超冷原子系综的周期性量子相干调制和潜在应用

吴金辉　　吉林大学

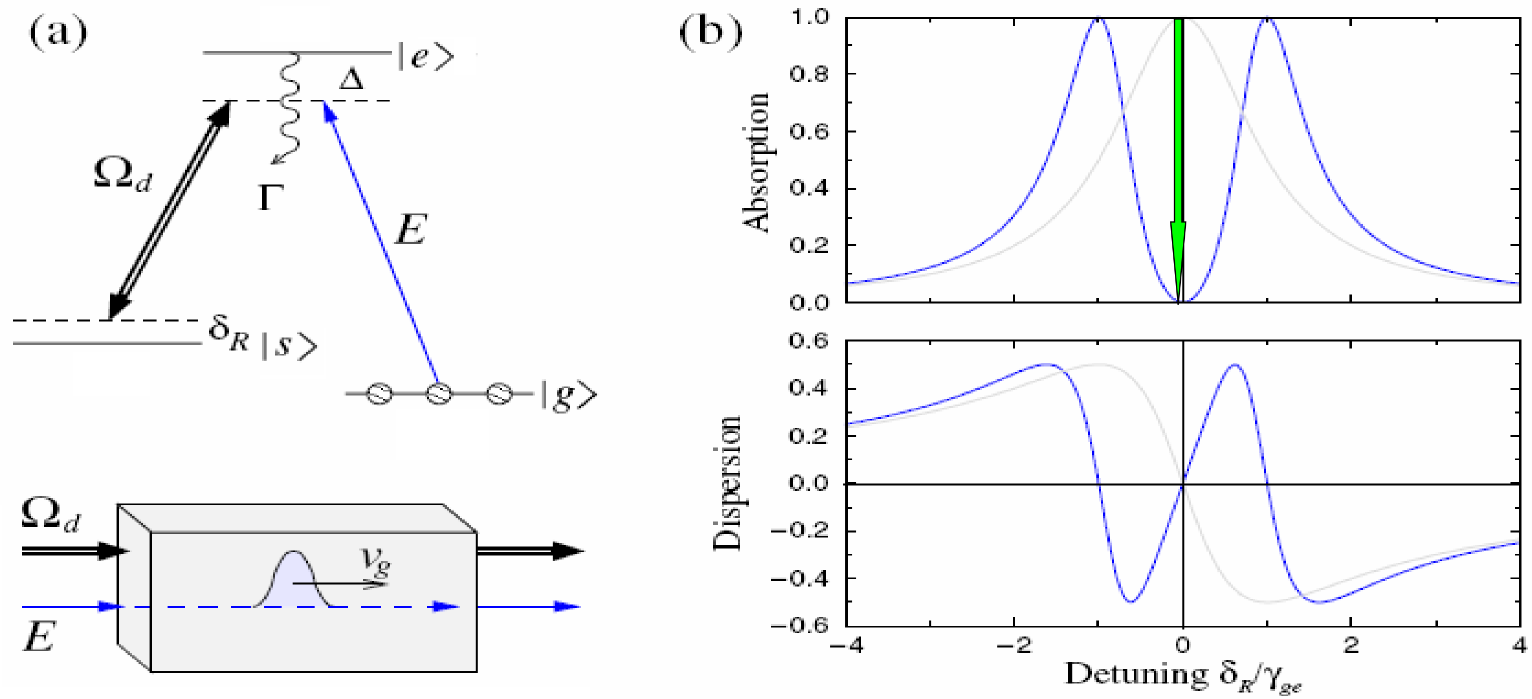
2012-08-18

第六届全国冷原子物理和量子信息青年学者学术讨论会　　浙江金华

Outlines: Homogeneous Cold Atoms

- I: Background
- II: All-Optically Induced Photonic Band-Gaps
- III: Dynamic Generation of Stationary Light Pulses
- IV: Conclusions

Electromagnetically Induced Transparency:

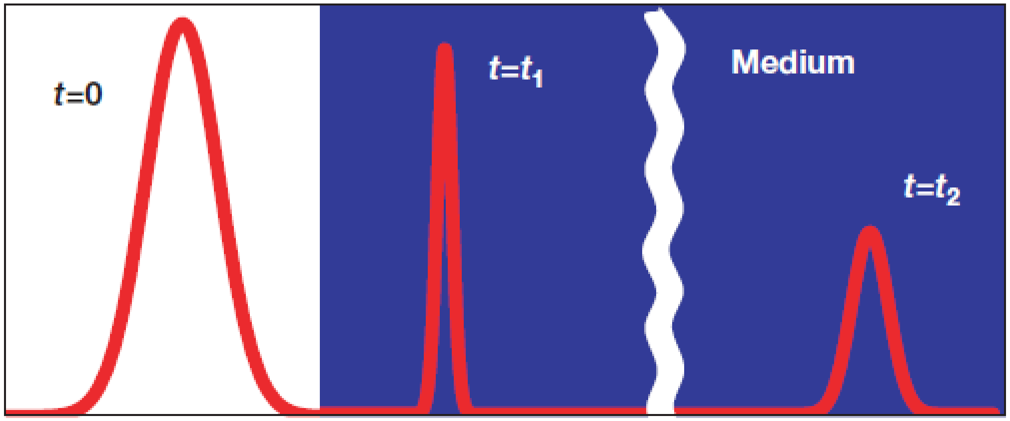


Susceptibility: $$\chi_p = \frac{N|d_{13}|^2}{2\varepsilon_0\hbar}\frac{\rho_{13}}{\Omega_p} = \frac{N|d_{13}|^2}{2\varepsilon_0\hbar}\frac{i(\gamma_{12}-i\Delta_p)}{(\gamma_{12}-i\Delta_p)(\gamma_{13}-i\Delta_p)+\Omega_c^2}$$

Group Velocity: $$\upsilon_g = \frac{c}{n_g(\omega_p)} = \frac{c}{n(\omega_p)+\omega_p\dfrac{dn(\omega_p)}{d\omega_p}}$$

$$n=\sqrt{1+\chi_p}$$

Slow Light and Spatial Compression:

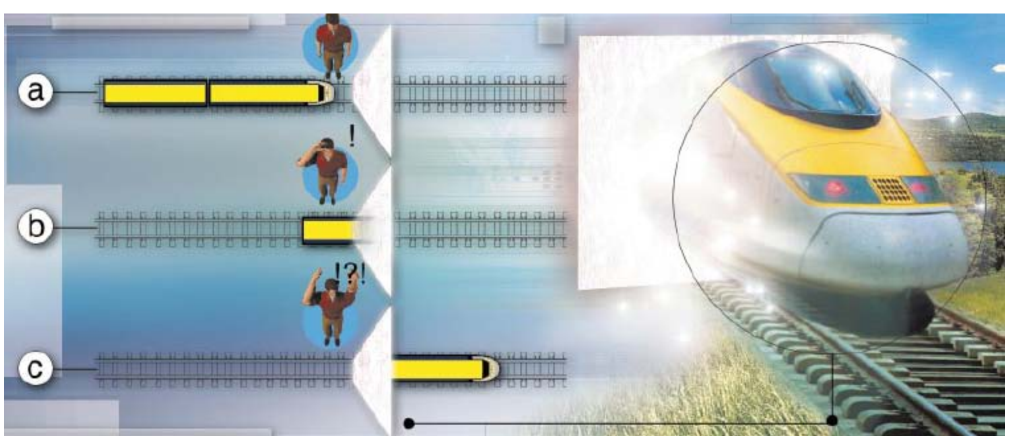


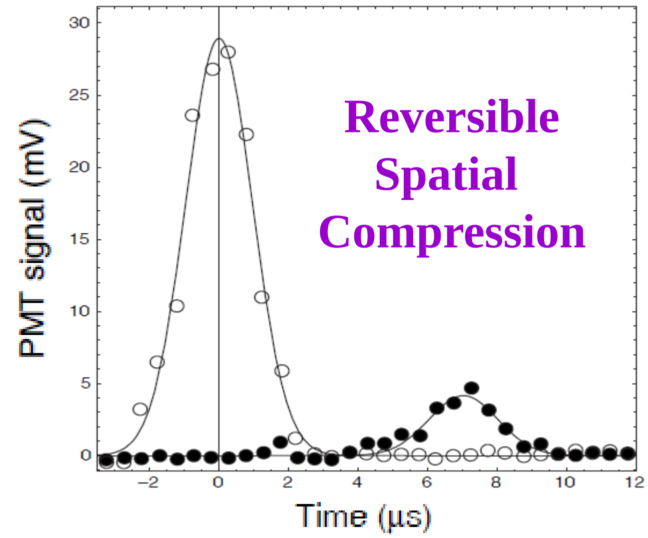
$$\frac{L_{in}}{L_{out}} = \frac{\upsilon_g}{c} \approx 10^{-8}$$

Photons

⇕

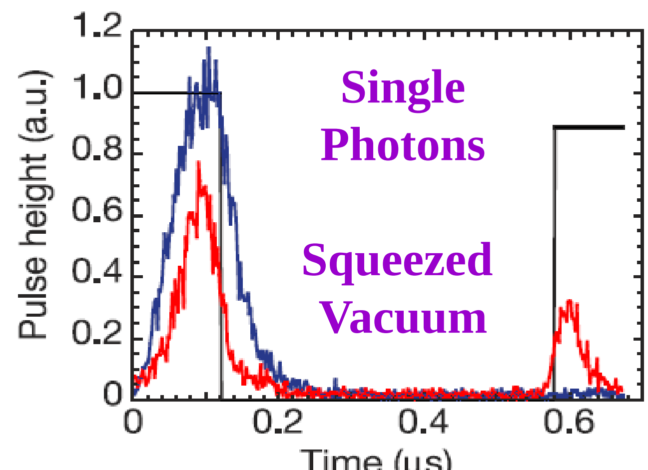
Spins





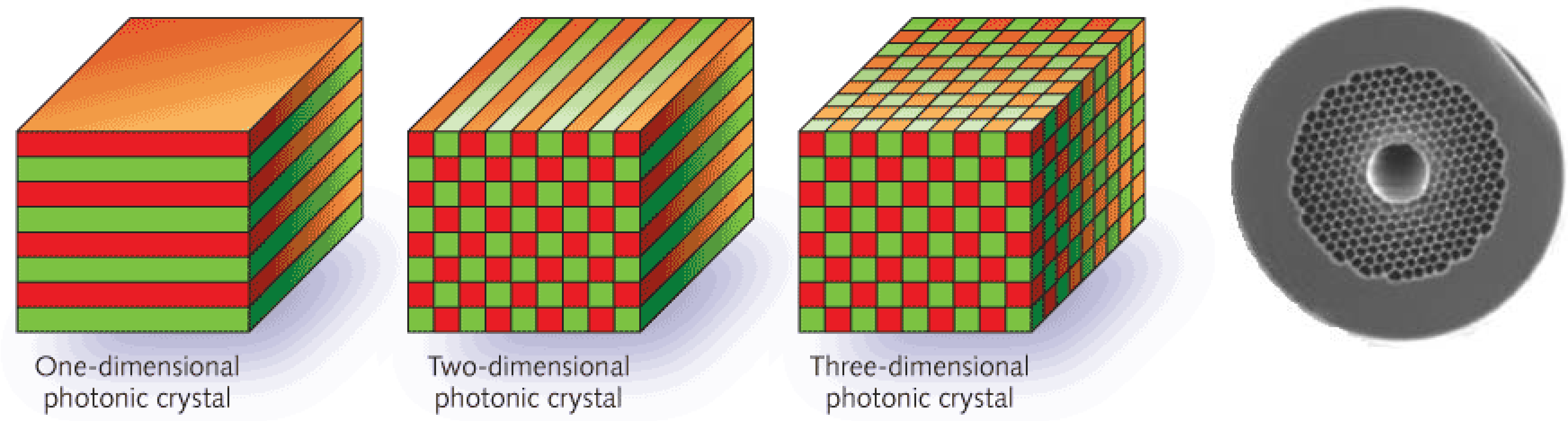
Nature 397, 594-598 (1999)

Nature 438, 833-836 (2005)

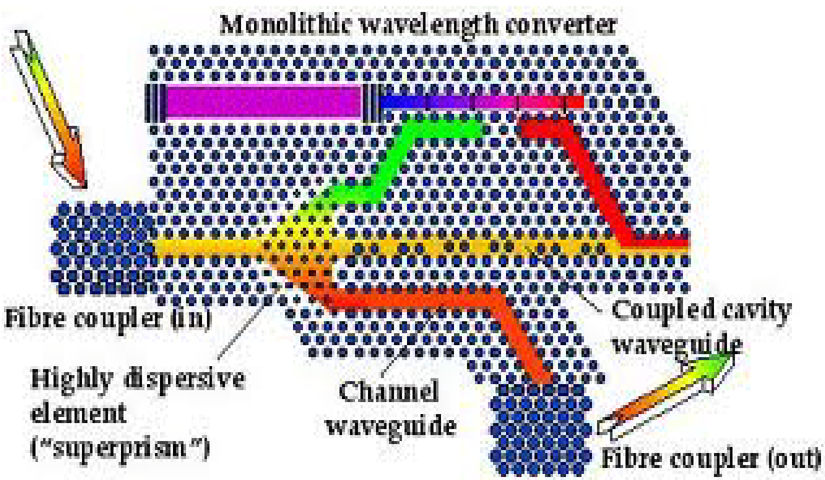


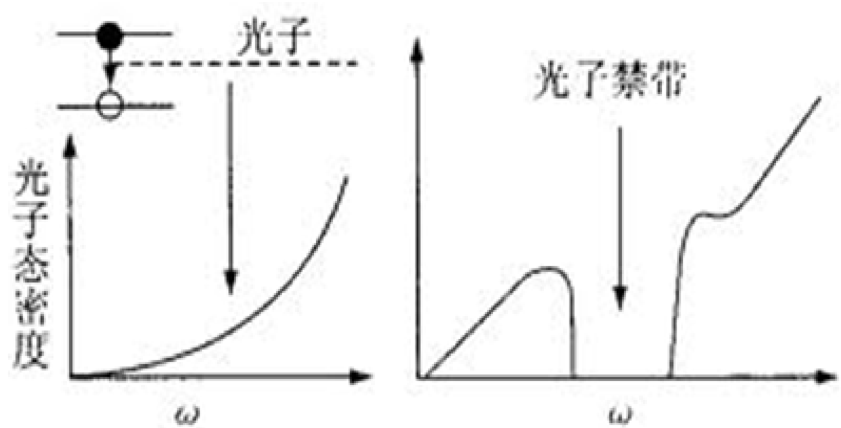
I. Background __ PBG

Photonic Crystals and Band-Gaps:



电子晶体：Nano-Meters in Period; 光子晶体：Micro-Meters in Period.





