# Radio-frequency spectroscopy
# and
# Clock shifts of optical transitions
# In atomic gases

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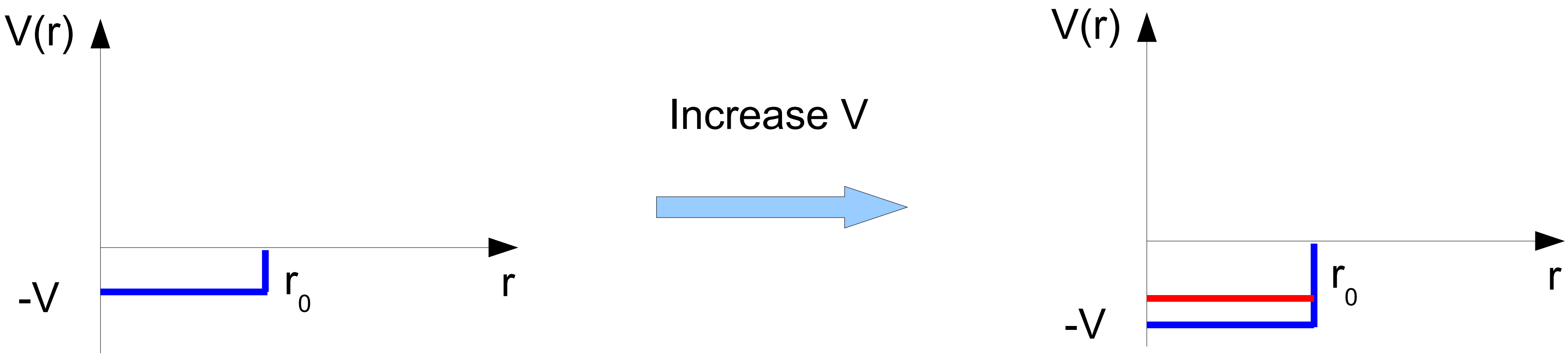
17 August, 2012 Jinhua

# RF & BEC-BCS crossover

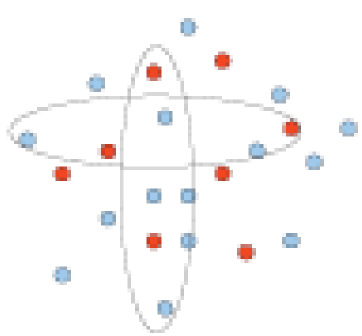
In a two component fermion gas



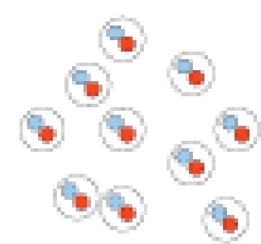

BCS limit

BEC limit

Experimentally realized by Feshbach resonance



crossover

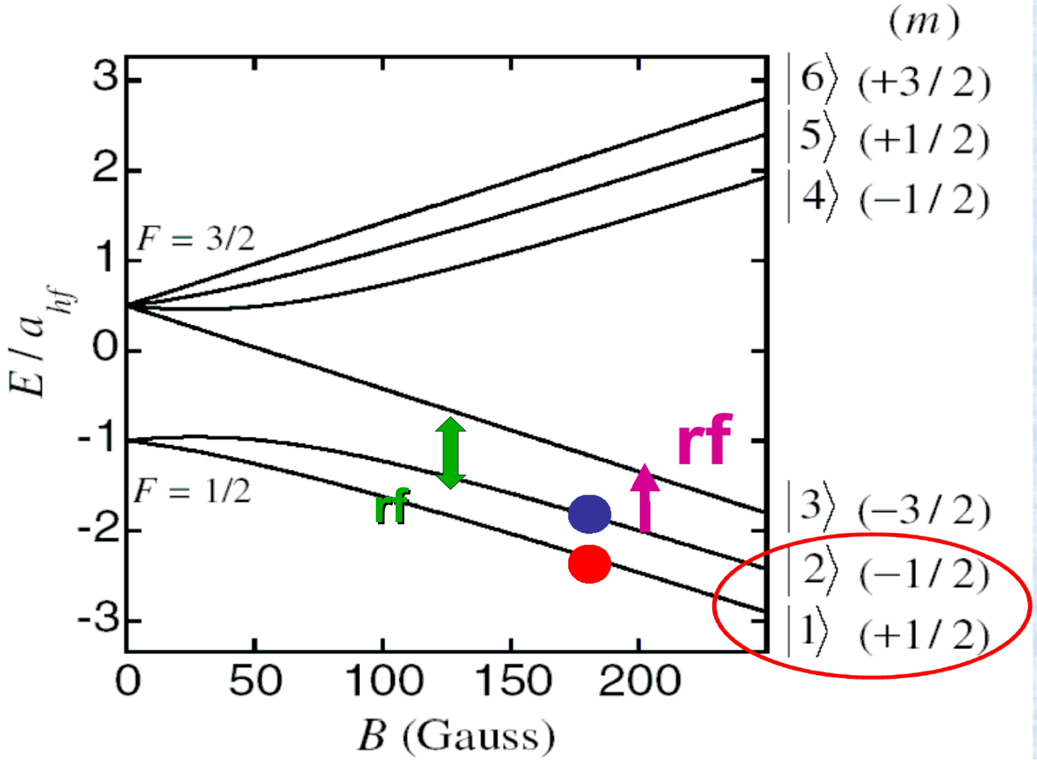


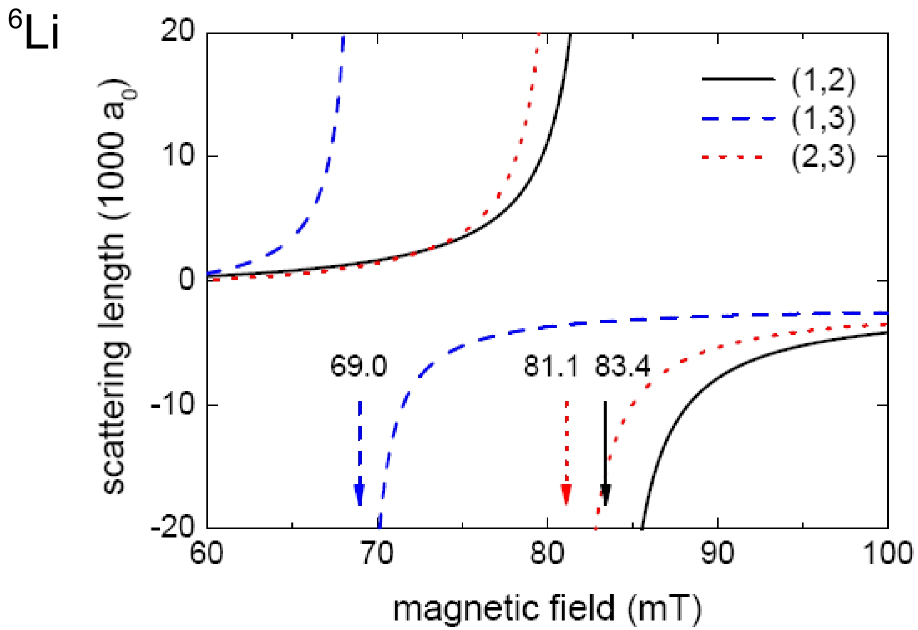
***How to detect the pairing?***

# RF spectroscopy

*J. Kinnunen, M. Rodriguez, and P. Torma, Science 305, 1131 (2004)*

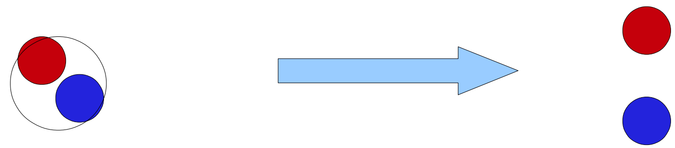
$^6$Li ground state in a magnetic field



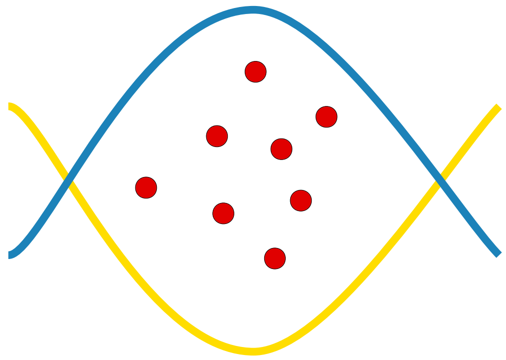



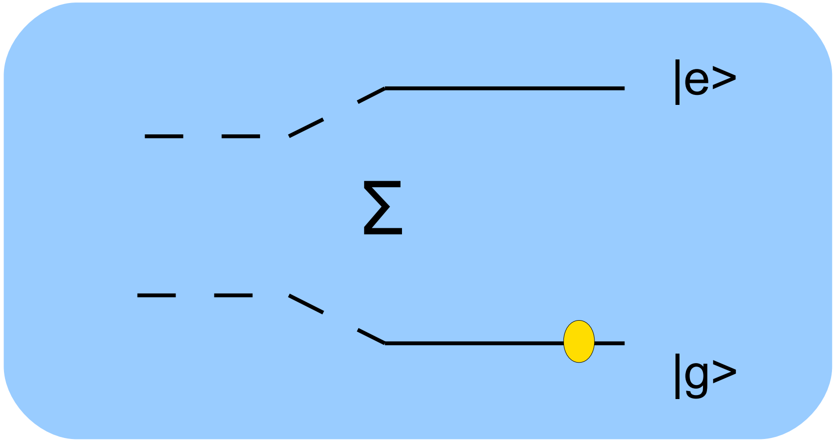

M. Bartenstein et al, Phys. Rev. Lett. 94, 103201 (2005)

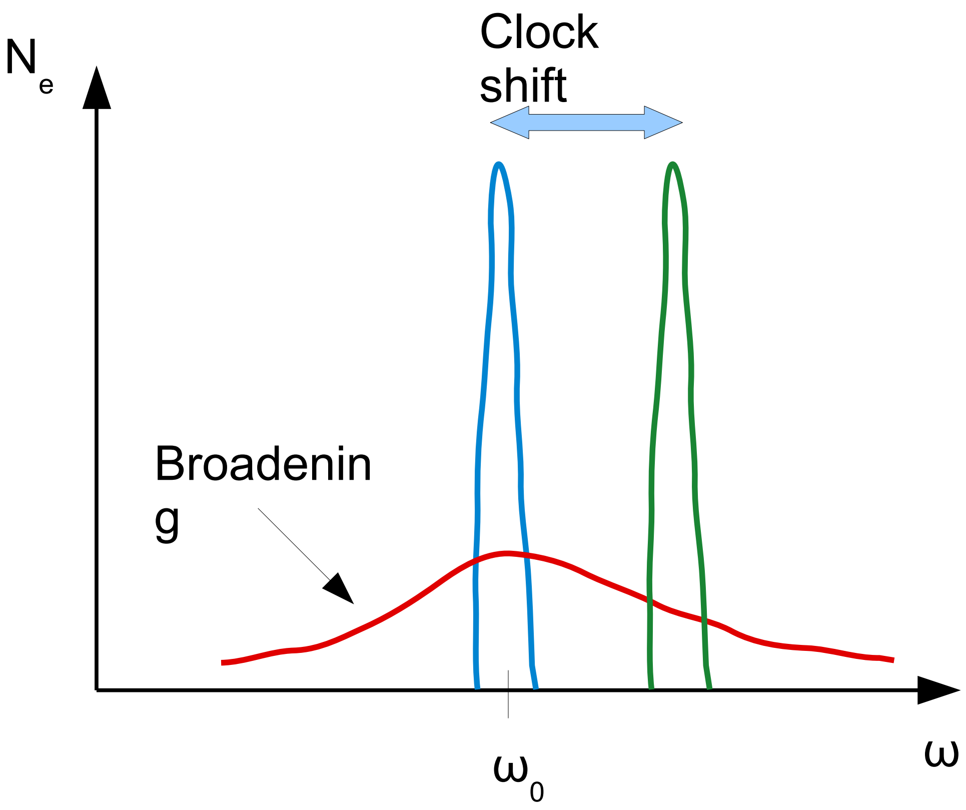
Noninteracting, flipping hyperfine spin pays Zeeman energies;
With pairing, breaking pairing pays extra energy.



# However, interactions affect atomic levels

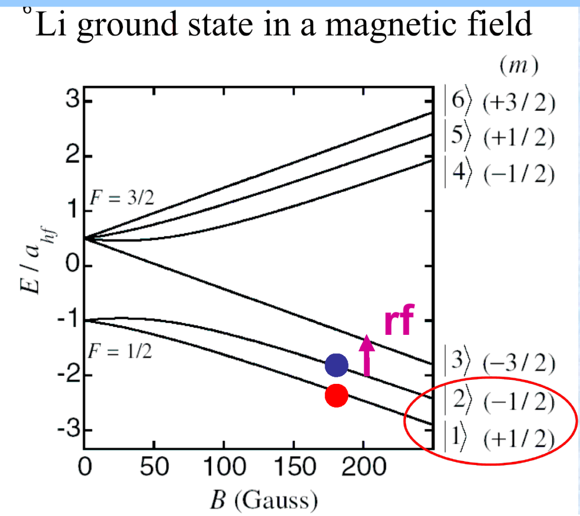





## Hartree-Fock mean field ~ n, ...

# Detection of gap by breaking pairs via rf excitation

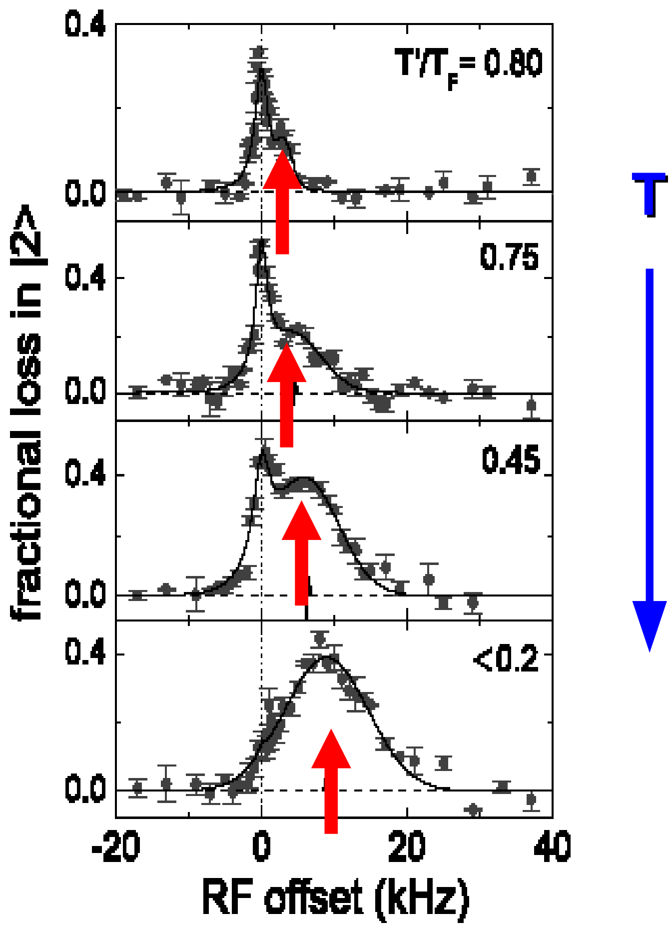


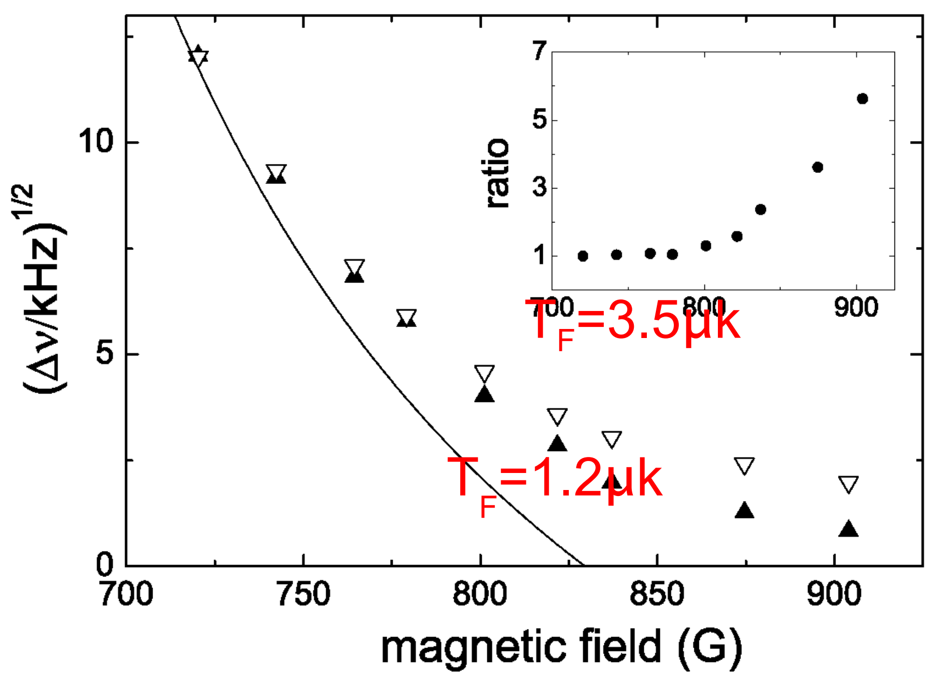

Chin et al., Science **305**, *1128* (2004)

Initially $n_1 = n_2$, $n_3 = 0$, Pairing between |1> and |2>
Excite |2> to |3> by rf.

