cdots,N-1} \quad (24)$$

with $b_{j,k}(x,y) = \int_{-\infty}^{\infty} dtexp(-t^2) e^{i\theta(\epsilon(t-x)-\epsilon(t-y))/2}(t-x)(ty)t^{j+k-2}$. $b_{j,k}(x,y)$ depend on Gamma function and confluent hypergeometric function–easy to evaluate!.

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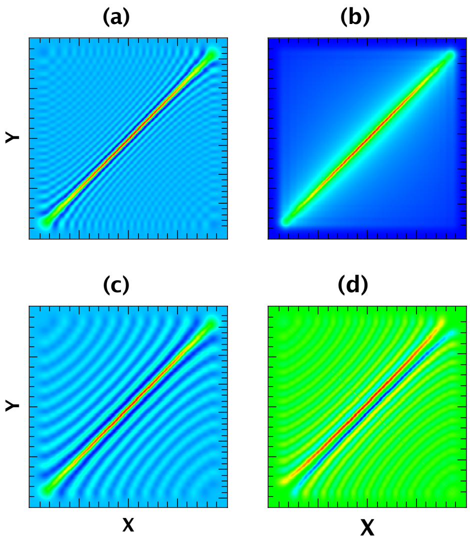

Figure: ROBDM for 40 anyons in a harmonic trap. (a) Fermions; (b) Bosons; (c) χ=0.5, Re[$\rho(x,y)$]; (d) χ=0.5, Im[$\rho(x,y)$].

momentum distribution

$$n(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \rho(x,y) e^{-ik(x-y)}.$$

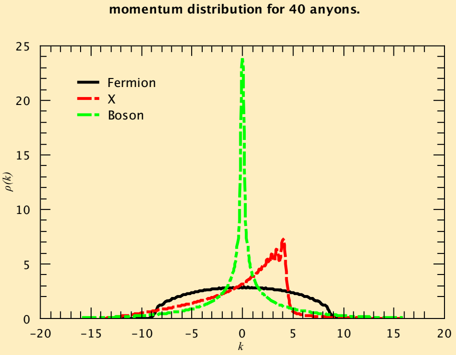

Figure: Momentum distribution for 40 anyons.

Outline

1. Trapped in optical lattice combined with a weak harmonic trap
   - Model and Method
   - Ground State
   - closing harmonic trap
   - harmonic trap becoming weaker
2. Hard core anyons in a harmonic trap
   - method for ground state
   - dynamics–similar to the case of ground state
3. Conclusion

a time-dependent harmonic potential $V_{ext} = m\omega^2(t)\, x^2/2$

$$\rho(x,y,t) = N\int dx_2\cdots dx_N \Phi_T^*(x_1,\cdots,x;t)\Phi_T(x_1,\cdots,y;t) = \frac{1}{b}\rho(\frac{x}{b},\frac{y}{b},0)\exp[-\frac{i\dot{b}}{b\omega_0}\frac{x^2-y^2}{2l_0^2}]. \quad (25)$$

Here the concise expression of ROBDM $\rho(x,y,0)$ at $t=0$

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Rescaling parameter satisfy $\ddot{b} + \omega^2(t)b = \omega_0^2/b^3$ with the initial condition $b(0)=1$ and $\dot{b}(0)=0$.

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Fermionization of anyons

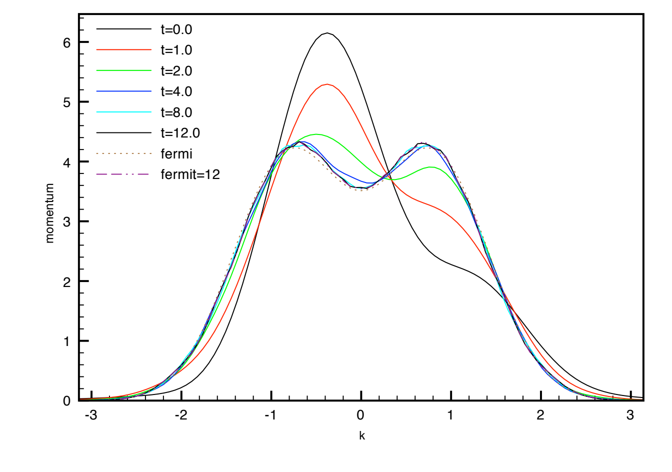
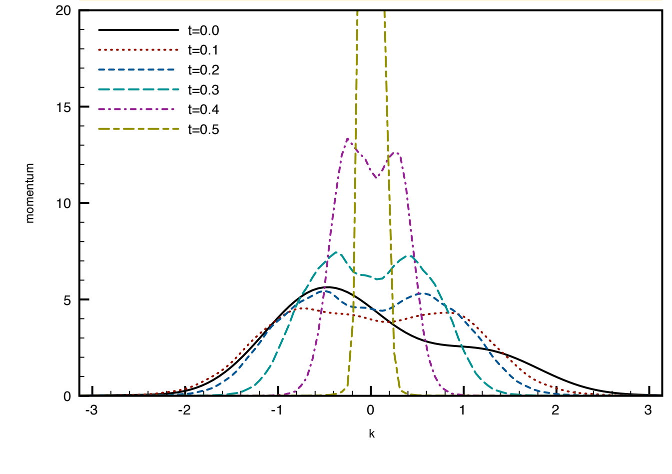


Figure: Momentum distribution for 2 anyons.



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Conclusion

- The exact numerical method to deal with the hard-core anyons confined in the optical lattice superimposed with a weak harmonic potential.
- The physical properties of density profile does not depend on the statistical parameter .
- The physical properties of ROBDM and momentum distribution depend on the statistical parameter .
- The momentum distribution of anyons are asymmetry about the zero momentum and anyon can bosonize and ferminize.

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谢谢!