集成光路：波长转换、光纤耦合输入和输出、超级透镜、耦合腔波导！

I. Background __ PBG

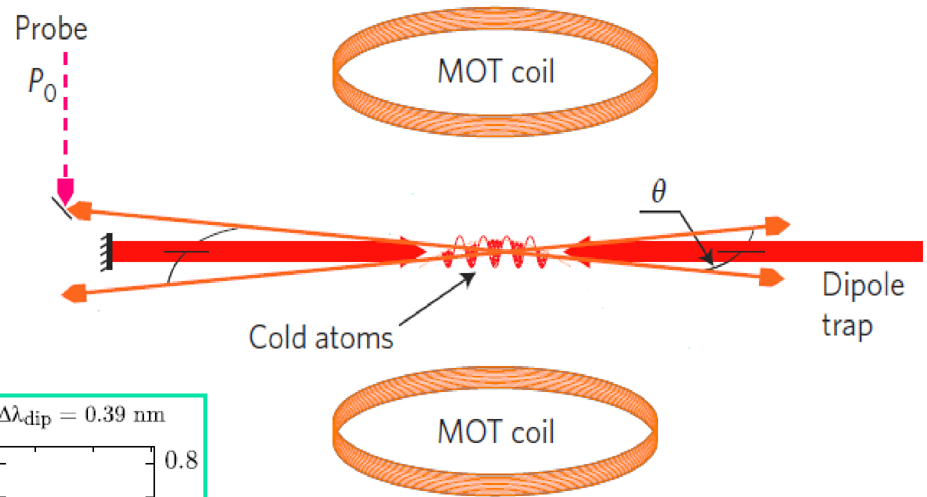
吉林大学

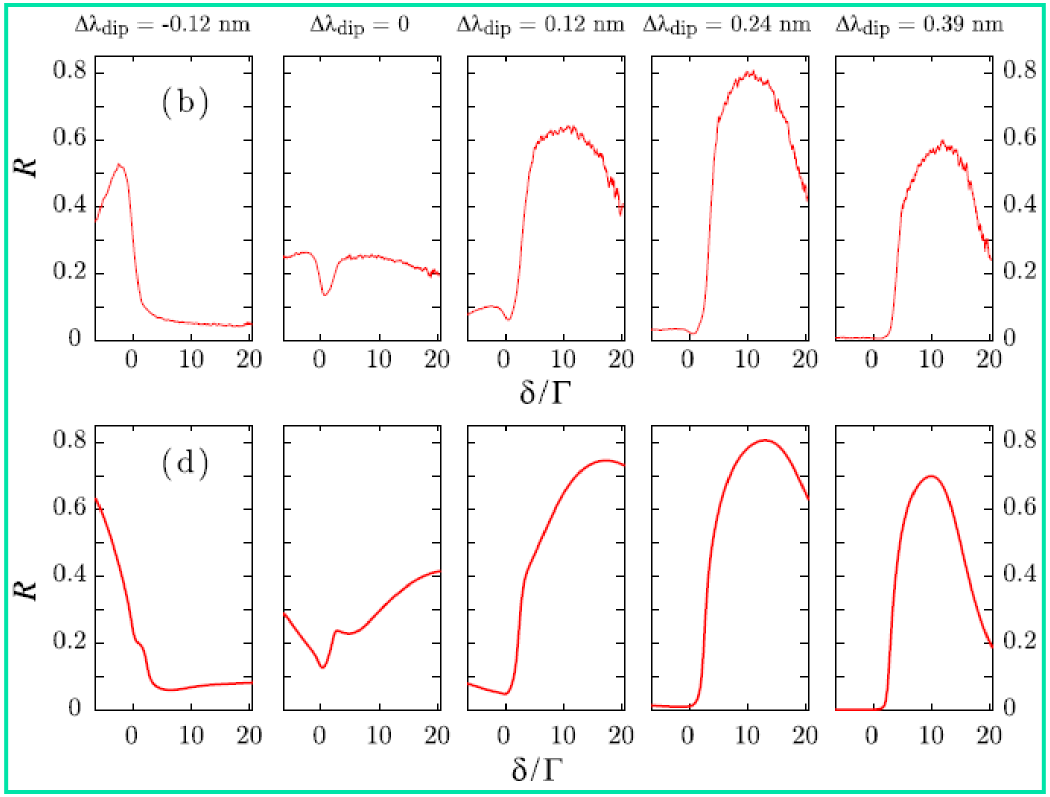
Photonic Band-Gaps in Atomic Lattice:

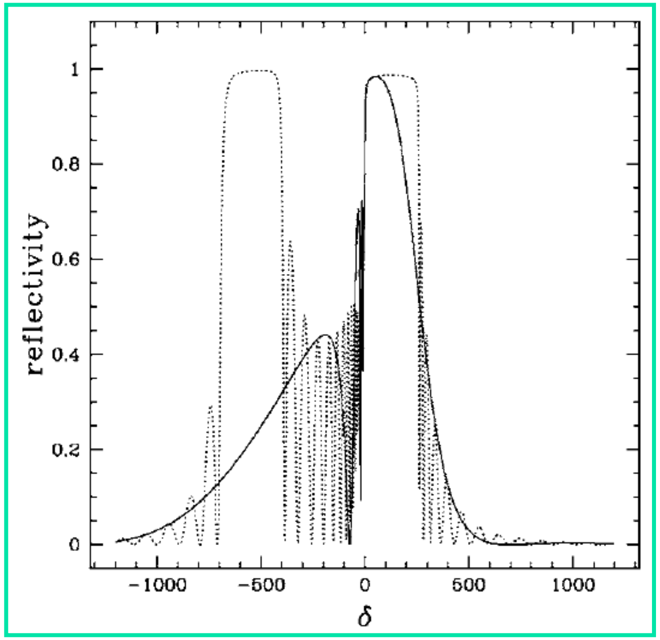
A Resonantly Absorbing Medium!

Phys. Rev. E 72, 046604 (2005)

Phys. Rev. Lett. 106, 223903 (2011)



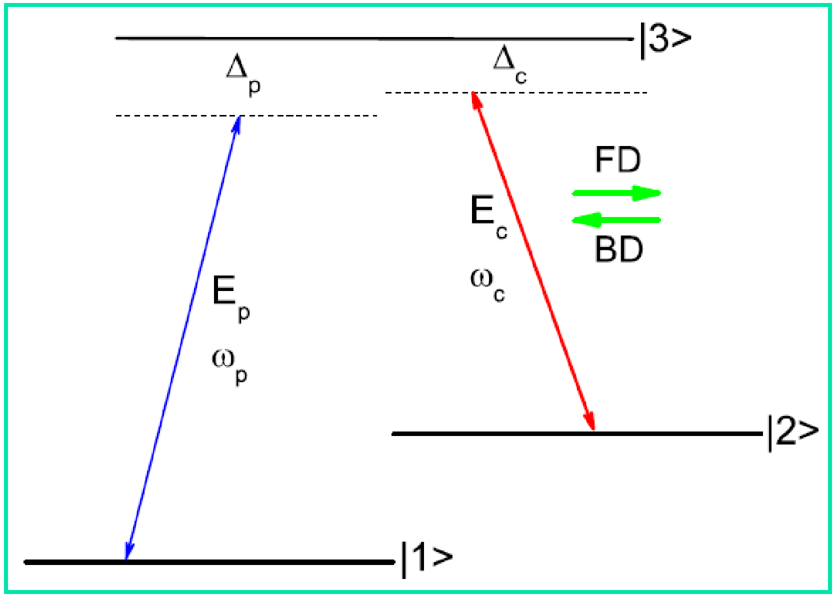


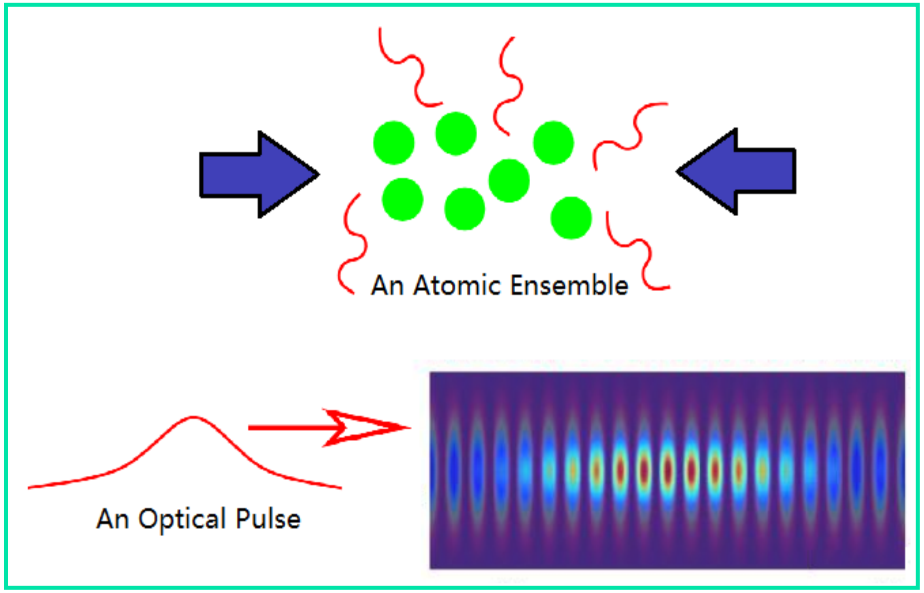


Outlines: Homogeneous Cold Atoms

- I: Background
- **II: All-Optically Induced Photonic Band-Gaps**
- III: Dynamic Generation of Stationary Light Pulses
- IV: Conclusions

II. All-Optically Induced Photonic Band-Gaps





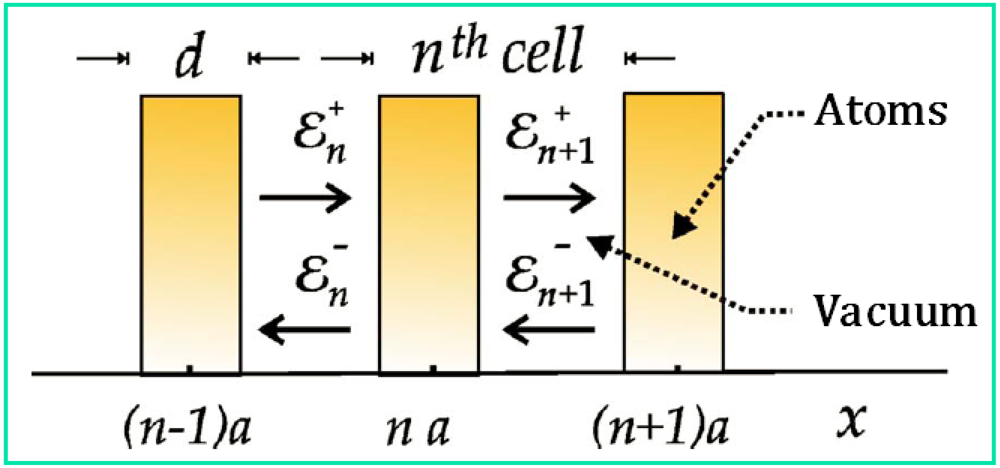
Refractive index is periodic in space! ::: Atomic density is homogeneous!