Naively expect that to excite |2> to |3> by rf, breaking pair, requires twice gap energy, $2\Delta$

$$E_p = \sqrt{\xi_p^2 + \Delta^2}$$



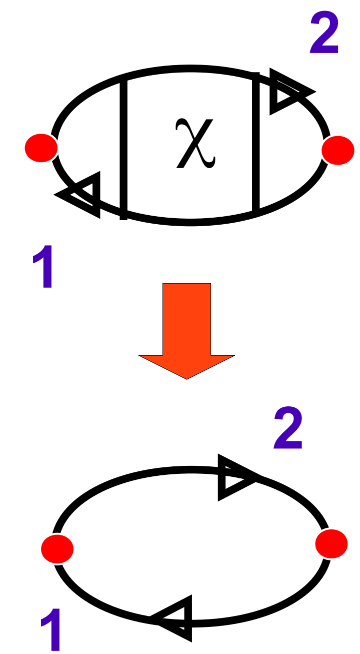

Measured frequency shift

# First attempt of theoretical calculation

J. Kinnunen, M. Rodriguez, and P. Torma, Science **305**, 1131 (2004)

The rf field couples to the electronic dipole moment:

$$H_{rf} = \langle \sigma' | \vec{E} \cdot \vec{d}_e | \sigma \rangle \int d^3\vec{r}\, \psi^+_{\sigma'}(\vec{r})\, \psi_\sigma(\vec{r})$$




Since $\lambda_{rf} \sim 1\mathrm{m} \gg 100\,\mu m$, take $k_{rf} \approx 0$

Fermi golden rule ⟹ RF spectroscopy

$$\chi''(\omega) \sim \sum_{f,i} \rho_i \left| \langle f | H_{rf} | i \rangle^2 \right| \delta(\omega - E_f + E_i)$$

Time ordered correlation function

● = hyperfine (spin) vertex

$$\chi(t-t') \sim \int d^3\vec{r} \int d^3\vec{r}\,' \langle T \psi^+{}_\sigma(\vec{r},t)\, \psi_{\sigma'}(\vec{r},t)\, \psi^+{}_{\sigma'}(\vec{r}\,',t')\, \psi_\sigma(\vec{r}\,',t') \rangle$$

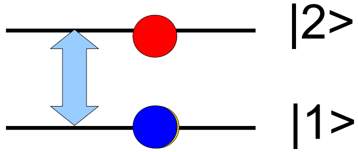
inducing particle-hole excitations

$$\approx \int d^3\vec{r} \int d^3\vec{r}\,' \langle T \psi^+{}_\sigma(\vec{r},t)\, \psi_\sigma(\vec{r}\,',t') \rangle \langle \psi_{\sigma'}(\vec{r},t)\, \psi^+{}_{\sigma'}(\vec{r}\,',t') \rangle$$

# Not good enough

Hartree-Fock approximation: $\Sigma_\sigma = \sum_{\sigma' \neq \sigma} g_{\sigma,\sigma'} n_{\sigma'}$

## Binary fermions

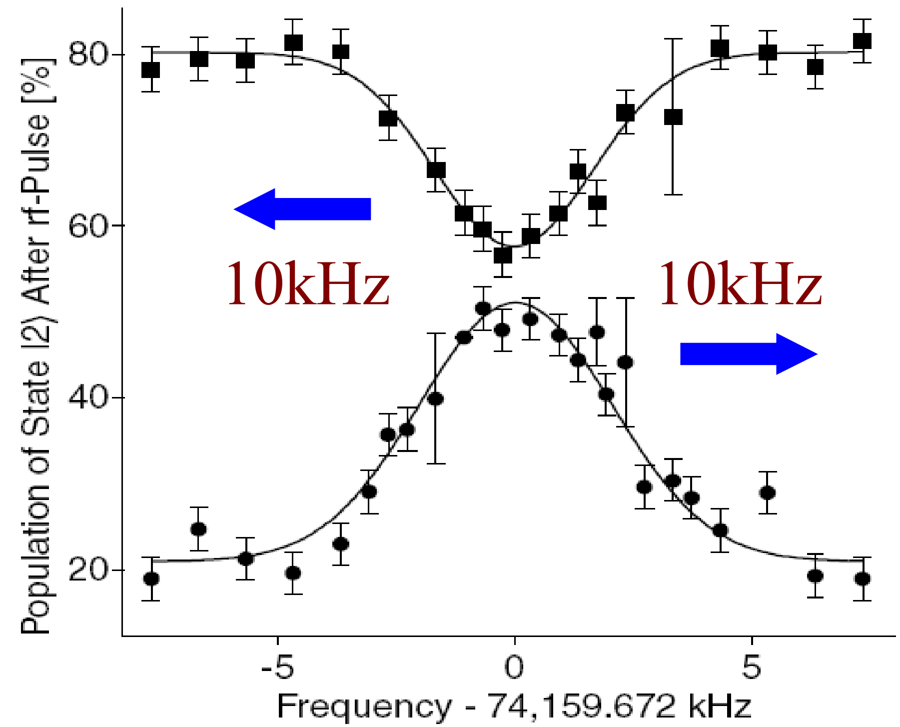



$$\chi \sim G_1 G_2 \sim \frac{n_1 - n_2}{\omega + g_{12}(n_2 - n_1)}$$

Predicts frequency shift:

$$\Delta\omega = g_{12}(n_1 - n_2)$$

$$g_{\sigma\sigma'} = 4\pi\hbar^2 a_{\sigma\sigma'}/m$$

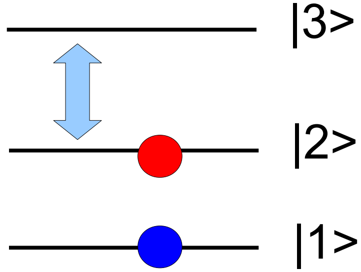



Initially, $n_1/n_2 = 20/80,\ \ 80/20$

But, $\Delta\omega = 0$

*M. W. Zwierlein, Z. Hadzibabic, S. Gupta, and W. Ketterle, PRL* ***91****, 250404 (2003)*

# Triple fermions

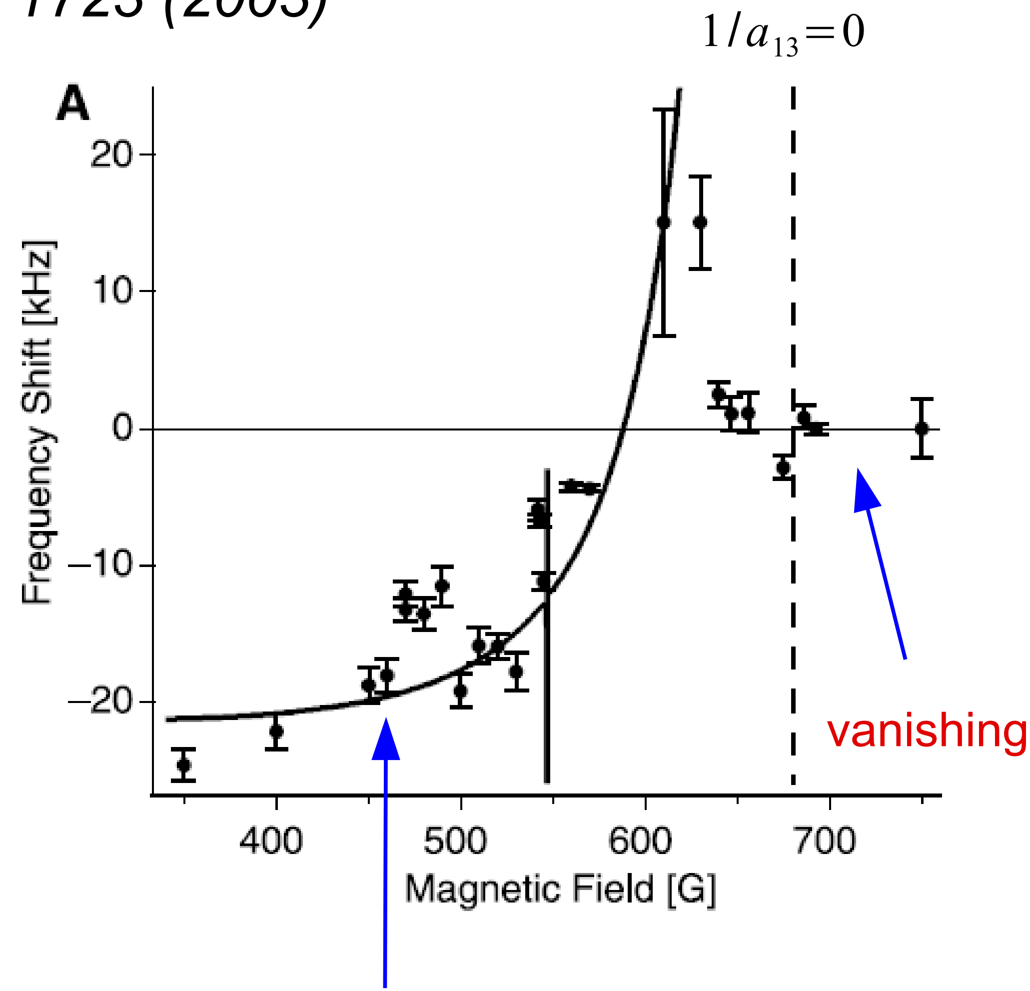



$$\chi \sim G_2 G_3 \sim \frac{n_2}{\omega - (g_{13} - g_{12}) n_1 - g_{23} n_2}$$

Predicts frequency shift:

$$\Delta\omega = (g_{13} - g_{12}) n_1 + g_{23} n_2$$

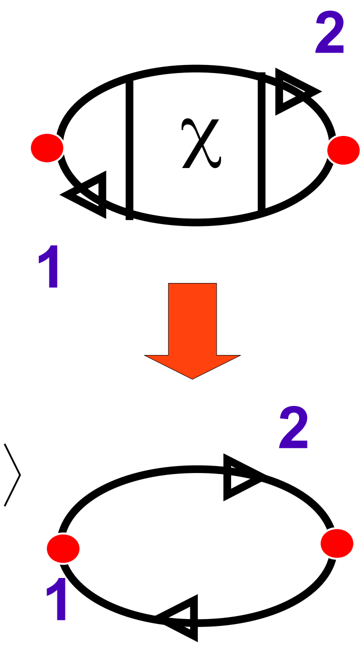
*S. Gupta, et al, Science **300**, 1723 (2003)*




$$\Delta\omega = 4\pi\hbar^2 n_1 (a_{13} - a_{12})/m$$

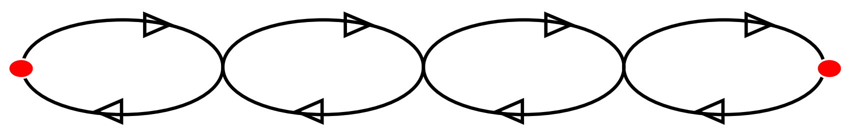
# A quick save:

Direct factorization approximates




$$\chi(t-t')$$
$$=\int d^3\vec{r}\int d^3\vec{r}\,'\langle T\psi^{+}_{\sigma}(\vec{r},t)\psi_{\sigma'}(\vec{r},t)\psi^{+}_{\sigma'}(\vec{r}\,',t')\psi_{\sigma}(\vec{r}\,',t')\rangle$$
$$\approx\int d^3\vec{r}\int d^3\vec{r}\,'\langle T\psi^{+}_{\sigma}(\vec{r},t)\psi_{\sigma}(\vec{r}\,',t')\rangle\langle\psi_{\sigma'}(\vec{r},t)\psi^{+}_{\sigma'}(\vec{r}\,',t')\rangle$$

**Must self-consistently sum bubbles to all orders:**

=  + . . .  Vertex correction

$$\chi_{12}=G_1G_2-G_1G_2g_{12}G_1G_2+G_1G_2g_{12}G_1G_2G_1G_2g_{12}G_1G_2+\ldots$$
$$=\frac{G_1G_2}{1+g_{12}G_1G_2}\sim\frac{n_1-n_2}{\omega}$$

no mean field shift

$$\chi_{23}=\frac{G_2G_3}{1+g_{23}G_2G_3}\sim\frac{n_2}{\omega-(g_{13}-g_{12})n_1}$$

$$\Delta\omega=4\pi\hbar^2n_1(a_{13}-a_{12})/m$$

# Constraint by symmetry

G. Baym and L.P. Kadanoff, PR 124, 287 (1961)
G. Baym, PR 127, 1391 (1962)

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

Equation of motion:

$$i \frac{\partial \rho}{\partial t} = [\rho, H]$$

$$H = T + U, T \sim \psi^+ \psi, U \sim \psi^+ \psi^+ \psi \psi$$

$$[\rho, T] + [\rho, U] - i \nabla \cdot \vec{j} = 0$$

Ward identity

$$\langle [\rho, T] \rangle \sim \langle \psi^+ \psi \rangle, \langle [\rho, U] \rangle \sim \langle \psi^+ \psi^+ \psi \psi \rangle, \langle j \rangle \sim \langle \psi^+ \psi \rangle$$

SU(2) matters for RF spectrum!