$$H = \hbar\omega_{21}|2\rangle\langle 2| + \hbar\omega_{31}|3\rangle\langle 3| - [\hbar\Omega_p e^{-i\Delta_p t}|3\rangle\langle 1| + \hbar\Omega_c(x)e^{-i\Delta_c t}|3\rangle\langle 2| + h.c.]$$

$$\chi_p = \frac{N_0 d_{13}^2}{2\varepsilon_0\hbar}\frac{\gamma_{12} - i(\Delta_p - \Delta_c)}{[\gamma_{12} - i(\Delta_p - \Delta_c)](\gamma_{13} - i\Delta_p) + \Omega_c^2(x)} \qquad n(x) = \sqrt{1 + \chi_p(x)}$$

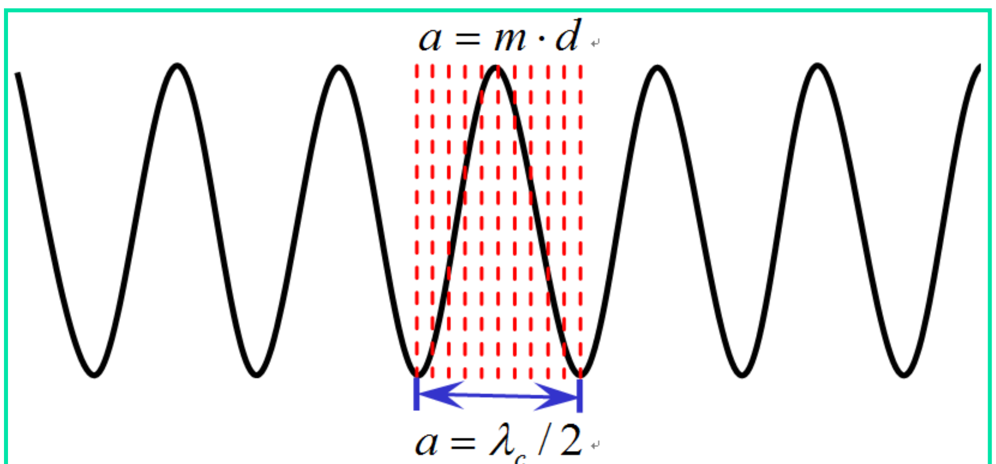
$$\Omega_c^2(x) = \Omega_0^2[(1+\sqrt{R_m})^2\cos^2(k_c x) + (1-\sqrt{R_m})^2\sin^2(k_c x)]$$

II. All-Optically Induced Photonic Band-Gaps



Consider first a simple periodic structure that alternate between a wide vacuum slab and a thin atom slab.

$$M=\frac{1}{4n}\begin{pmatrix} e^{+ik(a-d)} & 0 \\ 0 & e^{-ik(a-d)} \end{pmatrix}\begin{pmatrix} (n+1)^2 e^{+inkd}-(n-1)^2 e^{-inkd} & (n^2-1)^2 e^{+inkd}-(n^2-1)e^{-inkd} \\ (n^2-1)e^{-inkd}-(n^2-1)e^{+inkd} & (n+1)^2 e^{-inkd}-(n-1)^2 e^{+inkd} \end{pmatrix}$$



单个周期的传输矩阵：

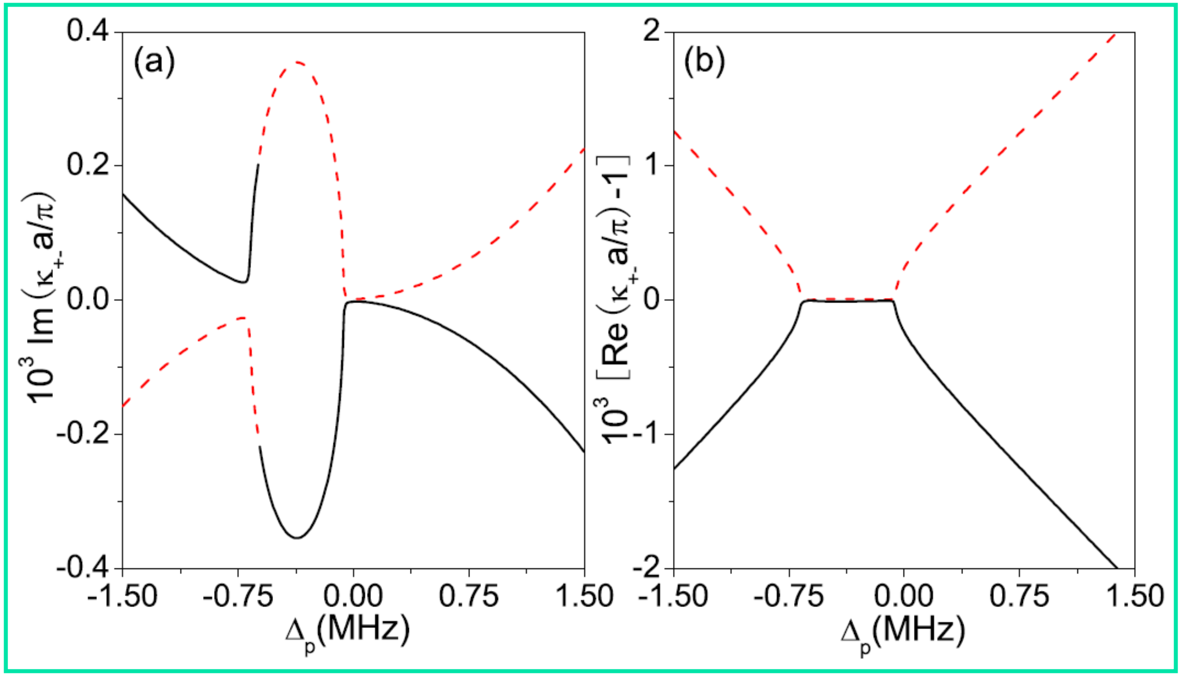
$$M=M_1(n_1)\cdot M_2(n_2)\cdot\ldots\ldots\cdot M_m(n_m)$$

整个样品的传输矩阵：

$$M^N=\frac{\sin N\kappa a}{\sin\kappa a}M-\frac{\sin(N-1)\kappa a}{\sin\kappa a}I$$

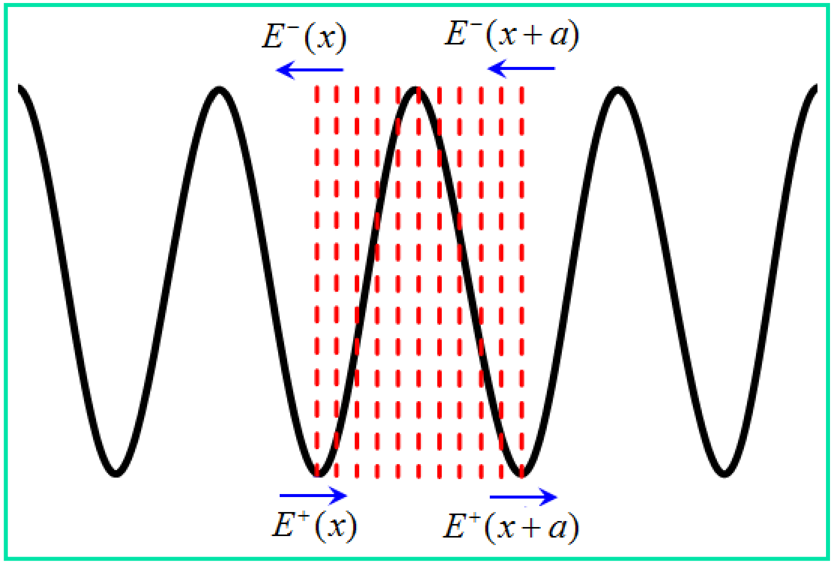
II. All-Optically Induced Photonic Band-Gaps

吉林大学



Imaginary and real parts of the Bloch wave-vector

Extinction and dispersion rates of the incident light



JOSAB 25, 1840-1849 (2008)

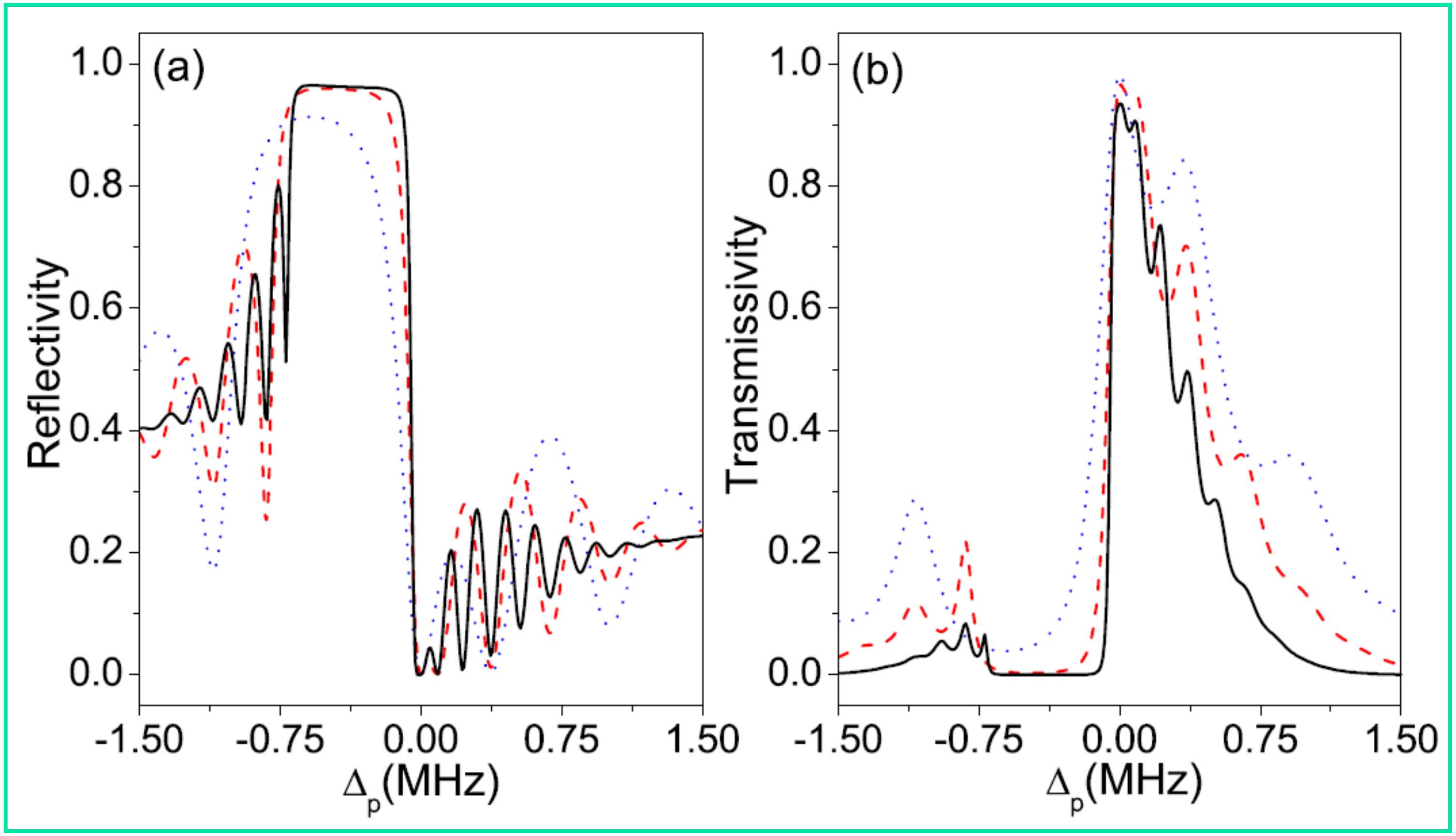
$$\begin{bmatrix} E^+(x+a) \\ E^-(x+a) \end{bmatrix} = M \begin{bmatrix} E^+(x) \\ E^-(x) \end{bmatrix} = \begin{bmatrix} e^{2i\kappa a}E^+(x) \\ e^{2i\kappa a}E^-(x) \end{bmatrix}$$

$$e^{2i\kappa a} - Tr(M)e^{i\kappa a} + 1 = 0$$

II. All-Optically Induced Photonic Band-Gaps

吉林大学

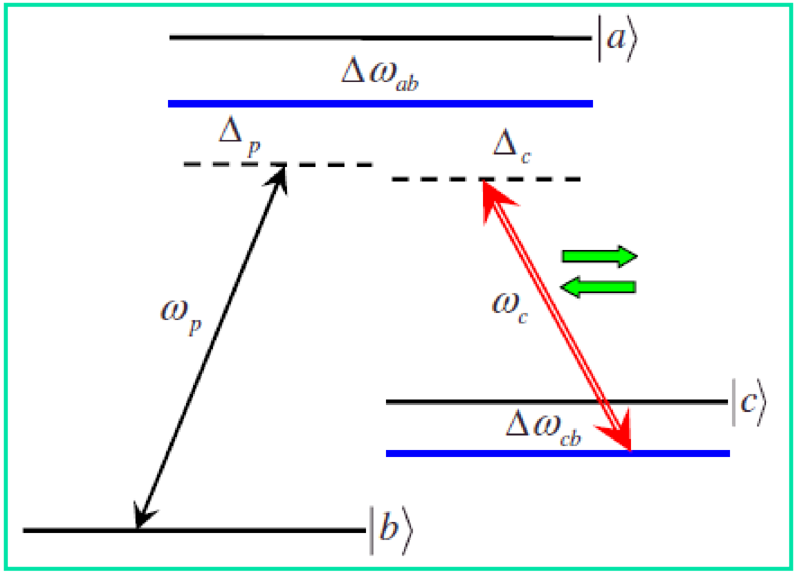
Probe reflectivity and transmissivity: one tunable PBG is generated within an EIT window whose width and depth change periodically in space!



$$r = \frac{M_{N(12)}}{M_{N(22)}} = \frac{M_{12}\sin(N\kappa a)}{M_{22}\sin(N\kappa a) - \sin[(N-1)\kappa a]} \qquad t = \frac{1}{M_{N(22)}} = \frac{\sin(\kappa a)}{M_{22}\sin(N\kappa a) - \sin[(N-1)\kappa a]}$$

II. All-Optically Induced Photonic Band-Gaps

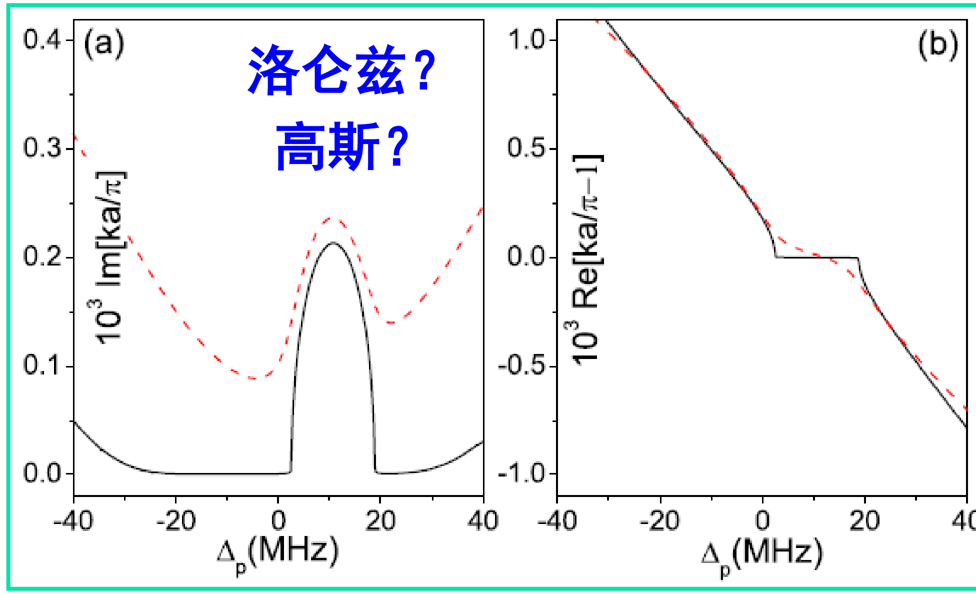
All-Optically Induced PBGs in Diamond with NV centers!

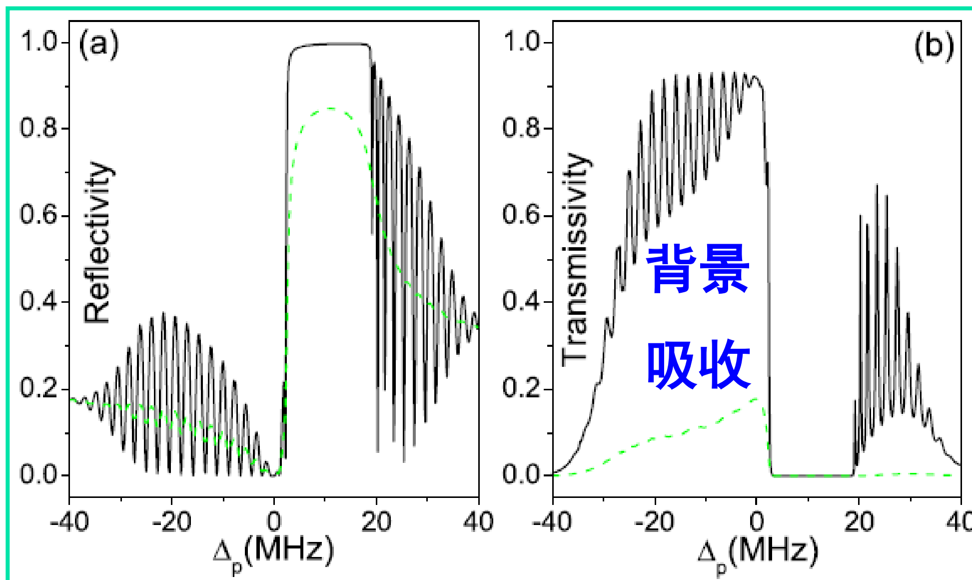


Phys. Rev. B 77, 113106 (2008)

$$\chi_p = \frac{1}{\pi W_{cb} W_{ab}} \int e^{-\Delta\omega_{cb}^2/W_{cb}^2} d(\Delta\omega_{cb})$$

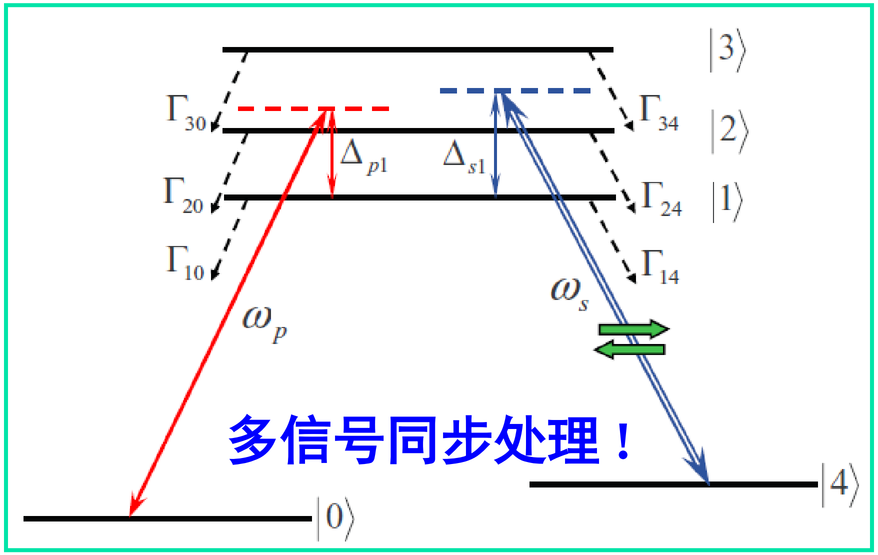
$$\times \int e^{-\Delta\omega_{ab}^2/W_{ab}^2} \frac{N_0 d_{ab}^2}{2\varepsilon_0 \hbar} \frac{\rho_{ab}}{\Omega_p} d(\Delta\omega_{ab})$$





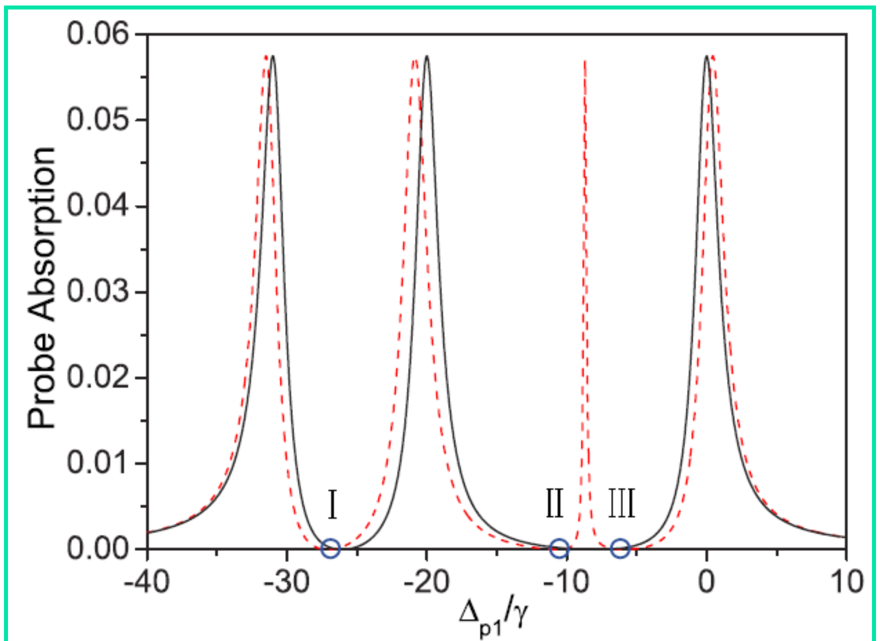
II. All-Optically Induced Photonic Band-Gaps

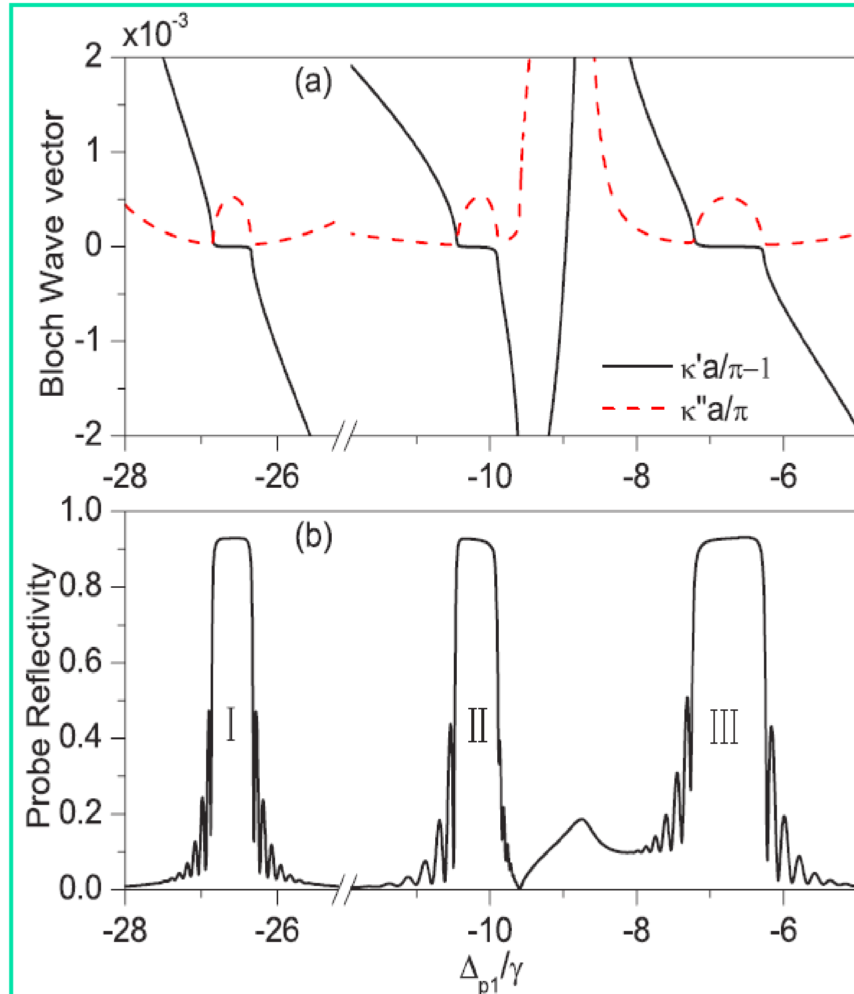
吉林大学



Opt. Lett. 35, 709-711 (2010)

Phys. Rev. A 83, 053815 (2011)