# Calculation of RF spectrum

ZY & G.Baym,
Phys. Rev. A 73, 063601 (2006)

RF probe couples to atomic magnetic moment (essentially valence e):

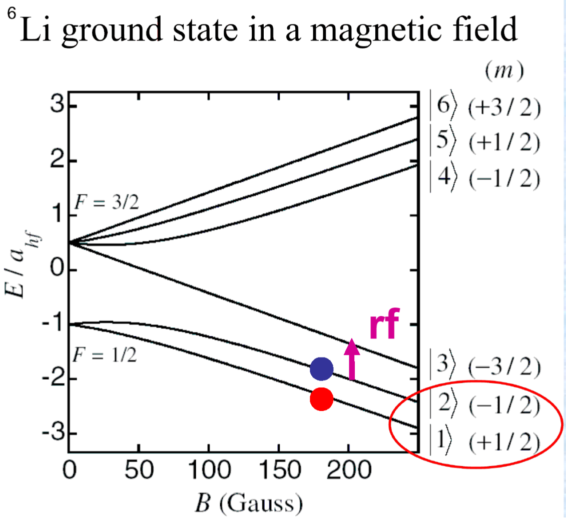
$$H_{probe} = \gamma_e \vec{B}_{rf} \cdot \vec{S}_e$$

Thus measure long-wavelength transverse spin-spin correlation function:

$$\chi''(\omega) \sim \langle \vec{S}_e \vec{S}_e \rangle(\omega)$$

Calculate χ"(ω) in manifold of three lowest hyperfine states (|1>, |2>, |3>) :

$$H = \sum_{i=1}^{3} \int d^3r \left( \epsilon_i \psi_i^\dagger(\mathbf{r}) \psi_i(\mathbf{r}) + \frac{\hbar^2}{2m} \nabla \psi_i^\dagger(\mathbf{r}) \cdot \nabla \psi_i(\mathbf{r}) \right)$$
$$+ \sum_{i<j} \int d^3r d^3r' v_{ij}(\mathbf{r}-\mathbf{r}') \psi_i^\dagger(\mathbf{r}) \psi_j^\dagger(\mathbf{r}') \psi_j(\mathbf{r}') \psi_i(\mathbf{r})$$




Construct L=1 pseudospin space:

$$Y = \Psi^+ \sigma_x \Psi, \quad \Psi = (\psi_1, \psi_2, \psi_3)$$

Then the measured corresponds to $\langle YY \rangle(\omega)$ ($B_{rf}$ along x direction)

# Constraints by SU(2) symmetry

Interaction SU(2) invariant: long wavelength response of χ'' must be at the Larmor frequency (cf. $^3$He, Leggett), no mean field or pairing corrections.

$$\chi(t) = -i\langle T\, Y(t)\, Y(0)\rangle \qquad \chi(\Omega) = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\chi''(\omega)}{\Omega - \omega}$$

Equation of operator $$i\frac{\partial}{\partial t} Y(t) = [Y, T + H_z + U]$$

$$[Y, T] = 0$$

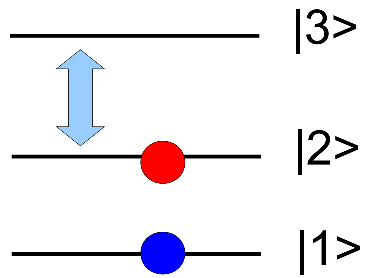
$$H_z \sim Y_z, [Y, H_z] \sim Y$$

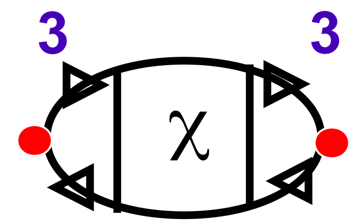
If interaction is SU(2) invariant $[Y, U] = 0$

The equation of motion for Y is closed and energy scales Associated with U drop out. Zeeman energies are the only Energy scales left.

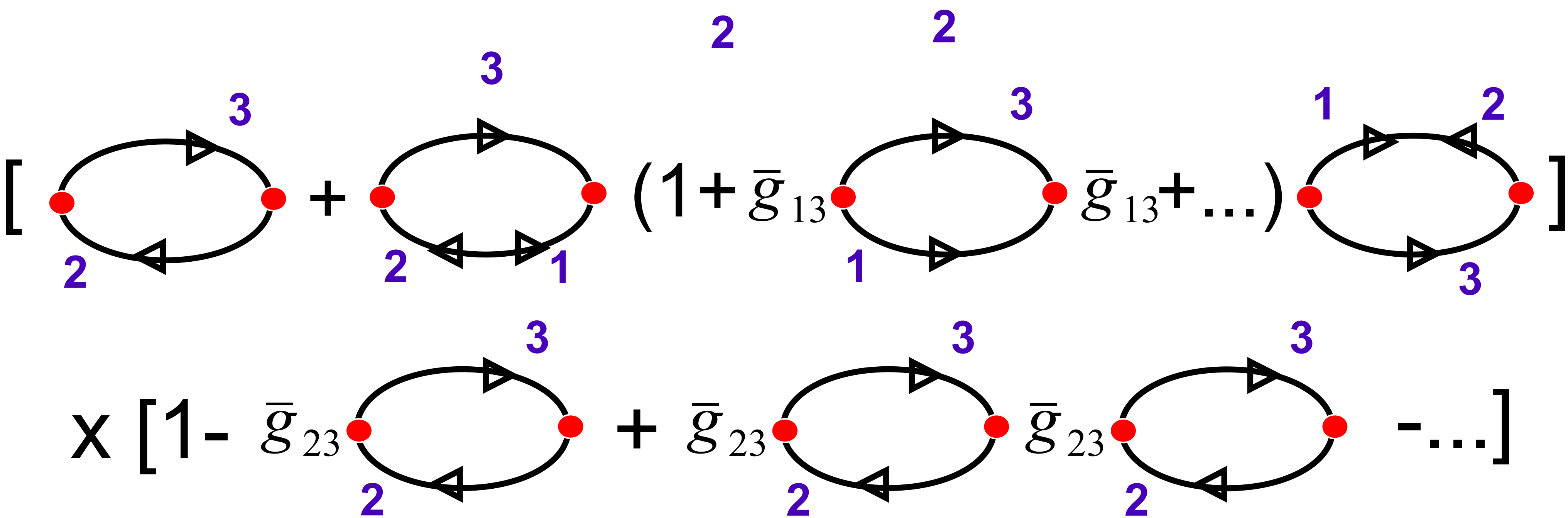
Experimentally, U is not SU(2) invariant.

# Self-consistent Hartree-Fock-BCS calculation



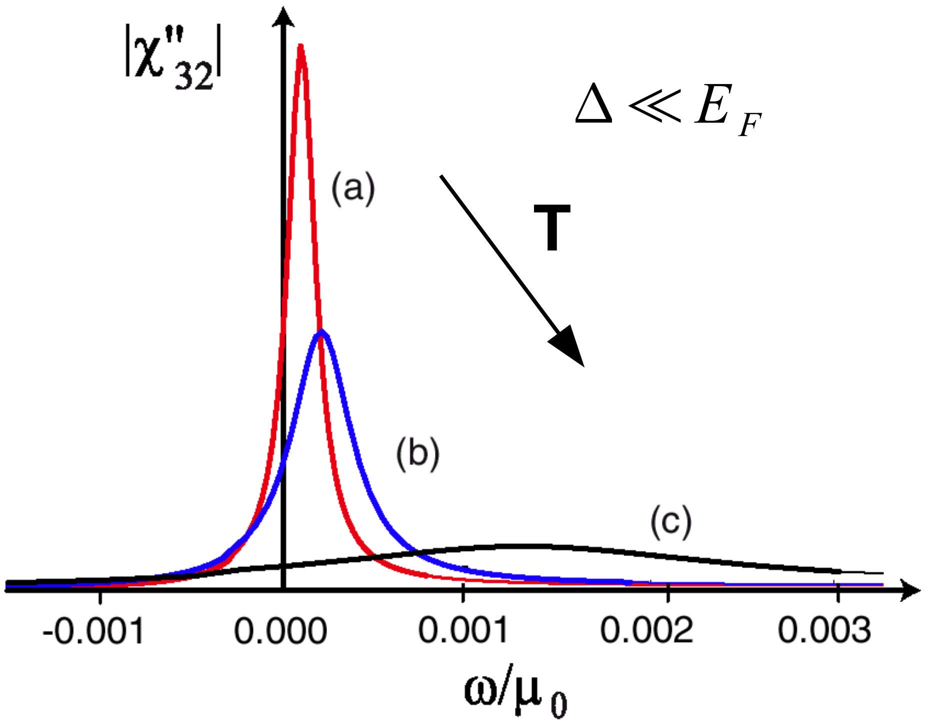

$\chi_{32} =$ 

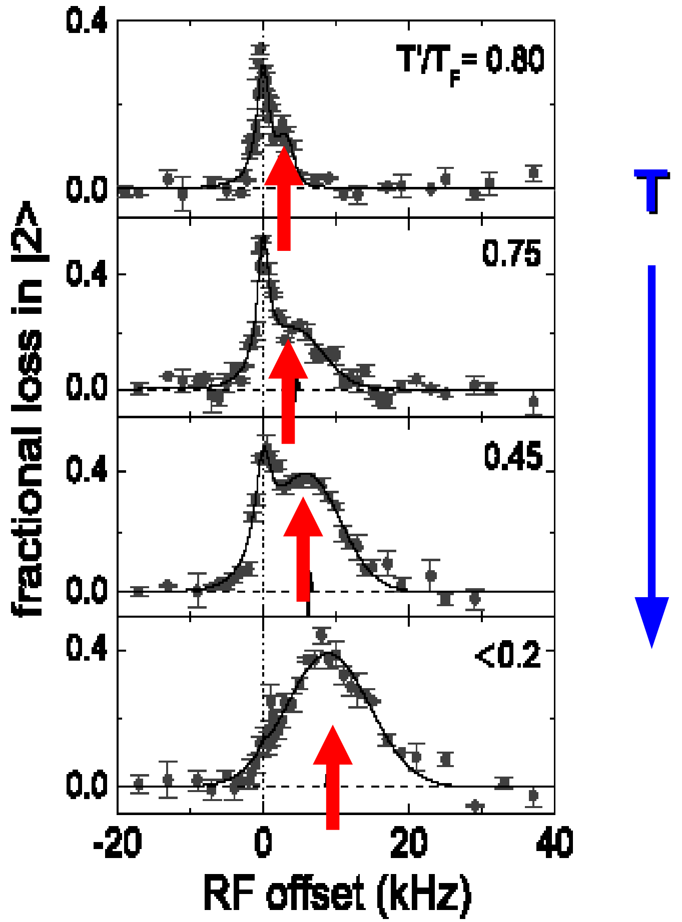

$$v_{ij}(r) = \bar{g}_{ij}\,\delta(\vec{r})$$



# Self-consistent Hartree-Fock-BCS calculation

Shift of "pairing" peak from the normal peak:



# Mean frequency shift by sum rules

$$\bar{\omega} = \frac{\int_{-\infty}^{\infty} d\omega\,\omega\chi''(\omega)}{\int_{-\infty}^{\infty} d\omega\,\chi''(\omega)} \equiv \omega_0 + \Omega_c$$

$$\omega_0 = \epsilon_3 - \epsilon_2$$

$\Omega_c$ = mean clock shift

For atoms initially in |1> and |2>

F-sum rule:

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\omega\chi''(\omega) = \frac{1}{2}\langle[[Y,H],Y]\rangle$$

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\chi''(\omega) = \int d^3r d^3r' \langle\psi_2^\dagger(\mathbf{r})\psi_3(\mathbf{r})\psi_3^\dagger(\mathbf{r}')\psi_2(\mathbf{r}')\rangle \Rightarrow N_2 \quad given \quad N_3=0$$

$$\Omega_c = \frac{1}{2N_2}\langle 12|[[Y,H_{\rm int}],Y]|12\rangle$$

$$\omega_0 = E_Z^3 - E_Z^2$$

$$= \frac{1}{n_2}\int d^3r[v_{13}(r) - v_{12}(r)]\langle\psi_1^\dagger(\mathbf{r})\psi_2^\dagger(0)\psi_2(0)\psi_1(\mathbf{r})\rangle$$

in terms of bare interactions

Zero if U SU(2) invariant

# Mean frequency shift by sum rules

Weak coupling HF-BCS:

HF BCS

$$\frac{1}{N}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\omega\chi''(\omega)=\epsilon_{32}-\frac{1}{n}(g_{12}-g_{13})(n^2+\Delta^2/g_{12}^2) \qquad (n_1=n_2=n)$$

correction vanishes if $g_{12}=g_{13}$ (SU(2) symmetric)

microscopic calculation requires summing bubles

$$\Delta\omega_H=(g_{13}-g_{12})n$$

$$\Delta\omega_{BCS}=(g_{13}-g_{12})\Delta^2/n\,g_{12}^2$$

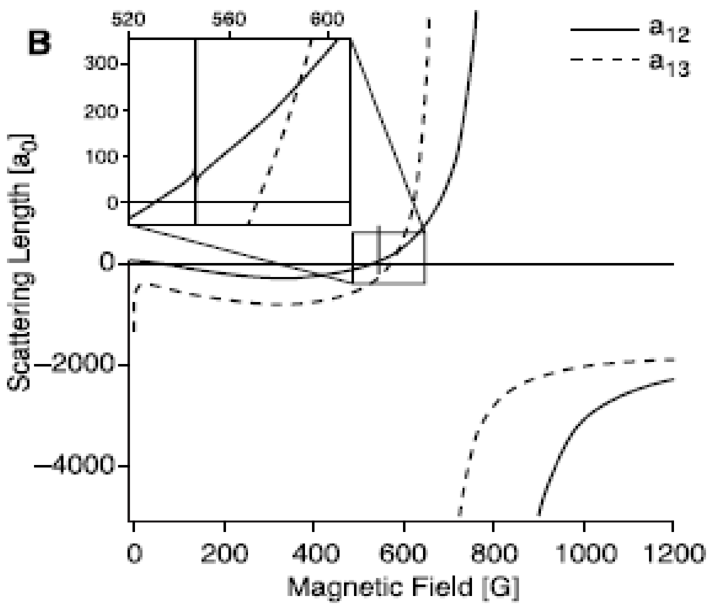
# Unitary regime

$$\Omega_c = \frac{1}{n_2}\int d^3r[v_{13}(r) - v_{12}(r)]\langle\psi_1^\dagger(\mathbf{r})\psi_2^\dagger(0)\psi_2(0)\psi_1(\mathbf{r})\rangle$$

$$\Omega_c = \frac{1}{n_2}(\bar{g}_{13} - \bar{g}_{12})\langle\psi_1^\dagger(0)\psi_2^\dagger(0)\psi_2(0)\psi_1(0)\rangle$$

Calculate correlation function in terms of coupling dependence of the free energy:

$$\frac{\partial(F/V)}{\partial\bar{g}_{12}} = \frac{1}{V}\langle\partial H/\partial\bar{g}_{12}\rangle = \langle\psi_1^\dagger(0)\psi_2^\dagger(0)\psi_2(0)\psi_1(0)\rangle$$




In this case: $\bar{a}_{13}/\bar{a}_{12} \simeq 1$

$$\Omega_c = \left(\frac{\bar{a}_{13}}{\bar{a}_{12}}\right)\left(\frac{1}{g_{13}} - \frac{1}{g_{12}}\right)\frac{1}{n_2}\frac{\partial F/V}{\partial g_{12}^{-1}}$$

mean clock shift in terms of renormalized quantities

# Physical meaning of the mean frequency shift

Interaction with rf field (long wavelength)

$$H_{\rm int} = \mathcal{B}(t)Y \quad \text{with} \quad Y = i\int d^3r \left(\psi_3^\dagger(\mathbf{r})\psi_2(\mathbf{r}) - \psi_2^\dagger(\mathbf{r})\psi_3(\mathbf{r})\right)$$

H_int rotates spin states from $|2\rangle \longrightarrow |\beta\rangle = \cos\theta|2\rangle + \sin\theta|3\rangle$

with spatial wavefunction unchanged

Rotated many particle state: $|1\beta\rangle = e^{-i\theta Y}|12\rangle$

If on average only one single atom promoted from |2> to |3>:

$$\theta = 1/\sqrt{N_2}$$ (realized by short rf pulse)