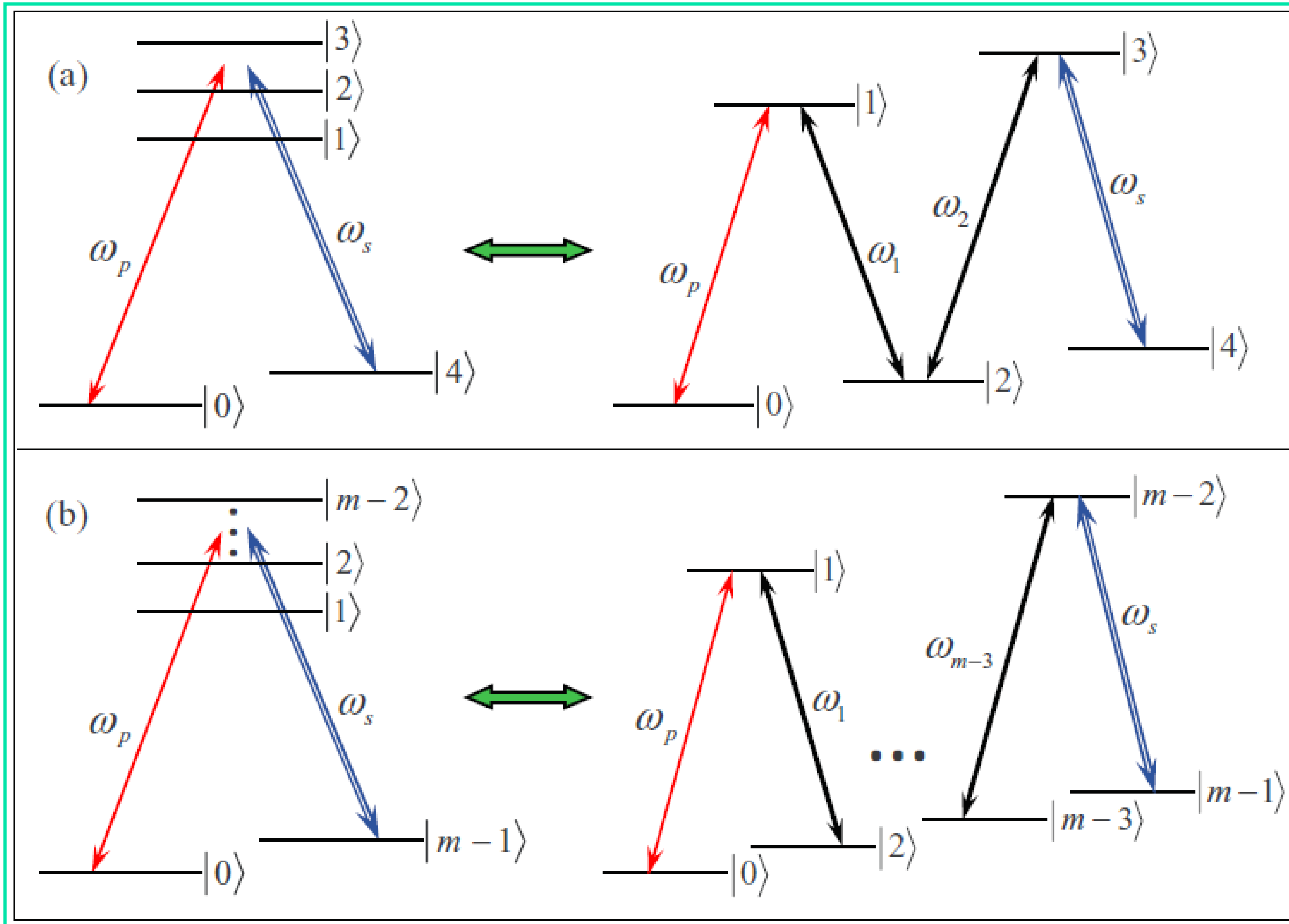




II. All-Optically Induced Photonic Band-Gaps

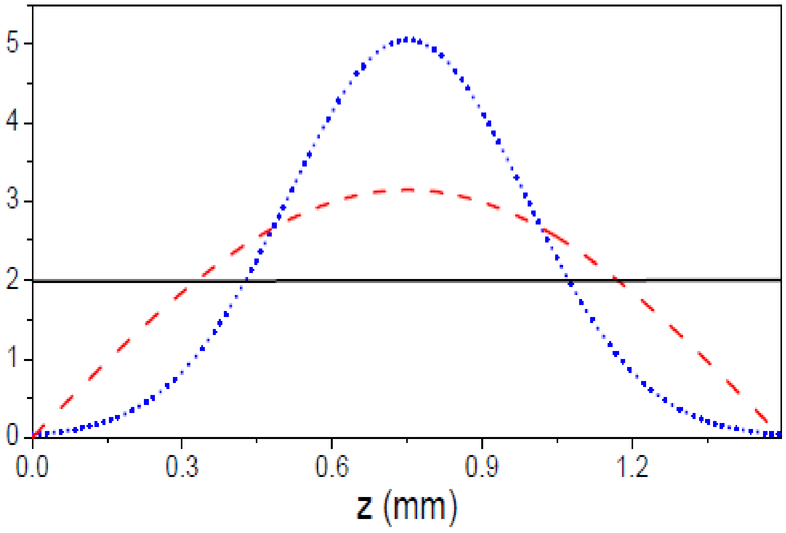
吉林大学

More PBGs may be optically induced around an absorption line in principle!



II. All-Optically Induced Photonic Band-Gaps

吉林大学

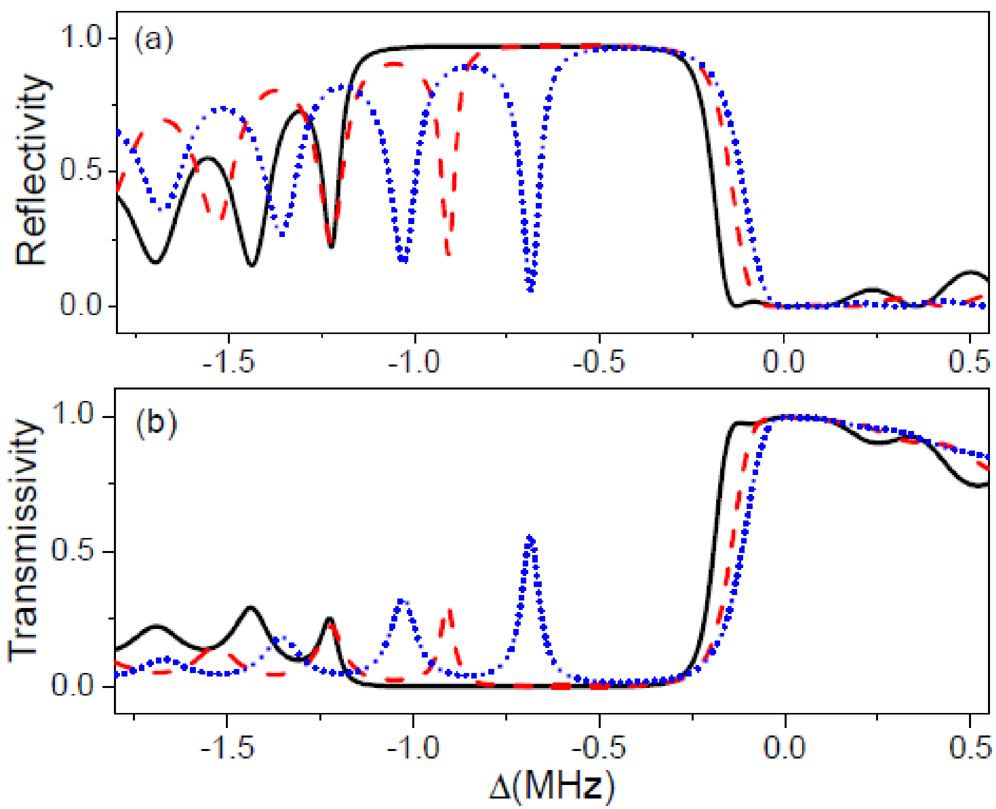


$$N_{a1}(z) = N_0$$

$$N_{a2}(z) = N' \sin(\pi z/L)$$

$$N_{a3}(z) = N'' \exp[-20(z-L/2)^2/L^2]$$

Opt. Express 19, 2111-2119 (2011)



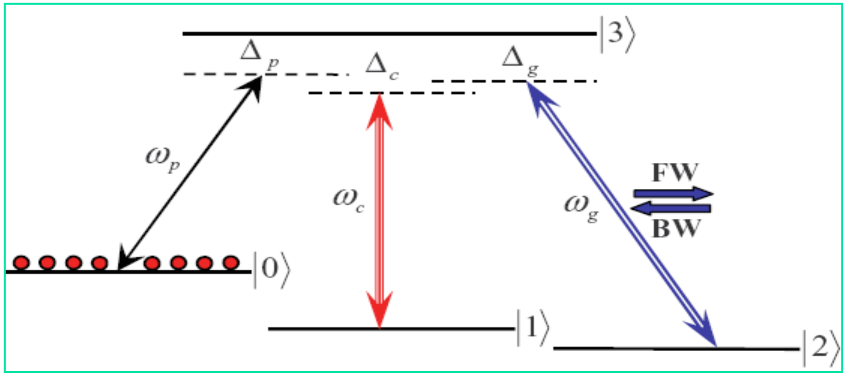
原子系综密度的非周期性不均匀性会对光子带隙的形成有什么影响？

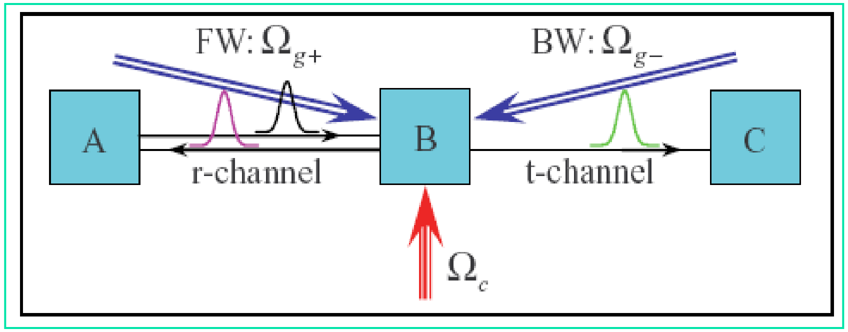
不存在 布洛赫波矢的恰当描述，但是对应的高反射频带依然存在！

II. All-Optically Induced Photonic Band-Gaps

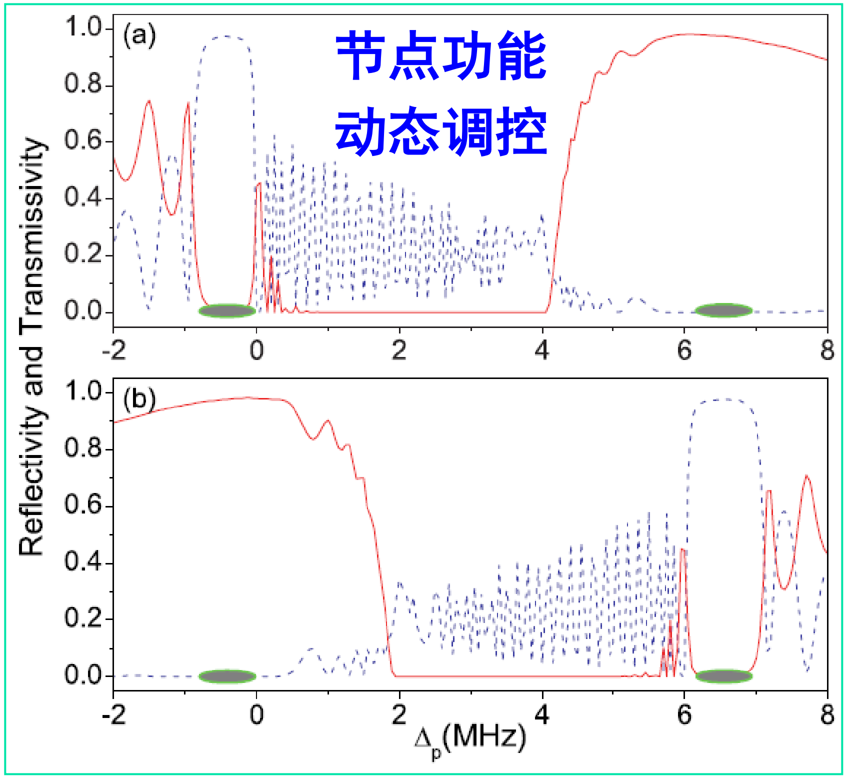
吉林大学

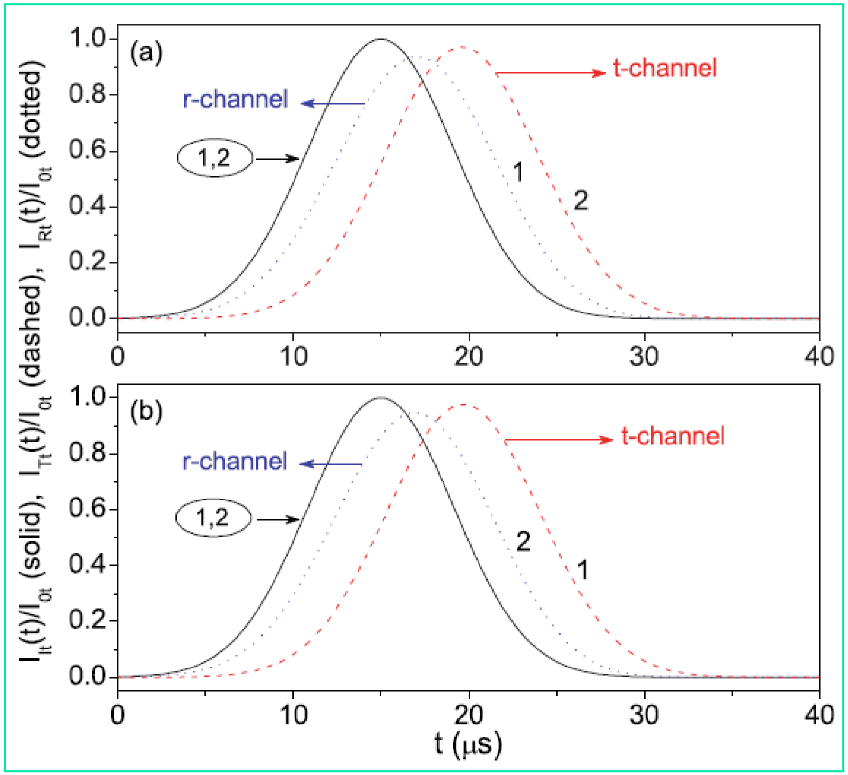
PBG + EIT: All-Optical Routing!