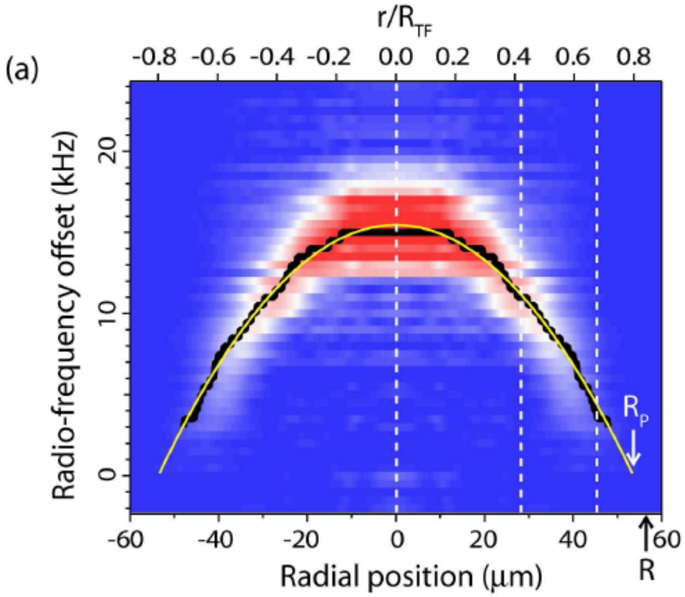
Energy difference of two states:

$$\delta E = \langle 1\beta|H|1\beta\rangle - \langle 12|H|12\rangle$$

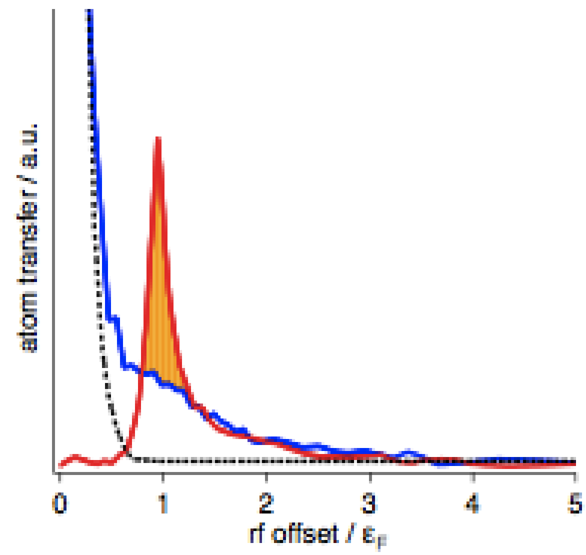
$$= \frac{1}{2N_2}\langle 12|[[Y,H],Y]|12\rangle = \omega_0 + \Omega_c$$

Mean clock shift is the difference in interaction energy of coherently rotated state |1β> and initial state |12>.

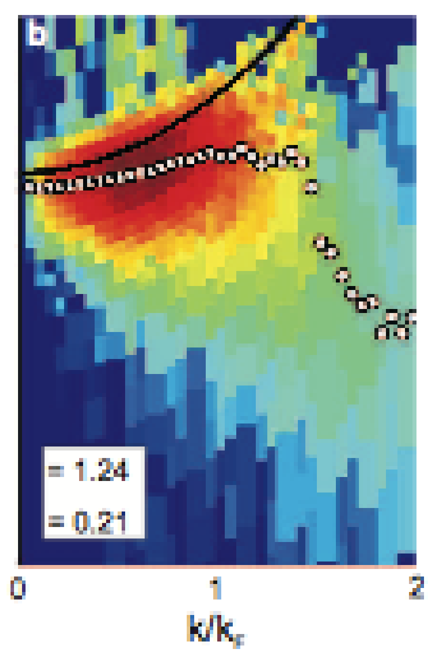
# Further development and application:



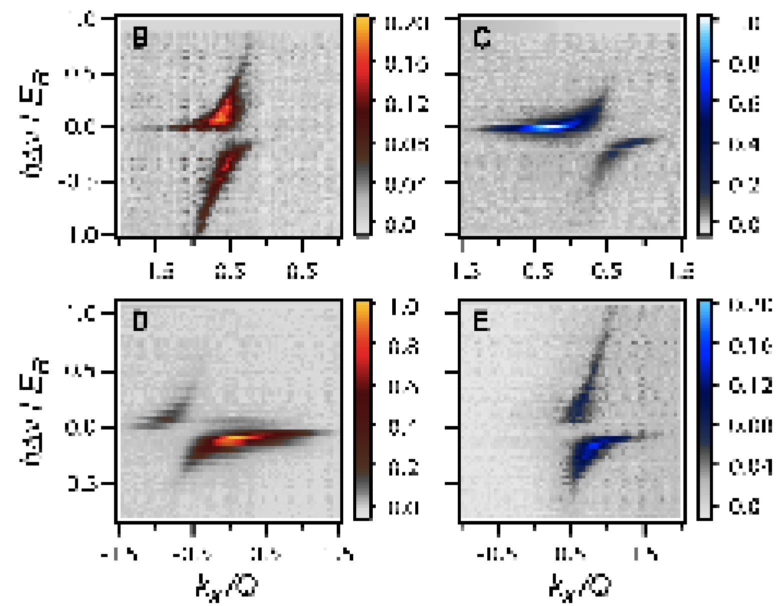

Tomography




Imbalanced Fermi gases




Pseudogap

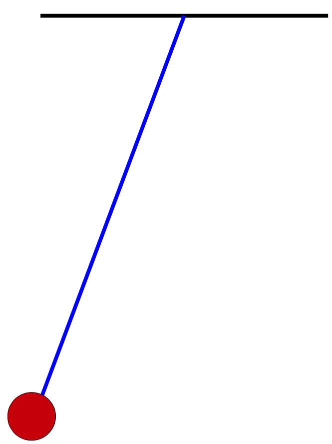



Spin-orbit coupled gases

# Atomic clocks

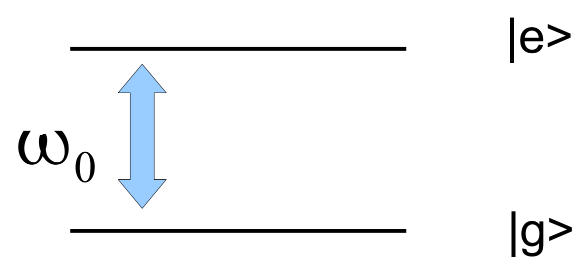
The period of any periodic physical process can be used as a Standard for time. Any physical system that undergoes such a Periodic process can be used as a clock.

Pendulum clocks



$$\omega_0 \sim \sqrt{g/L}$$

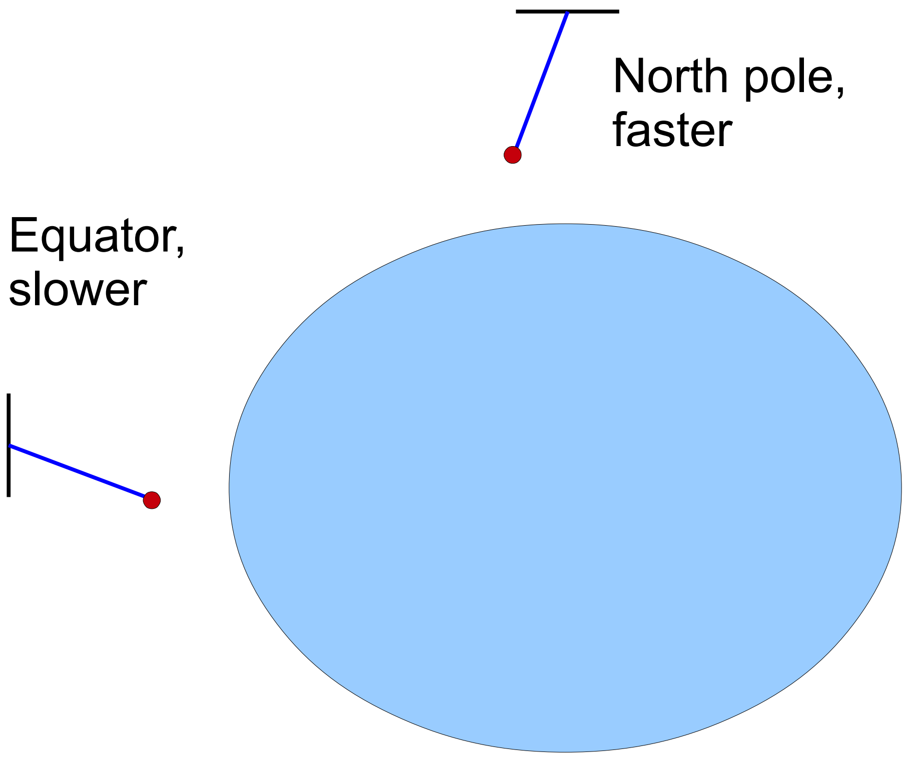
Atomic clocks




$$\psi(0) = \chi_g + \chi_e$$

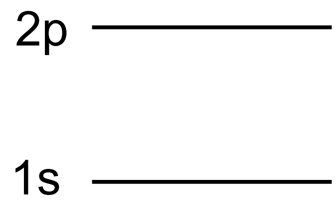
$$\psi(t) = \chi_g + \exp(-i\omega_0 t)\chi_e$$

# Pendulum is not a good clock.




# Atomic clocks

Hydrogen atoms




$$\omega_0 \sim e^2/a_0 \sim 10^{15}\ Hz$$

optical

Fine-structure splitting

$$\omega_0 \sim \alpha^2 e^2/a_0 \sim 10^{11}\ Hz$$

Hyperfine-structure splitting

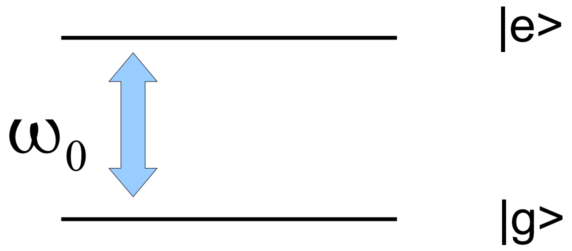
$$\omega_0 \sim 100\ MHz$$

microwave

Combination of fundamental physical constants.

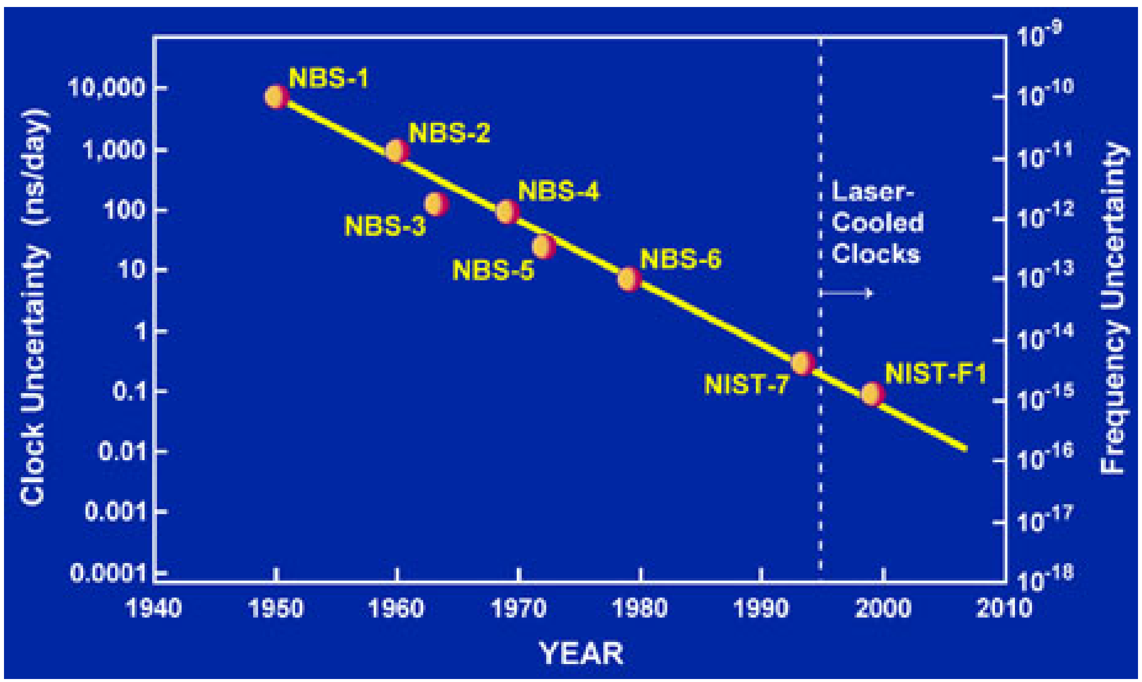
The problem is to measure the intrinsic frequency accurately.

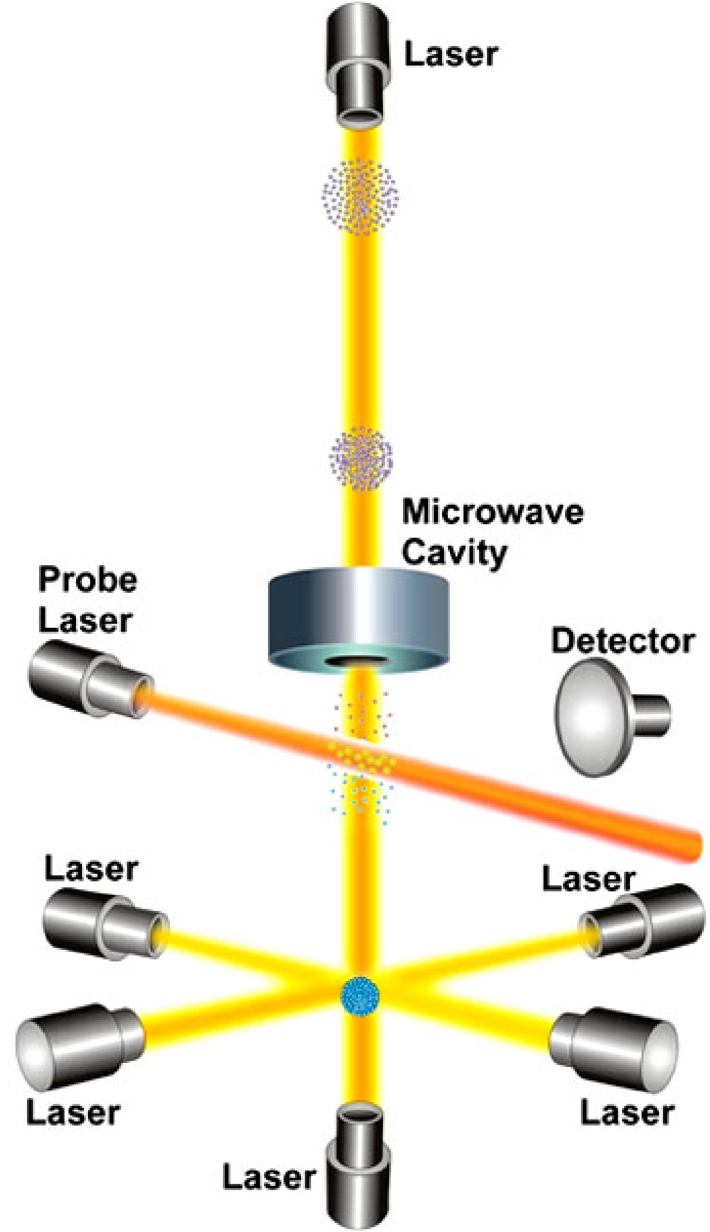
Atomic clocks



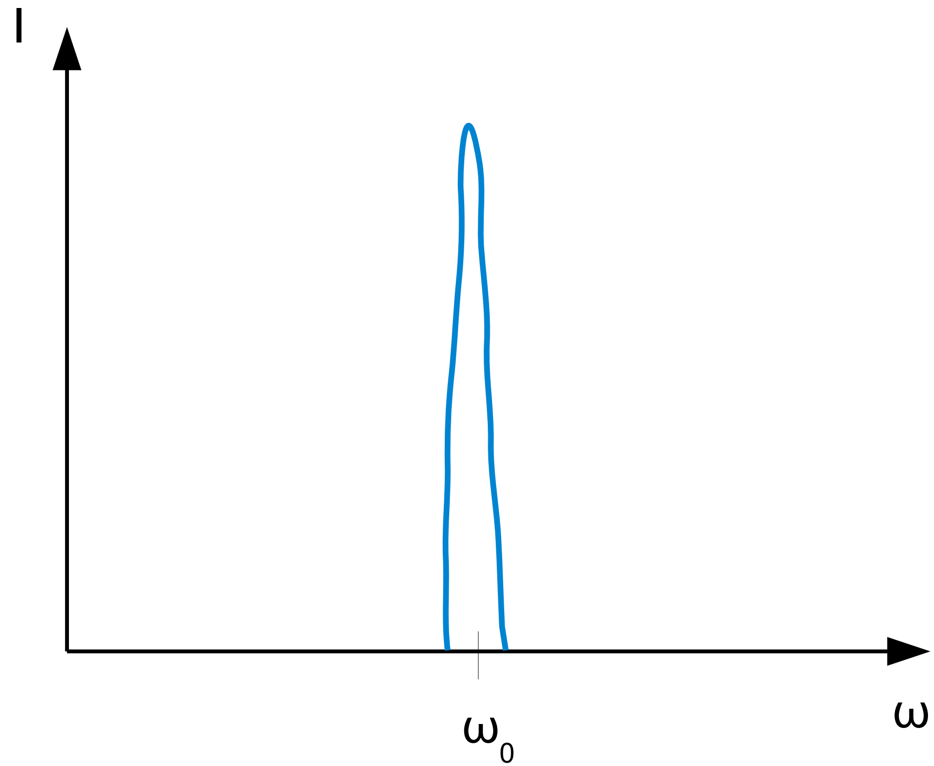

Clock shifts

$$\omega_{measure} - \omega_0$$

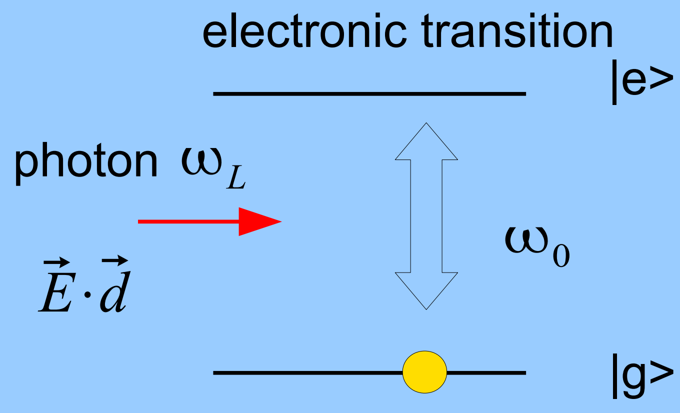


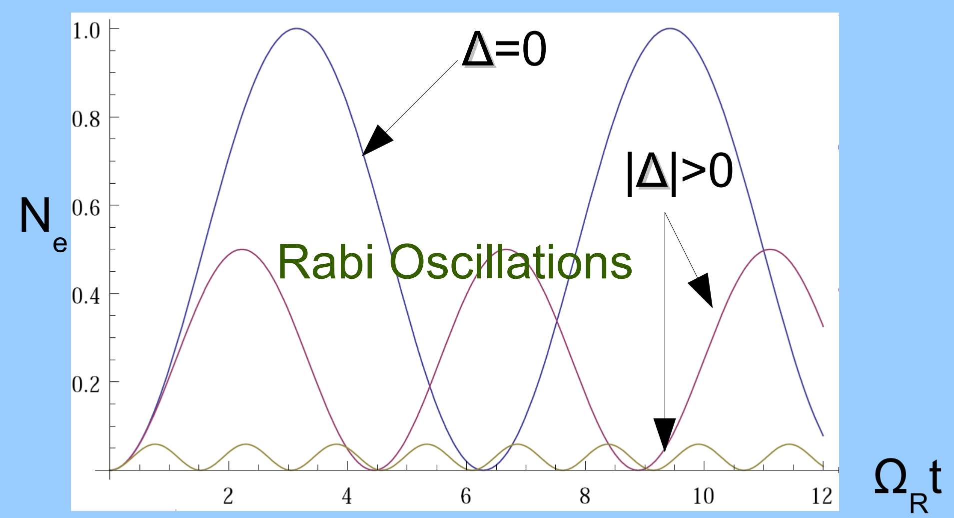

NIST-F1 Cesium ( 铯 ) fountain atomic clock of microwave transition, serving as the US time and frequency standard, with an uncertainty of 5.10×10−16 (as of 2005).


I
ω
ω0

# Atomic Clock

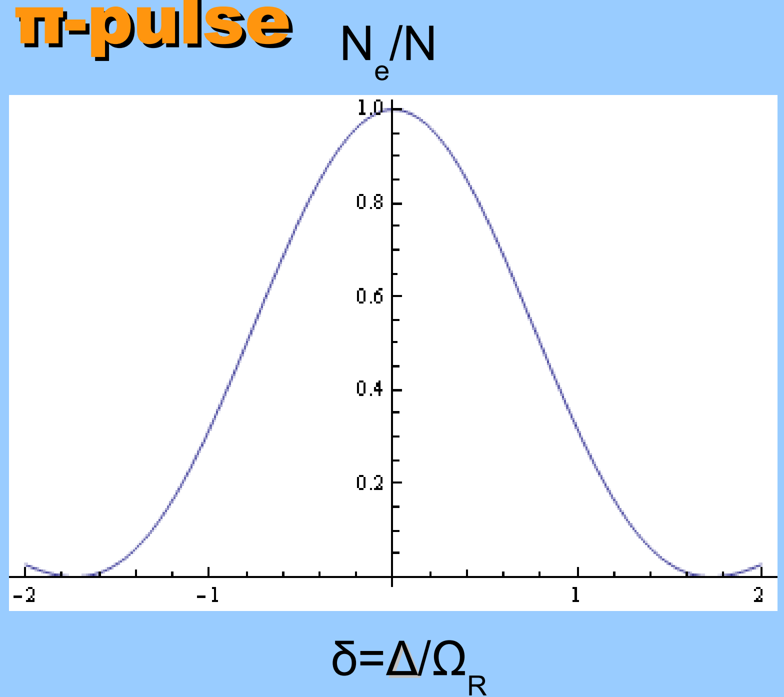



# Coherent Evolution




Rabi frequency $\Omega_R$ , Detuning $\Delta=\omega_L-\omega_0$

# π-pulse



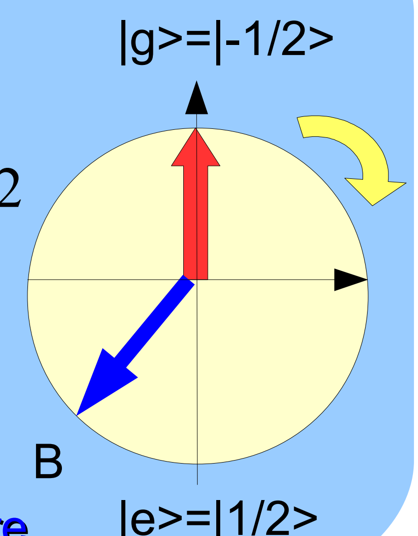

# Pseudospin

$$H_{probe}=B_+S_-+B_-S_+$$

$$S_i=[a_e^+,a_g^+]\sigma_i[a_e,a_g]^T/2$$

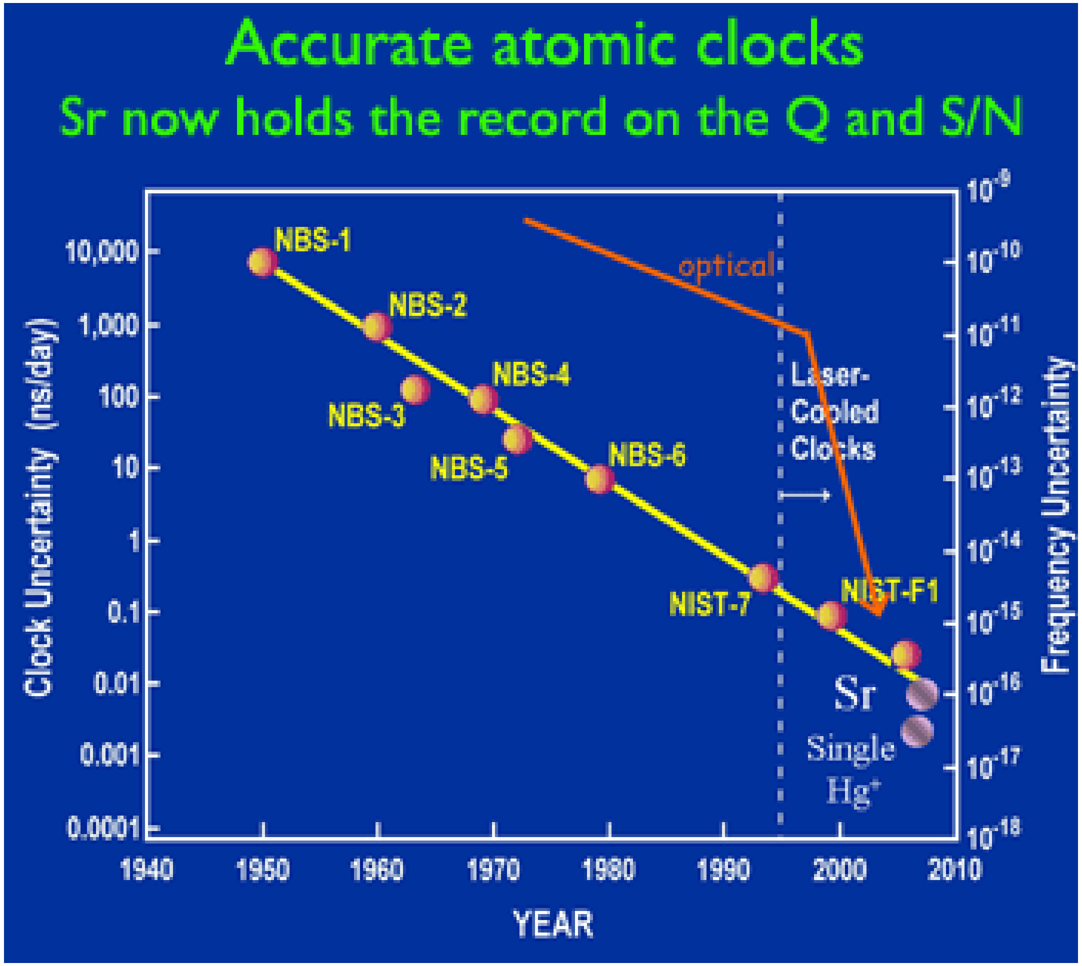
$$B_-=\langle g|\vec{E}\cdot\vec{d}|e\rangle$$

$$B_+=B_-^*$$




Bloch Sphere

# New generation: optical atomic clocks

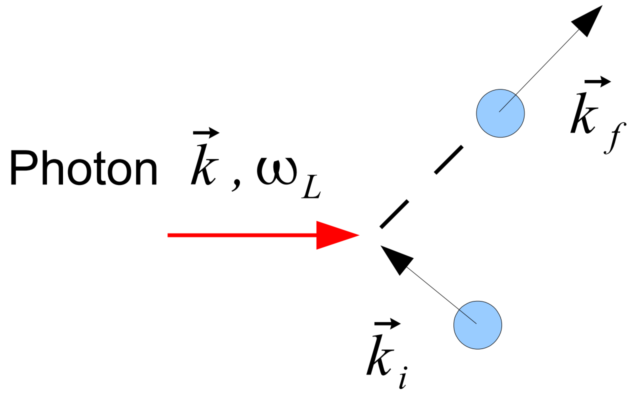



**Narrow line width and higher frequency good for high Q=f/df**
**Charge neutral good for high signal-noise ratio**

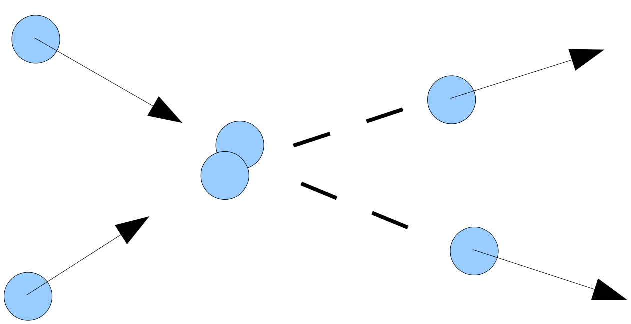
Sr ( 锶 )-87, 1S0->3P0 transition, df~1mHz

# Two obstacles to high accuracy:

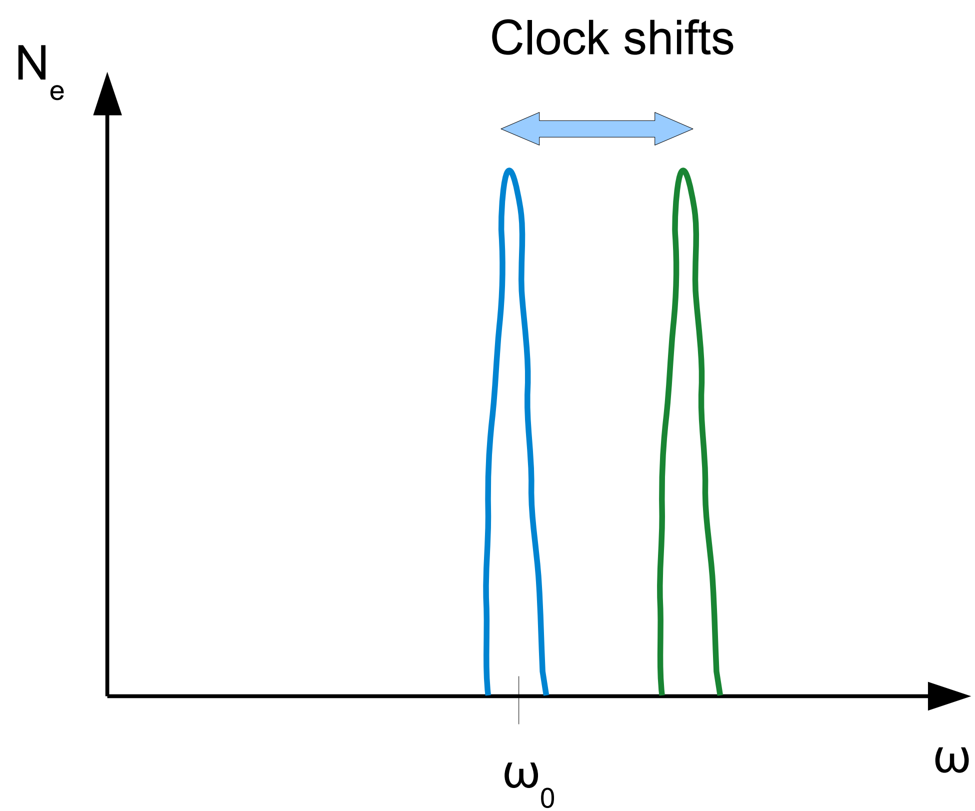
Translational motions and interatomic interactions