Phys. Rev. A 81, 013804 (2010)



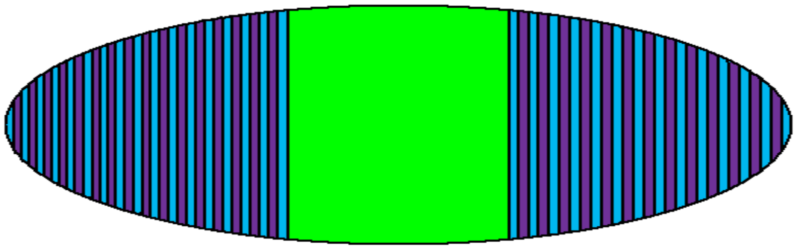


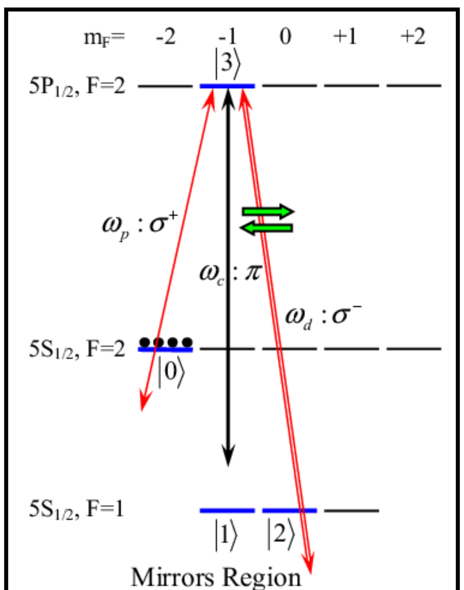
II. All-Optically Induced Photonic Band-Gaps

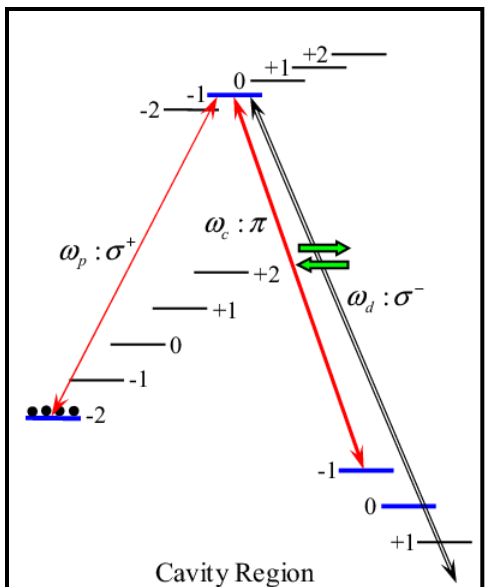
吉林大学

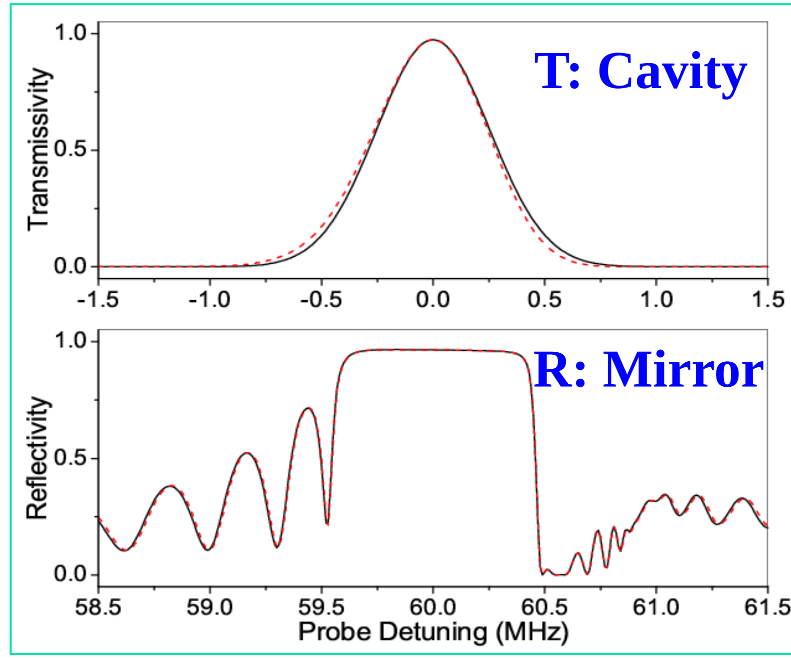
Phys. Rev. Lett. 103, 133601 (2009)

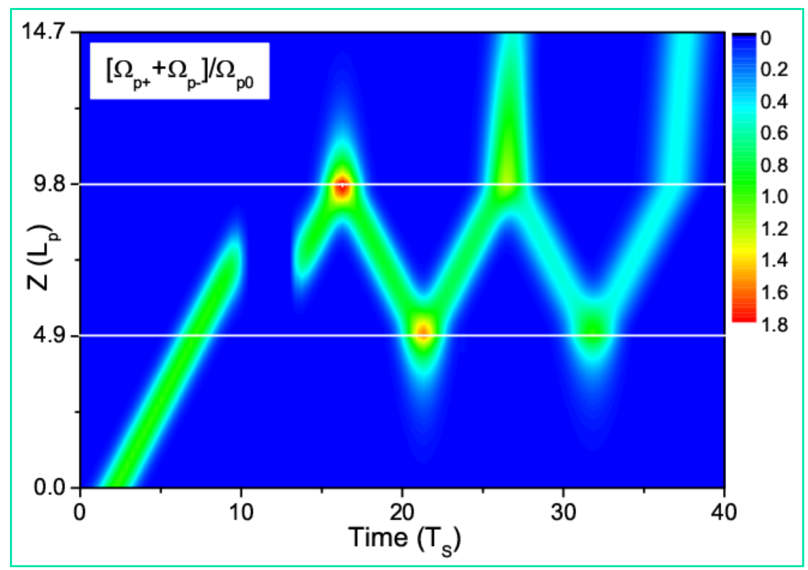
EIT + PBG + Zeeman: Dynamically Controlled Cavity in Cold Atoms, a new scheme for light storage!









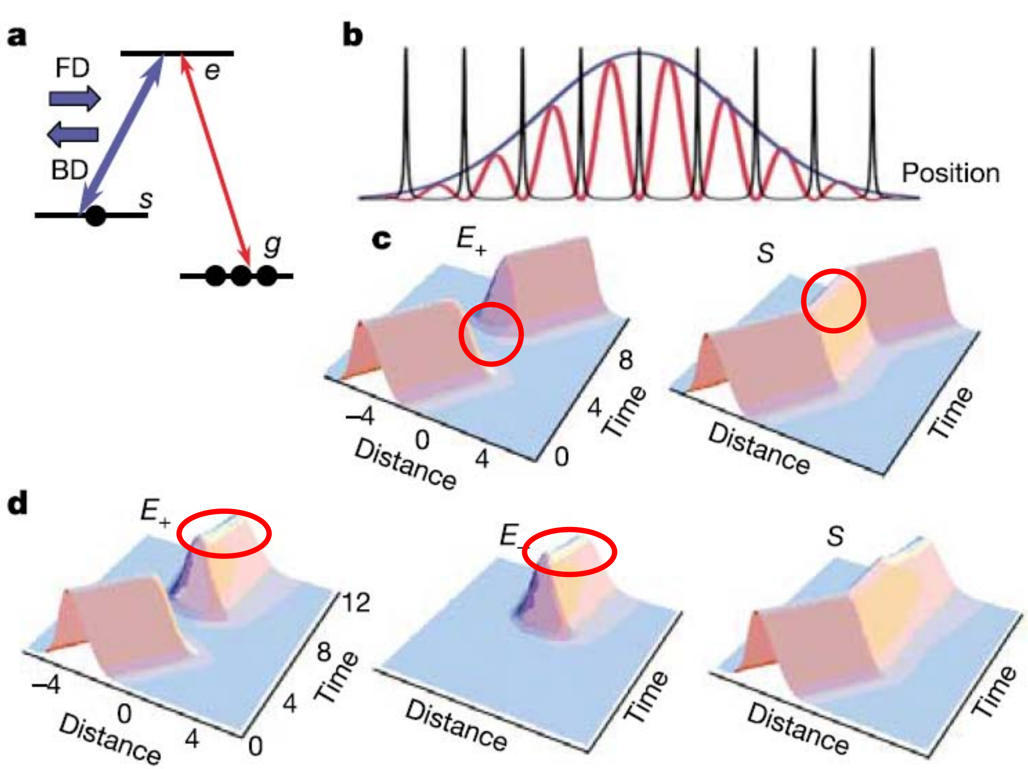


Outlines: Homogeneous Cold Atoms

- I: Background
- II: All-Optically Induced Photonic Band-Gaps
- III: Dynamic Generation of Stationary Light Pulses
- IV: Conclusions

III. Dynamic Generation of Stationary Light Pulses

Nature 426, 638-641 (2003)

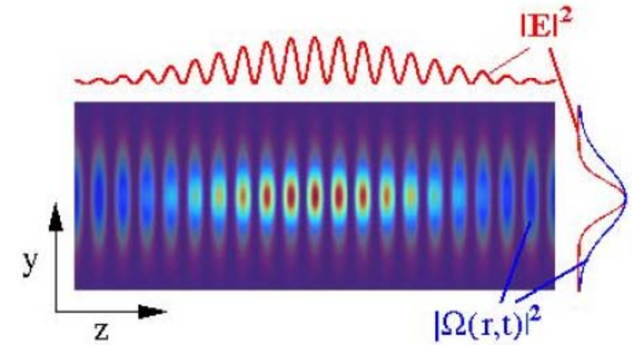


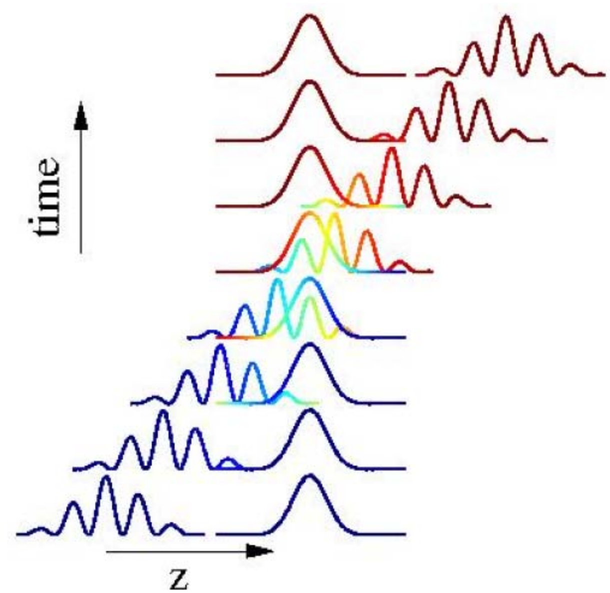
驻波光栅的反射作用导致了光脉冲的停滞？

$$\frac{\partial}{\partial z}\Psi_+ = -\alpha_-\xi(\Psi_+ - \Psi_-) - \frac{1}{c}\frac{\partial}{\partial \tau}(\alpha_+\Psi_+ + \alpha_-\Psi_-)$$

$$\frac{\partial}{\partial z}\Psi_- = -\alpha_+\xi(\Psi_+ - \Psi_-) + \frac{1}{c}\frac{\partial}{\partial \tau}(\alpha_+\Psi_+ + \alpha_-\Psi_-)$$

Phys. Rev. Lett. 94, 063902 (2005)

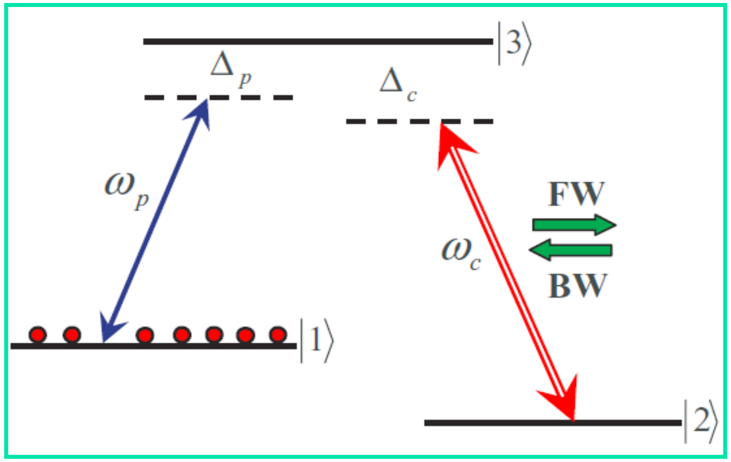




运动光和静止光之间的交叉相位调制，可用于实现通用量子相位门---光子量子逻辑运算！

III. Dynamic Generation of Stationary Light Pulses

吉林大学

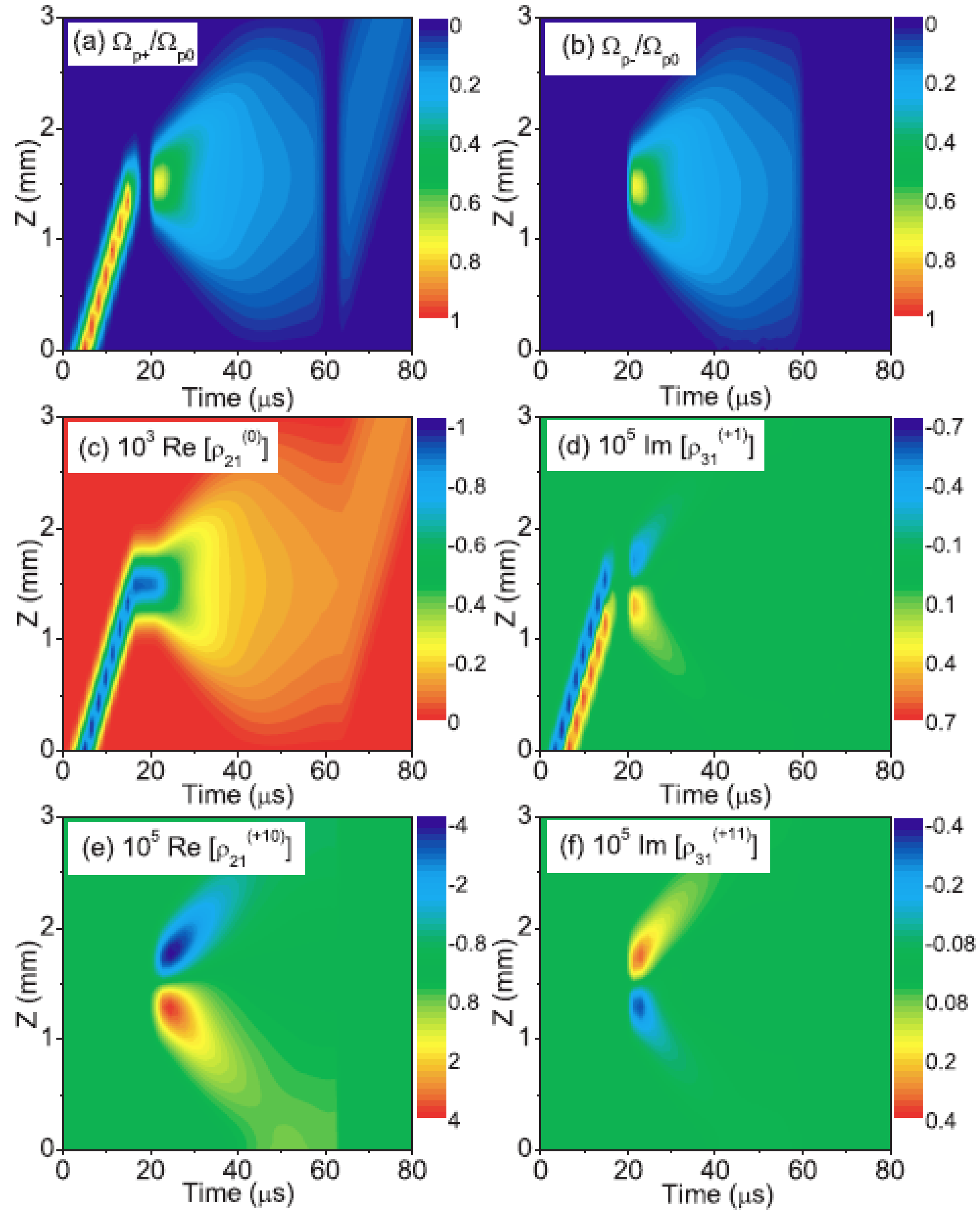


$$\rho_{21}^{(-4)} \quad \rho_{21}^{(-2)} \quad \rho_{21}^{(0)} \quad \rho_{21}^{(+2)} \quad \rho_{21}^{(+4)}$$

$$\rho_{21}^{(-5)} \quad \rho_{21}^{(-3)} \quad \rho_{21}^{(-1)} \quad \rho_{21}^{(+1)} \quad \rho_{21}^{(+3)} \quad \rho_{21}^{(+5)}$$

Phys. Rev. A 81, 033822 (2010)

Stationary light pulses in cold atoms experience very fast decay and diffusion for the truncated expansion at |n|=30 !



III. Dynamic Generation of Stationary Light Pulses

吉林大学

$$\partial_t \rho_{21} = -[\gamma_{21} - i(\Delta_p - \Delta_c)]\rho_{21} + i\Omega_c^* \rho_{31}$$

$$\partial_t \rho_{31} = -[\gamma_{31} - i\Delta_p]\rho_{31} + i\Omega_c \rho_{21} + i\Omega_p$$

$$\Omega_c(z,t) = [\Omega_{c+}(t)e^{+ik_c z} + \Omega_{c-}(t)e^{-ik_c z}]$$

$$\Omega_p(z,t) = [\Omega_{p+}(z,t)e^{+ik_c z} + \Omega_{p-}(z,t)e^{-ik_c z}]$$

密度矩阵元空间傅立叶展开

$$\rho_{21}(z,t) = \sum_{n=-\infty}^{+\infty} \rho_{21}^{(2n)}(z,t)e^{+i2nk_c z}$$

$$\rho_{31}(z,t) = \sum_{n=0}^{-\infty} \rho_{31}^{(2n-1)}(z,t)e^{+i(2n-1)k_c z} + \sum_{n=0}^{+\infty} \rho_{31}^{(2n+1)}(z,t)e^{+i(2n+1)k_c z}$$

数值计算基本方程

$$\partial_t \rho_{21}^{(2n)} = -\gamma_{21}\rho_{21}^{(2n)} + i\Omega_{c+}^* \rho_{31}^{(2n+1)} + i\Omega_{c-}^* \rho_{31}^{(2n-1)}$$

$$\partial_t \rho_{31}^{(2n\pm1)} = -\gamma_{31}\rho_{31}^{(2n\pm1)} + i\Omega_{c+}\rho_{21}^{(2n-1\pm1)} + i\Omega_{c-}\rho_{21}^{(2n+1\pm1)} + i\Omega_{p\pm}\delta_{n,0}$$

$+$

$$\partial_z \Omega_{p+} = -\partial_t \Omega_{p+}/c + i\Delta k\Omega_{p+} + i\gamma_{31}\alpha\rho_{31}^{(+1)}/2$$

$$\partial_z \Omega_{p-} = +\partial_t \Omega_{p-}/c - i\Delta k\Omega_{p-} - i\gamma_{31}\alpha\rho_{31}^{(-1)}/2$$

III. Dynamic Generation of Stationary Light Pulses

Successive generation of higher-order components of spin and optical coherence!

$$\rho_{21}^{(0)}$$

$$\underleftrightarrow{\Omega_{c+}} \cdot \frac{\rho_{31}^{(+1)}}{+k_c}$$

$$\underrightarrow{Maxwell} \cdot \frac{\Omega_{p+}}{+k_c} \cdot \underrightarrow{Liouville} \cdot \frac{\rho_{31}^{(+1)}}{+k_c}$$

$$\underleftrightarrow{\Omega_{c-}^{*}} \cdot \frac{\rho_{21}^{(+2)}}{+2k_c} \cdot \underleftrightarrow{\Omega_{c+}} \cdot \frac{\rho_{31}^{(+3)}}{+3k_c} \cdot \underleftrightarrow{\Omega_{c-}^{*}} \cdot \frac{\rho_{21}^{(+4)}}{+4k_c} \cdot \underleftrightarrow{\Omega_{c+}} \cdot \frac{\rho_{31}^{(+5)}}{+5k_c} \cdots\cdots$$

Unwanted alternative absorption and emission of C+ and C-!

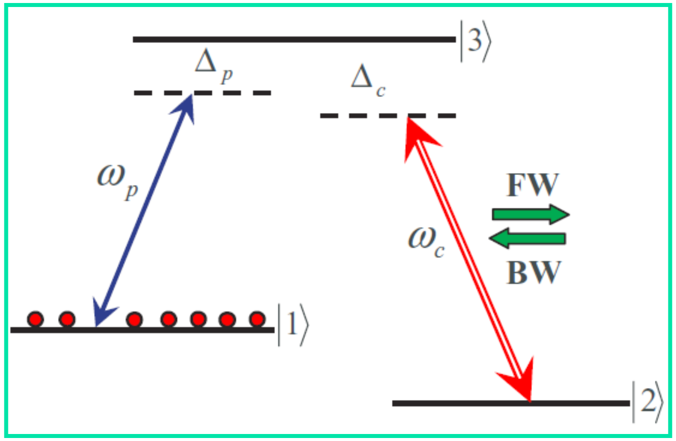
$$\underleftrightarrow{\Omega_{c+}^{*}} \cdot \frac{\rho_{31}^{(-2)}}{-2k_c} \cdot \underleftrightarrow{\Omega_{c-}} \cdot \frac{\rho_{31}^{(+3)}}{-3k_c} \cdot \underleftrightarrow{\Omega_{c+}^{*}} \cdot \frac{\rho_{21}^{(-4)}}{-4k_c} \cdot \underleftrightarrow{\Omega_{c-}} \cdot \frac{\rho_{31}^{(-5)}}{-5k_c} \cdots\cdots$$

$$\underleftrightarrow{\Omega_{c-}} \cdot \frac{\rho_{31}^{(-1)}}{-k_c}$$

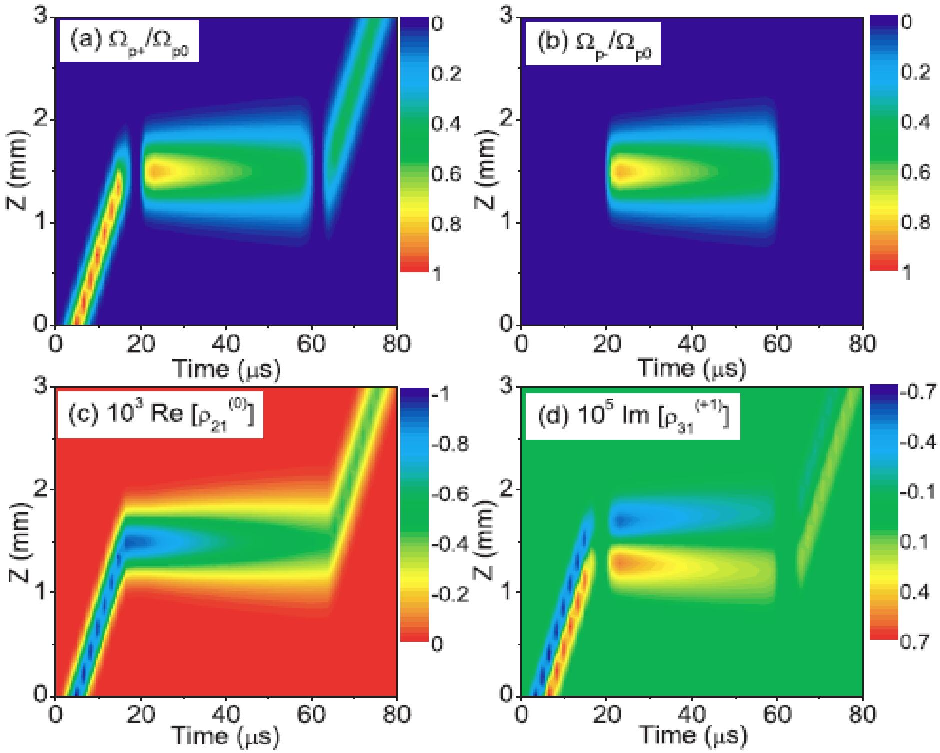
$$\underrightarrow{Maxwell} \cdot \frac{\Omega_{p-}}{-k_c} \cdot \underrightarrow{Liouville} \cdot \frac{\rho_{31}^{(-1)}}{-k_c}$$

$|n|=0$

$|n|=1$



III. Dynamic Generation of Stationary Light Pulses

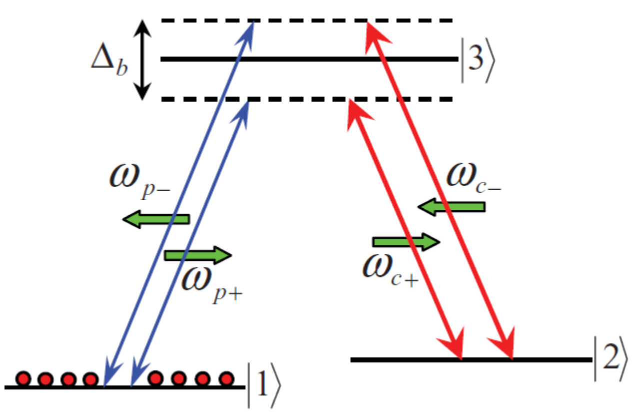


$$\rho_{21}(z,t) = \sum_{n=-\infty}^{+\infty} \rho_{21}^{(2n)}(z,t) e^{+i2nk_c z}$$