$$\sim\delta(\omega-\omega_0-k_f^2/2\mathrm{m}+k_i^2/2\mathrm{m})$$

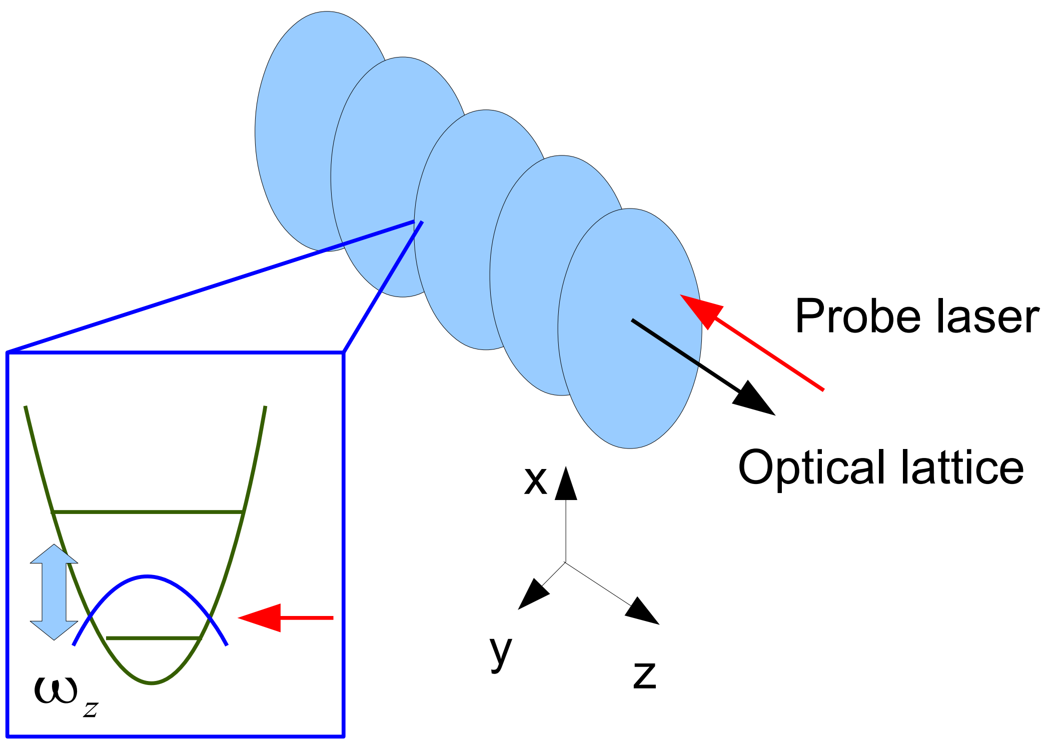


Collisions

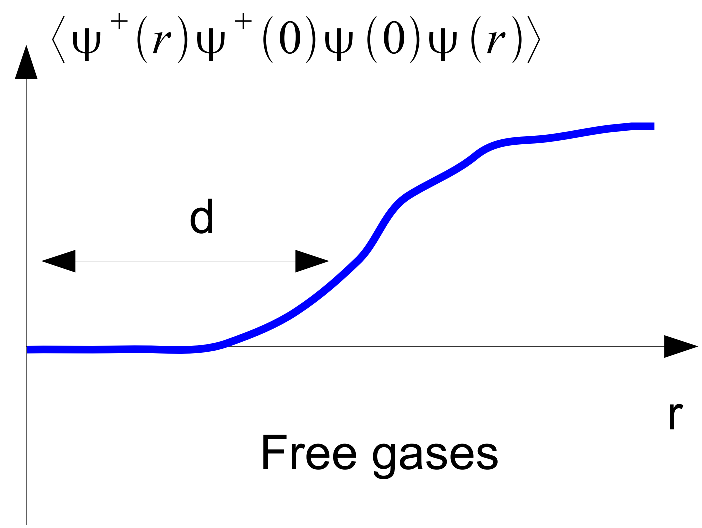


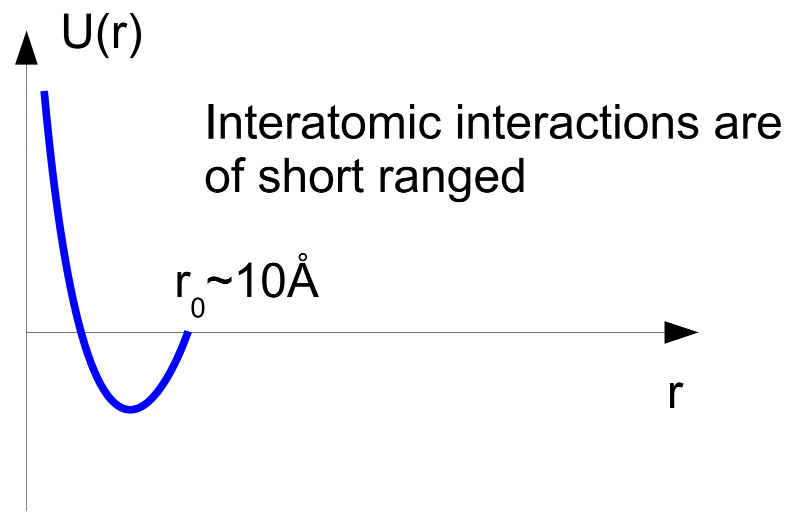
# Solutions:

Freeze the translational motion by loading atoms in optical lattices




Suppress interatomic interactions by using identical fermions



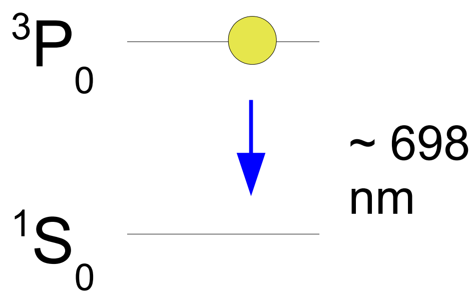



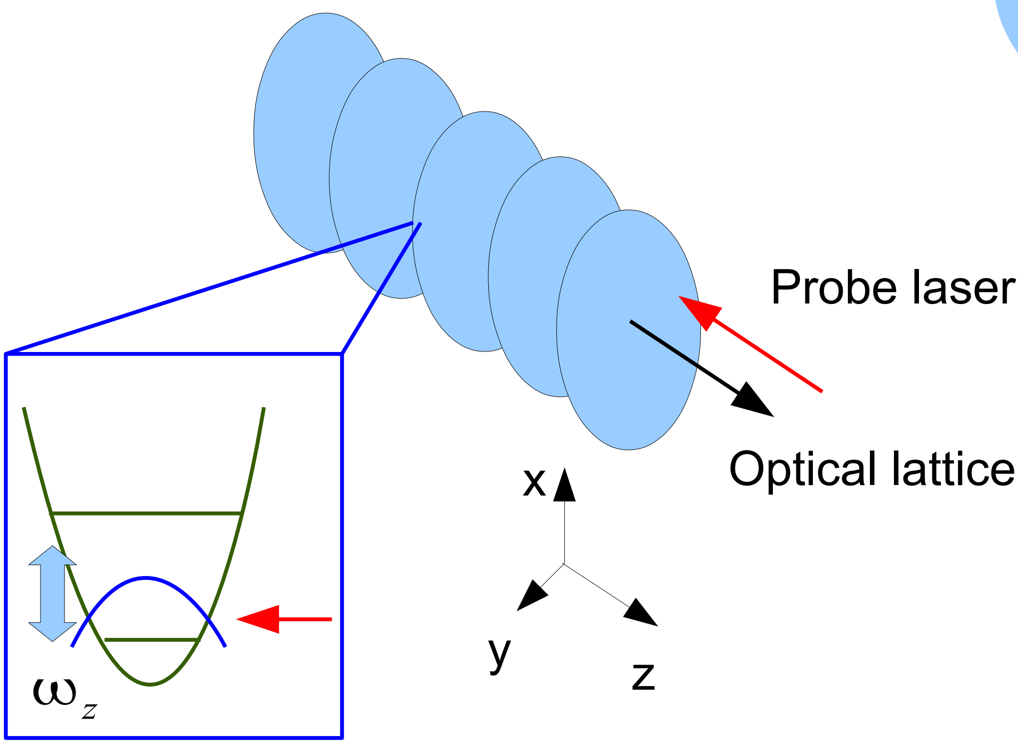

Dilute gases: $r_0/d \ll 1$

$$\int U(r)\langle\psi^+(r)\psi^+(0)\psi(0)\psi(r)\rangle \sim 0$$

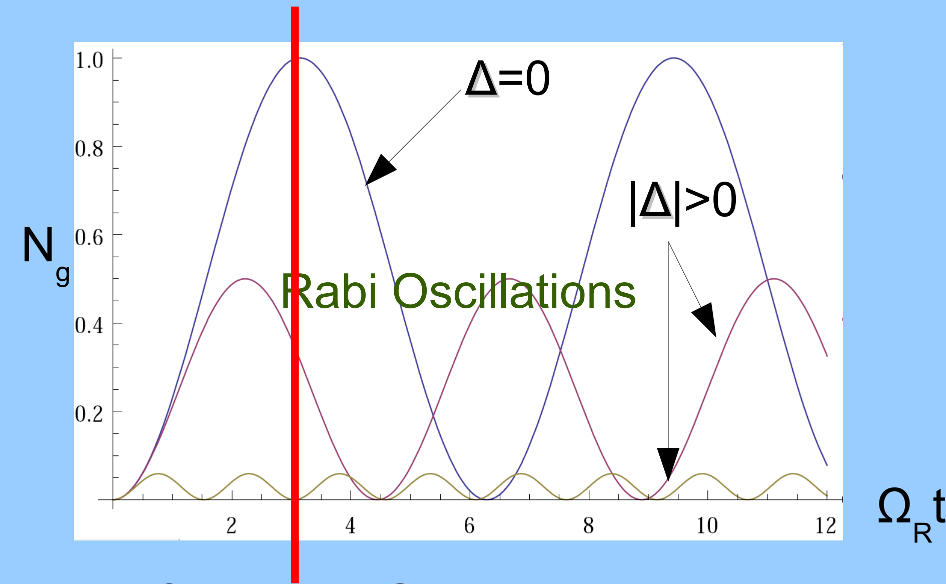
# Atomic clock of $^{87}$Sr @ JILA

*Ye's group, Science 324, 360 - 363 (2009)*



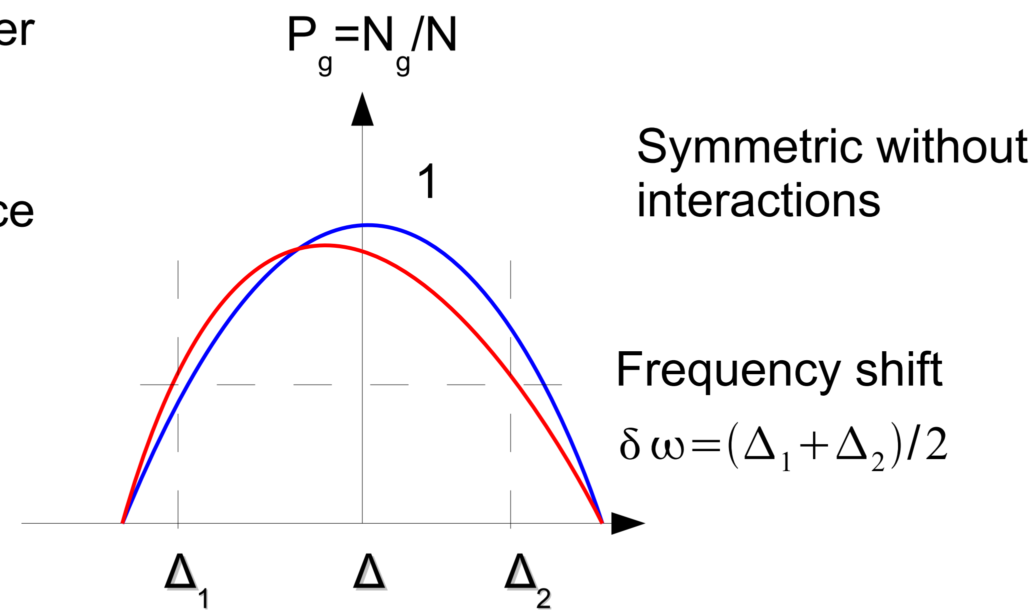



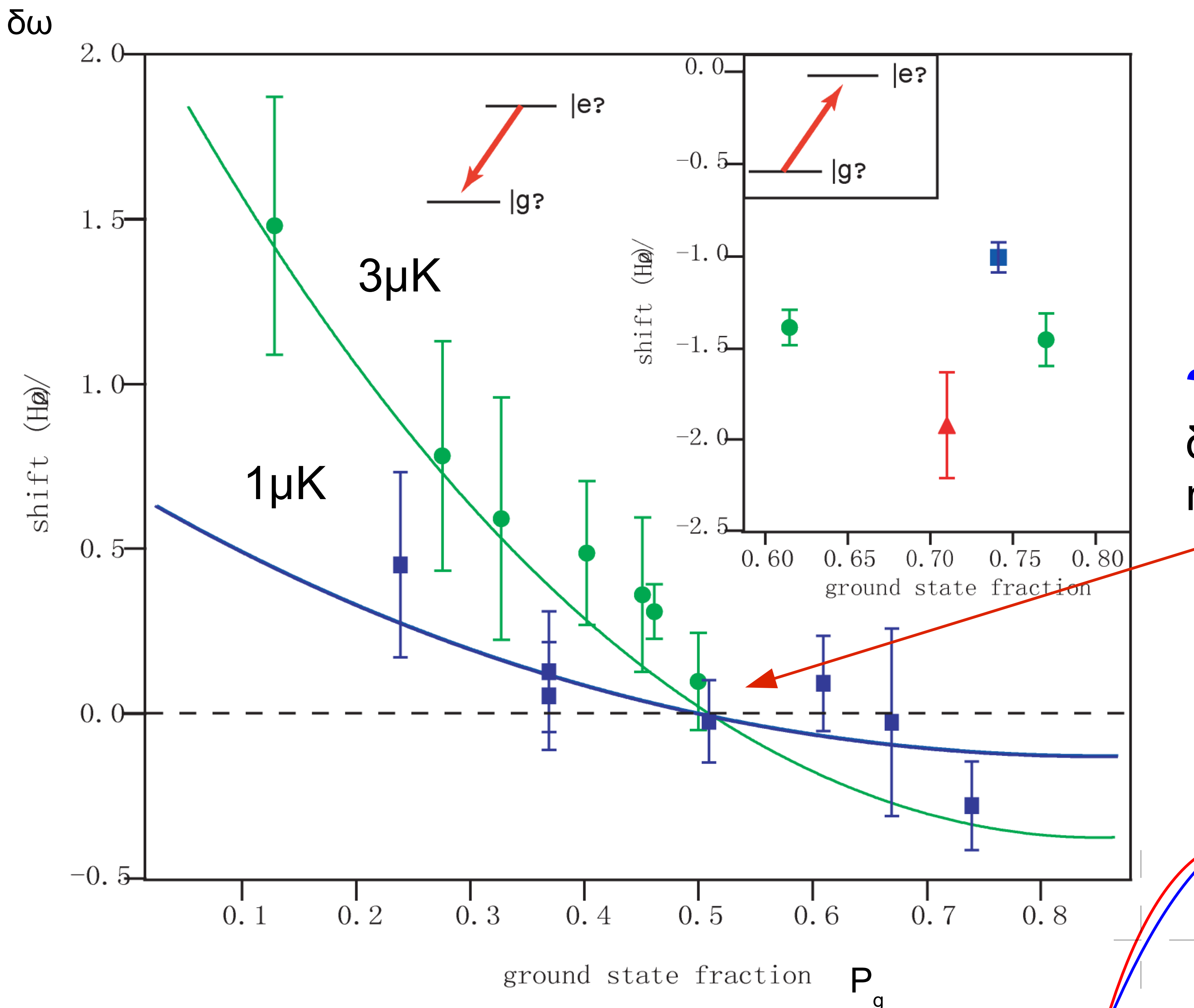

## Coherent Evolution

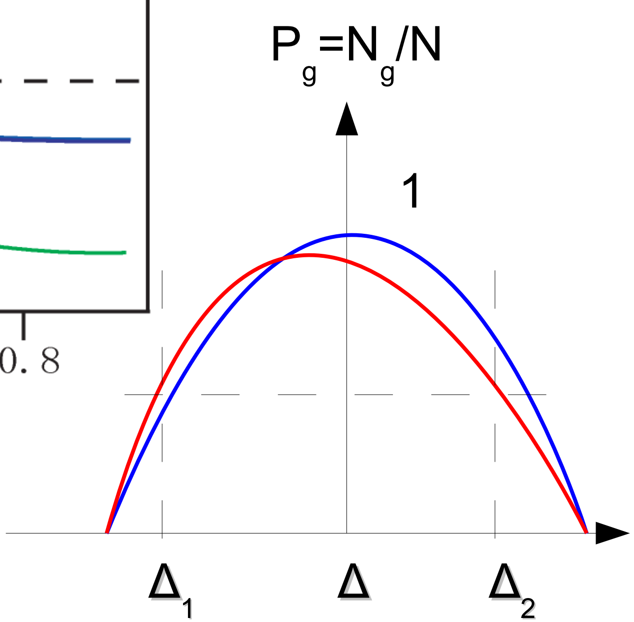



Rabi frequency $\Omega_R$ , Detuning $\Delta=\omega_L-\omega_0$

Measure N_g after a π pulse with various Δ.

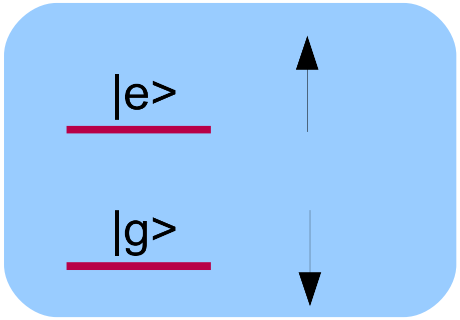





Density-related shifts observed

# Confusion?!




Construct S=1/2 pseudospin space:

$$\hat{\boldsymbol{S}} = \frac{1}{2}\int d\vec{r}\,[\psi_e^+(\vec{r}),\psi_g^+(\vec{r})]\,\boldsymbol{\sigma}\,[\psi_e(\vec{r}),\psi_g(\vec{r})]^T$$

$$H = H_0 + H_1$$

$$H_0 = \int d\vec{r} \sum_{\sigma=e,g} \psi_\sigma^{\ +}(\vec{r})\left[\frac{-\nabla^2}{2\mathrm{m}} + E_\sigma + V_\sigma(\vec{r})\right]\psi_\sigma(\vec{r})$$

external potential independent of σ

$$\omega_0 = E_e - E_g$$

$$H_1 = \int d\vec{r}_1\, d\vec{r}_2\, U(|\vec{r}_1 - \vec{r}_2|)\,\psi_e^{\ +}(\vec{r}_1)\,\psi_g^{\ +}(\vec{r}_2)\,\psi_g(\vec{r}_2)\,\psi_e(\vec{r}_1)$$

$$H_1 = \sum\nolimits_{i<j} U(|\vec{r}_i - \vec{r}_j|)(1/2 - 2\,\hat{s}_i\cdot\hat{s}_j) \qquad \hat{\boldsymbol{S}} = \sum\nolimits_i \hat{s}_i$$

Interaction SU(2) invariant $[H_{1,}\hat{\boldsymbol{S}}] = 0$

# Equations of pseudospin operators

To calculate N_g(t) or N_e(t), note S_z=(N_g-N_e)/2 and N=N_g+N_e

Homogeneous external probe field couples to the system as

$$H_{probe} = B_{+}(t)\hat{S}_{-} + B_{-}(t)\hat{S}_{+}$$

The coherent evolution of the pseudospin, governed by $K=H_0+H_1+H_{probe}$, is determined as

$$i\frac{d}{dt}\hat{\boldsymbol{S}} = [\hat{\boldsymbol{S}}, H_0 + H_1 + H_{probe}] = \omega_0[\hat{\boldsymbol{S}}, \hat{S}_z] + [\hat{\boldsymbol{S}}, H_{probe}]$$

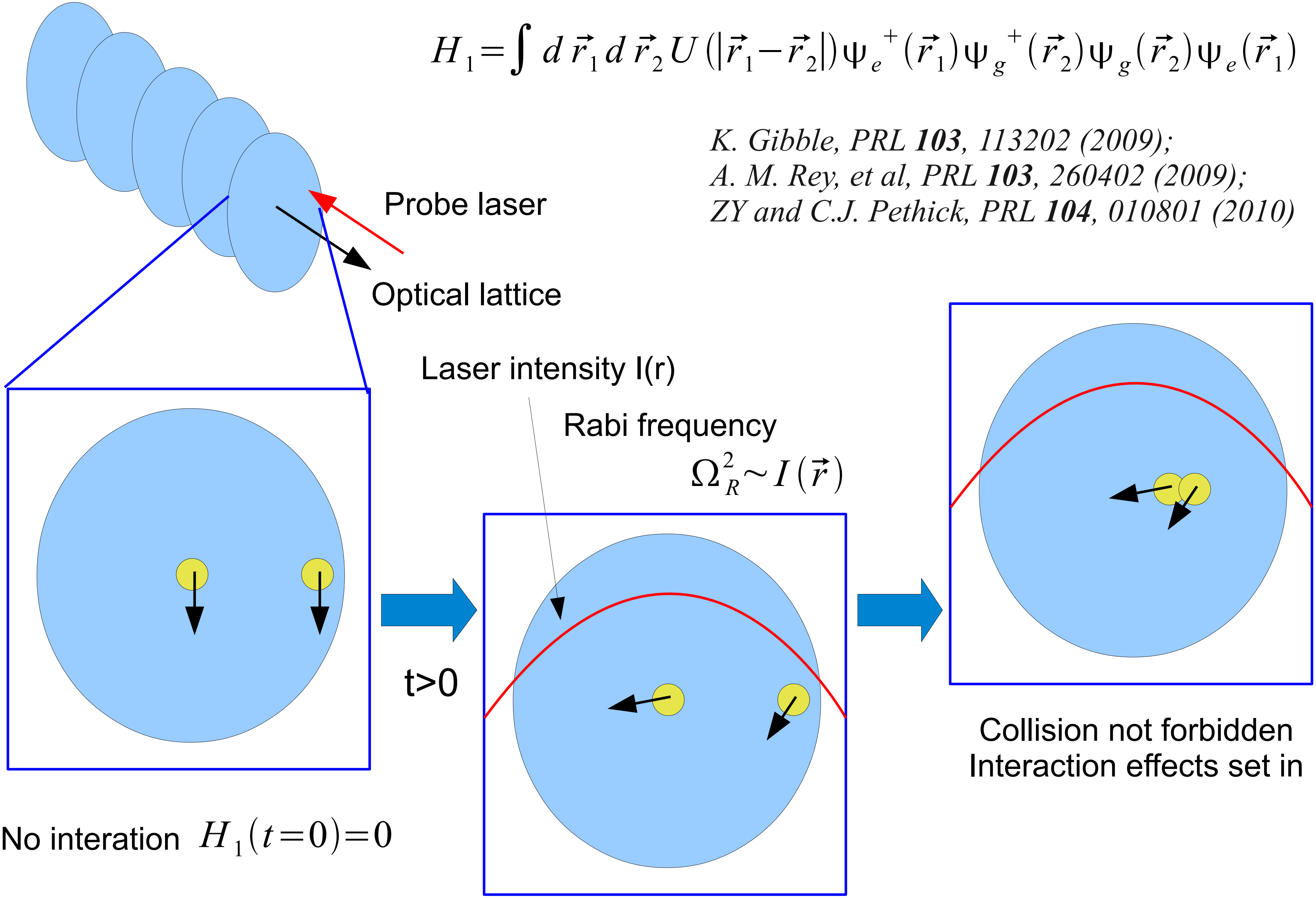
**S_z independent of U(r) since** $[H_{1,}\hat{\boldsymbol{S}}]=0$ **and** $[\hat{\boldsymbol{S}}, H_{probe}]$ **only depends on S**

**For a two-component Fermi gas, there should be no frequency shift for a homogenous probe field.**

# Inhomogeneity of probe fields

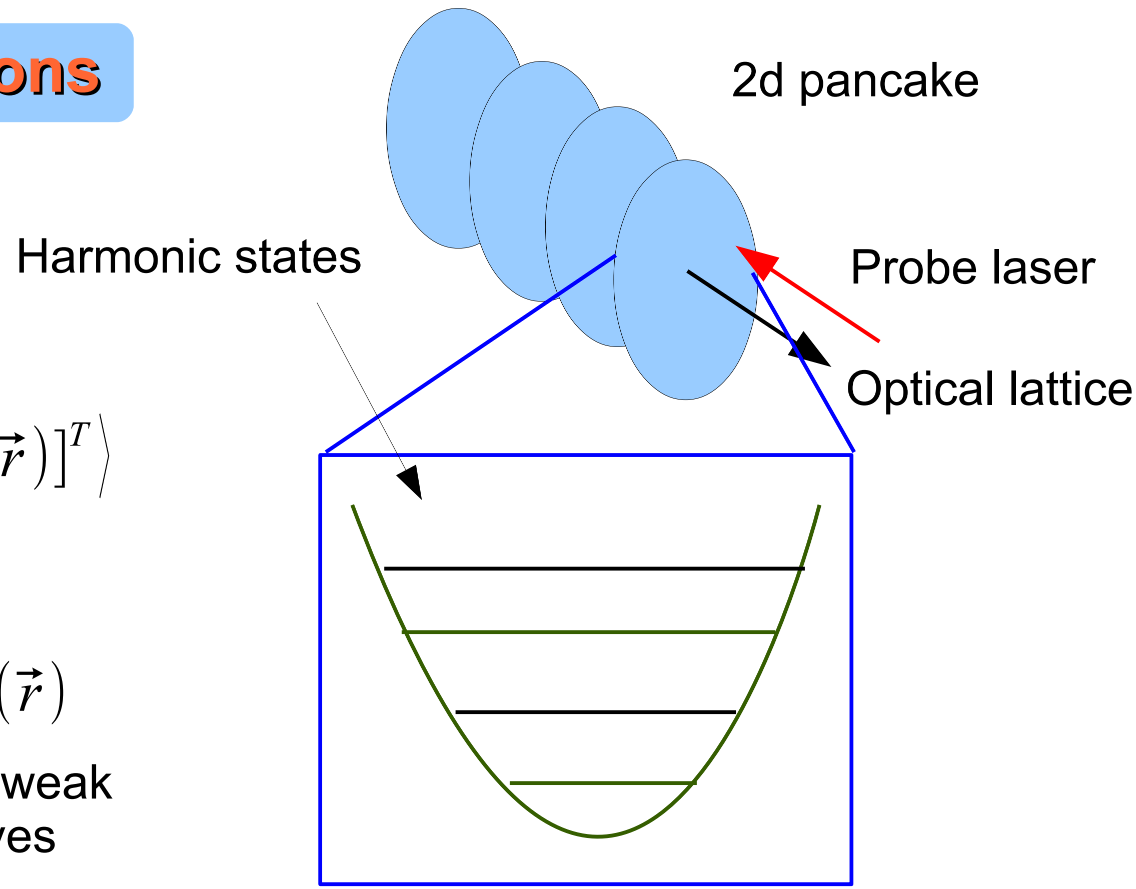
$$H_1 = \int d\vec{r}_1 d\vec{r}_2 U(|\vec{r}_1 - \vec{r}_2|) \psi_e^+(\vec{r}_1) \psi_g^+(\vec{r}_2) \psi_g(\vec{r}_2) \psi_e(\vec{r}_1)$$

*K. Gibble, PRL* ***103****, 113202 (2009);*
*A. M. Rey, et al, PRL* ***103****, 260402 (2009);*
*ZY and C.J. Pethick, PRL* ***104****, 010801 (2010)*



# Pseudospins & Bloch equations

Expand $\psi_\alpha(\vec{r}) = \sum_i a_{\alpha i} \phi_i(\vec{r})$




Pseudospin for each motional state

$$\vec{S}_i = \frac{1}{2} \left\langle [a_{ei}^{+}(\vec{r}), a_{gi}^{+}(\vec{r})] \boldsymbol{\sigma} [a_{ei}(\vec{r}), a_{gi}(\vec{r})]^T \right\rangle$$

The total pseudospin $\vec{S} = \sum_i \vec{S}_i$

$$H_1 = g \int d\vec{r}\, \psi_e^{+}(\vec{r}) \psi_g^{+}(\vec{r}) \psi_g(\vec{r}) \psi_e(\vec{r})$$

Within the lamb-Dicke regime and in the weak interaction limit, the mean field theory gives

$$\frac{d}{dt} \vec{S}_i = \vec{\Omega}_i \times \vec{S}_i + 2 \sum_j g_{ij} \vec{S}_i \times \vec{S}_j$$

Interaction: molecular field

$$g_{ij} = g \int d\vec{r} |\phi_i(\vec{r}) \phi_j(\vec{r})|^2$$

**Pseudospins driven out of phase**

$$g = 4\pi a / m$$

S-wave scattering length

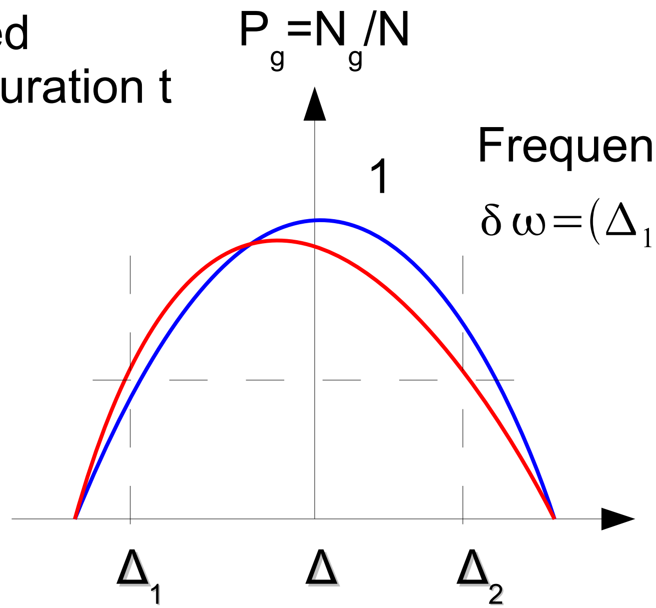
For laser field $\vec{E}(\vec{r}, t) = \vec{E}_-(\vec{r}) e^{-i\omega_L t} + C.C.$

$$\Omega_{iz} = \Delta = E_e - E_g - \omega_L \quad \Omega_{ix} = 2\Re(\Omega_{i-}) \quad \Omega_{iy} = -2\Im(\Omega_{i-})$$

$$\Omega_{i-} = -\int d\vec{r} |\phi_i(\vec{r})|^2 \left\langle e \middle| \vec{d} \cdot \vec{E}_-(\vec{r}) \middle| g \right\rangle$$

# Frequency shift

For fixed pulse duration t




Frequency shift

$\delta\omega = (\Delta_1 + \Delta_2)/2$

The symmetry of the Bloch equations

$$\frac{d}{dt}\vec{S}_i = \vec{\Omega}_i \times \vec{S}_i + 2\sum_j g_{ij}\vec{S}_i \times \vec{S}_j$$

For simplicity, assume $\Omega_{ix}$ nonzero

Noninteracting (g=0), invariant under the transformation

$$\{\Delta, S_{kx}, S_{ky}, S_{kz}\} \rightarrow \{-\Delta, -S_{kx}, S_{ky}, S_{kz}\}$$

$$S_z(\Delta_1) = S_z(\Delta_2)$$

$$S_z(\Delta) = S_z(-\Delta), \delta\omega = 0$$

Interacting, invariant under the transformation

$$\{\Delta, g, S_{kx}, S_{ky}, S_{kz}\} \rightarrow \{-\Delta, -g, -S_{kx}, S_{ky}, S_{kz}\}$$

For small shift:

$$\delta\omega = \frac{2\delta S_z(t, \Delta)}{dS_z^0(t, \Delta)/d\Delta}$$

# Pseudospins & Bloch equations

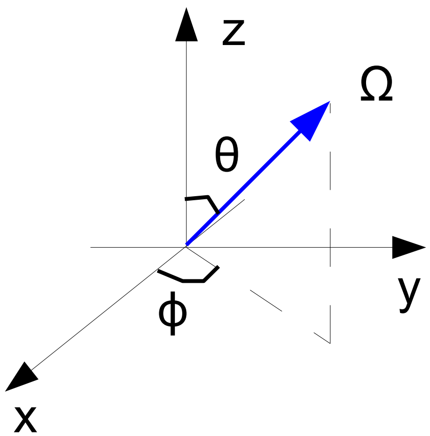
The Bloch equations can be converted into integral ones

$$\vec{S}_i(t) = G_i(t)\vec{S}_i(0) + 2\sum_j g_{ij}\int_0^t dt'\, G_i(t-t')\vec{S}_i(t')\times\vec{S}_j(t')$$

$$\left(\frac{d}{dt} - Q\right) G(t) = \delta(t)$$

Individual free precession

$$Q = \Omega\begin{pmatrix} 0 & -\cos\theta & \sin\theta\sin\phi \\ \cos\theta & 0 & -\sin\theta\cos\phi \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{pmatrix}$$



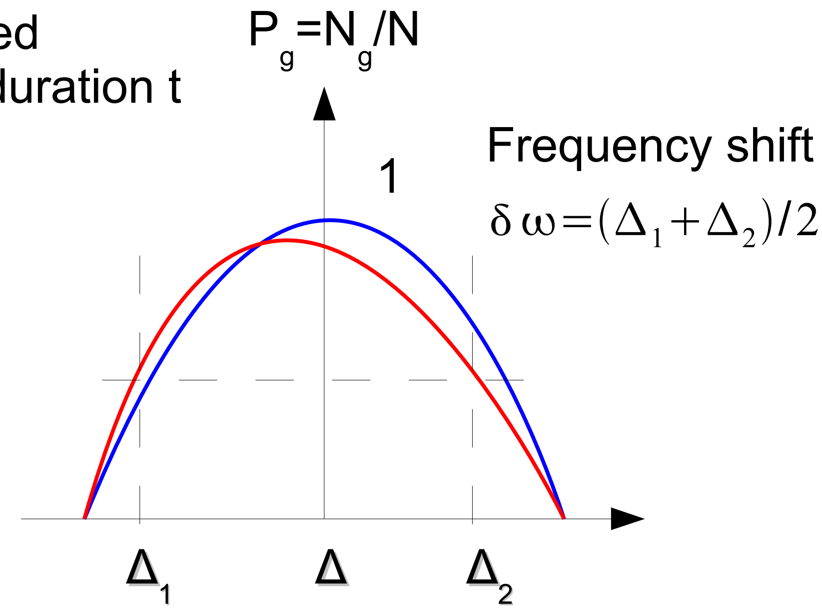

The change of the total pseudospin due to interaction is

$$\delta\vec{S}(t) = \sum_j g_{ij}\int_0^t dt'\, \underline{[G_i(t-t') - G_j(t-t')]}[\vec{S}_i(t')\times\vec{S}_j(t')]$$

Zero if probe field is homogeneous, i.e., $\Omega_i$ is independent of i

# Small dispersion limit

For fixed
pulse duration t




In experiment, small dispersion in driving frequencies:

$$|\vec{\Omega}| = |\vec{\Omega}_i| \sim 6\ Hz \qquad |\delta\vec{\Omega}_{ij}| = |\vec{\Omega}_i - \vec{\Omega}_j| \sim 0.1\ Hz$$

Since $P_g = N/2 - \sum_i S_{iz}$

$$\delta \mathrm{P}_g(t) = C_1 \Xi_1 + C_3 \Xi_3 + C_3 \Xi_3,$$

where

$$\Xi_1 = \sum_{i,j} \frac{4 g_{ij} S_i(0) S_j(0)}{\Omega_{xy}^3} (\delta \boldsymbol{\Omega}_{ij\|})^2$$

$$\Xi_2 = \sum_{i,j} \frac{4 g_{ij} S_i(0) S_j(0)}{\Omega_{xy}^3} (\delta \boldsymbol{\Omega}_{ij\perp})^2$$

$$\Xi_3 = \sum_{i,j} \frac{4 g_{ij} S_i(0) S_j(0)}{\Omega_{xy}^3} \mathbf{e}_z \cdot (\delta \boldsymbol{\Omega}_{ij\|} \times \delta \boldsymbol{\Omega}_{ij\perp})$$

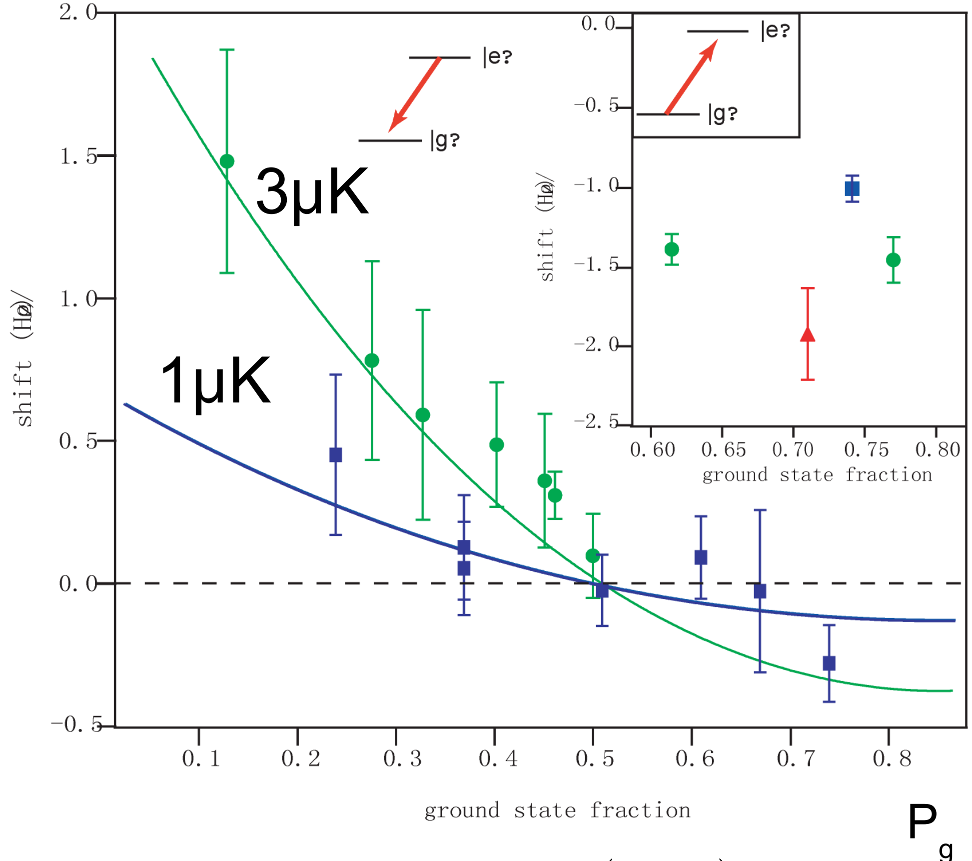
with $\delta = \Delta/\Omega_{xy}$, $\tau = \Omega_{xy} t$, $\boldsymbol{\Omega}_{xy} = \overline{\boldsymbol{\Omega}}_i - \Delta \mathbf{e}_z$, $\delta \boldsymbol{\Omega}_{ij\|} = (\delta \boldsymbol{\Omega}_{ij} \cdot \boldsymbol{\Omega}_{xy}) \boldsymbol{\Omega}_{xy} / \Omega_{xy}^2$, and $\delta \boldsymbol{\Omega}_{ij\perp} = \delta \boldsymbol{\Omega}_{ij} - \delta \boldsymbol{\Omega}_{ij\|}$

$$\begin{aligned} C_1 = & \frac{\delta}{6(1+\delta^2)^4} \Big\{ 3(2+\delta^2) \sin^2\left(\tau\sqrt{1+\delta^2}\right) \\ & - 3\left[\tau^2 + (3\tau^2 - 4)\delta^2 + 2(2+\tau^2)\delta^4\right] \\ & \times \cos\left(\tau\sqrt{1+\delta^2}\right) + 3\left[4\delta^4 - 4\delta^2 - \tau^2(1+\delta^2)\right] \\ & + \tau\sqrt{1+\delta^2}\left[\tau^2 + (\tau^2+9)\delta^2 - 6\delta^4\right] \\ & \times \sin\left(\tau\sqrt{1+\delta^2}\right) \Big\}, \end{aligned}$$

$$\begin{aligned} C_2 = & \frac{\delta}{2(1+\delta^2)^{7/2}} \Big\{ 4\delta^2\sqrt{1+\delta^2} - 4\delta^2\sqrt{1+\delta^2} \\ & \times \cos\left(\tau\sqrt{1+\delta^2}\right) - \sqrt{1+\delta^2} \sin^2\left(\tau\sqrt{1+\delta^2}\right) \\ & -\tau\left(-1+\delta^2+2\delta^4\right) \sin\left(\tau\sqrt{1+\delta^2}\right) \Big\}, \end{aligned}$$

$$\begin{aligned} C_3 = & \frac{2\sin^2\left(\tau\sqrt{1+\delta^2}/2\right)}{(1+\delta^2)^{5/2}} \\ & \times \left\{ -\tau\sqrt{1+\delta^2} + \sin\left(\tau\sqrt{1+\delta^2}\right) \right\}, \end{aligned}$$

# Frequency shift



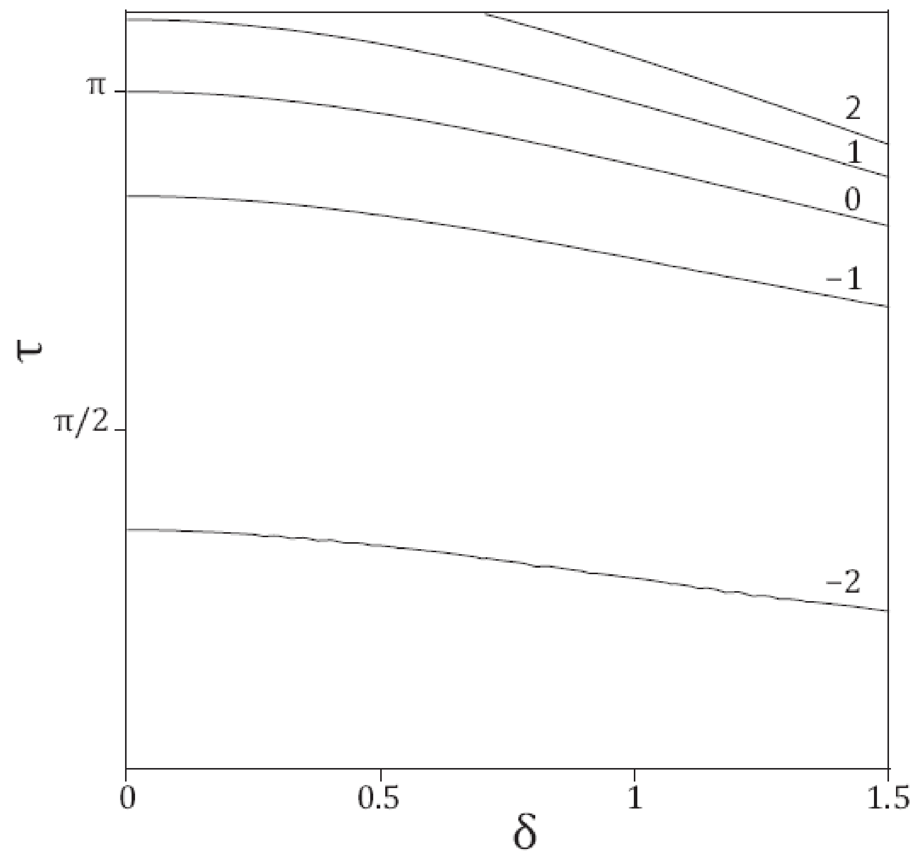

**The shift:** $$\delta\omega = \frac{2\delta S_z(t,\Delta)}{dS_z^0(t,\Delta)/d\Delta}$$

$$\mathbf{E}_-(\mathbf{r}) = E_0 \mathbf{e}_x \frac{w(0)}{w(z)} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \times \exp\left(i\frac{2\pi}{\lambda_L}z + i\frac{\pi(x^2+y^2)}{\lambda_L R(z)} - i\zeta(z)\right)$$

S-wave scattering length a unknown

Temperature T lower, atoms concentrate more at the center of the trap,
$\Omega_{xy}$ increases, τ increases
zero shift occurs at smaller δ, larger $P_g$

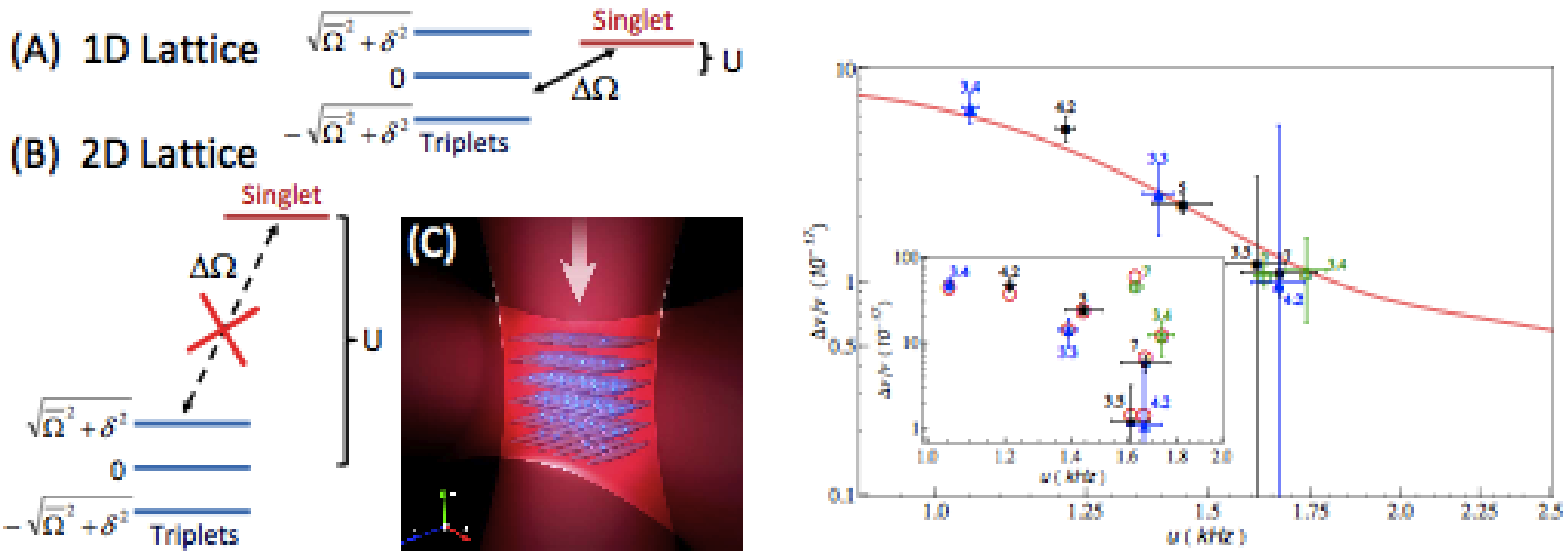
The magnitude of the shift $\sim T$




$$\delta = \Delta/\Omega_{xy}, \tau = \Omega_{xy} t$$

# Ongoing story

*Ye's group, Science 331, 1043 - 1046 (2011)*



Thank you