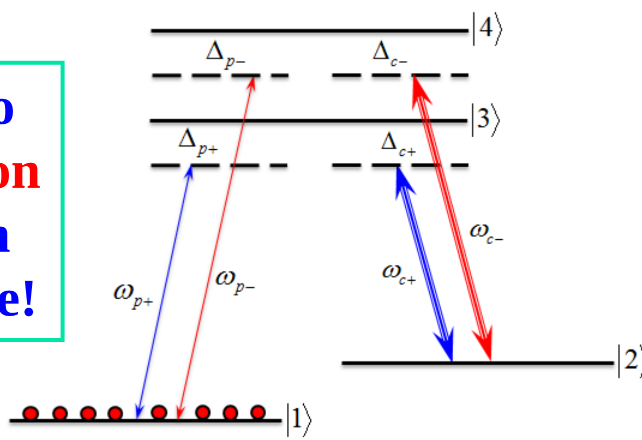
$$\rho_{31}(z,t) = \sum_{n=0}^{-\infty} \rho_{31}^{(2n-1)}(z,t) e^{+i(2n-1)k_c z} + \sum_{n=0}^{+\infty} \rho_{31}^{(2n+1)}(z,t) e^{+i(2n+1)k_c z}$$

?

$$\rho_{31}^{(-1)}(z,t) e^{-ik_c z} \quad \rho_{21}^{(0)}(z,t) \quad \rho_{31}^{(+1)}(z,t) e^{+ik_c z}$$

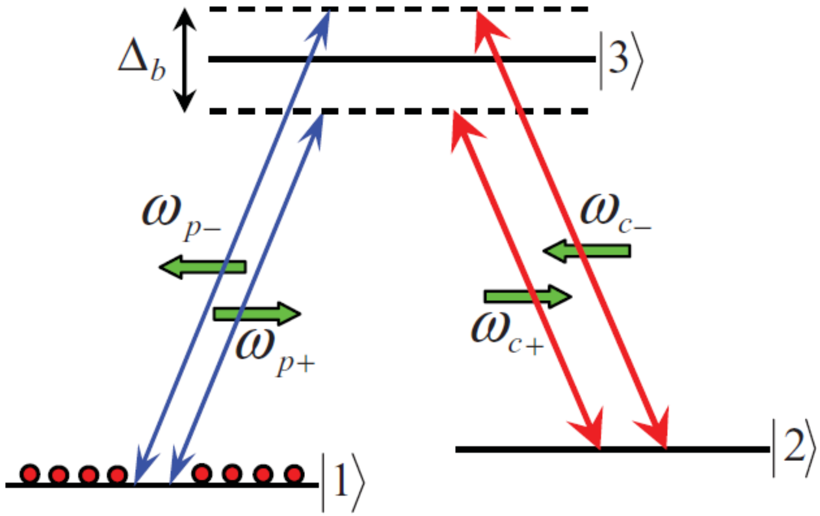


Two feasible ways to suppress the excitation of higher-order spin and optical coherence!



III. Dynamic Generation of Stationary Light Pulses

吉林大学



Phys. Rev. A 82, 013807 (2010)

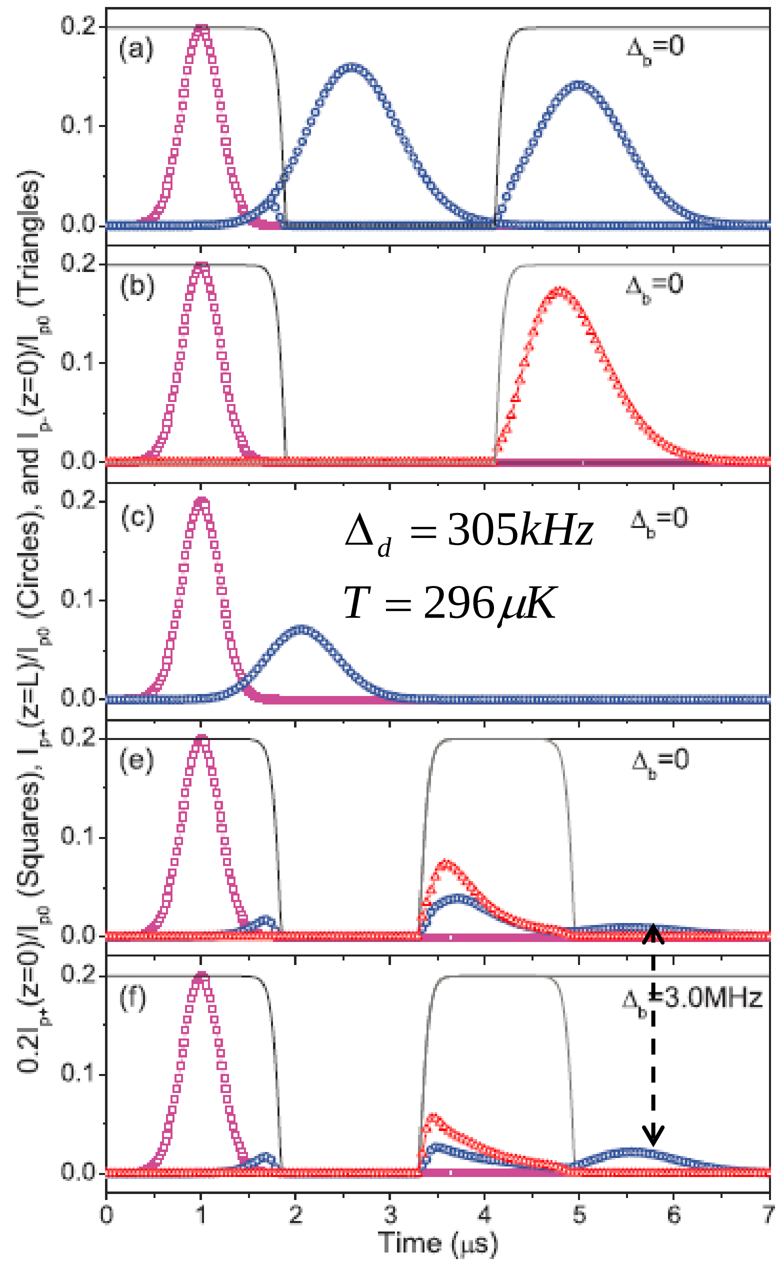
Phys. Rev. Lett. 102, 213601 (2009)

$$\rho_{31}^{(\pm1)}(z,t)=\int_{-\infty}^{+\infty}\rho_{31}^{(\pm1)}(z,t,\upsilon)N(\upsilon)d\upsilon$$

$$\gamma_{31}^{(2n+1)}=\gamma_{31}-i\Delta_{p+}+\underline{in\Delta_b}+\underline{i\upsilon(n+1)k_{c+}+i\upsilon nk_{c-}}$$

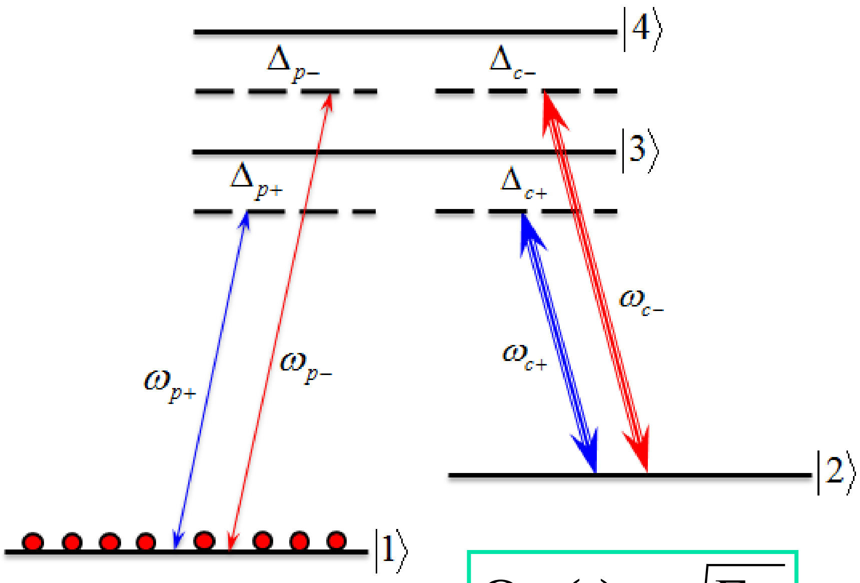
$$\gamma_{31}^{(2n-1)}=\gamma_{31}-i\Delta_{p-}+\underline{in\Delta_b}+\underline{i\upsilon nk_{c+}+i\upsilon(n-1)k_{c-}}$$

$$\gamma_{21}^{(2n)}=\gamma_{21}-i(\Delta_{p+}-\Delta_{c+})+\underline{in\Delta_b}+\underline{i\upsilon n(k_{c+}+k_{c-})}$$



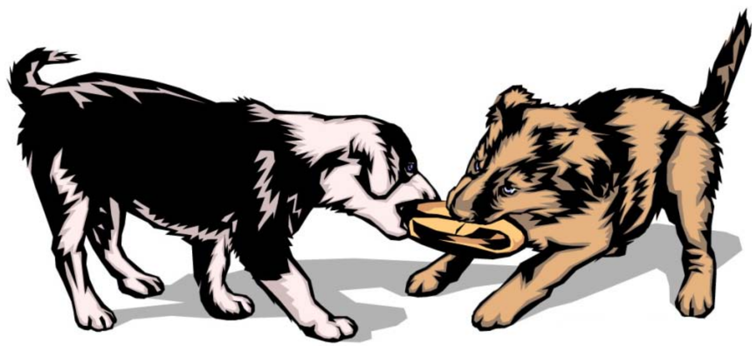
III. Dynamic Generation of Stationary Light Pulses

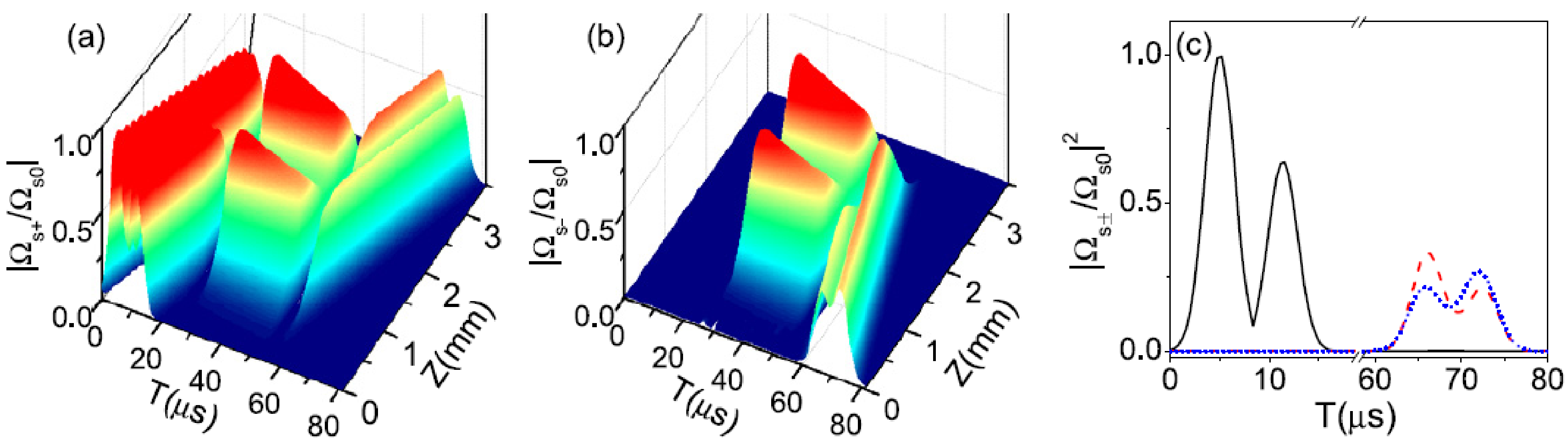
吉林大学



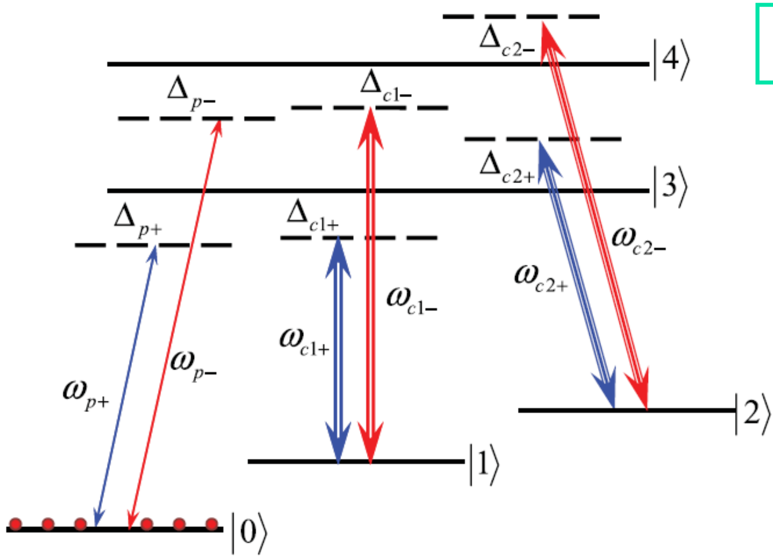
The balanced competition between the FW probe and the BW probe (sharing a common spin coherence) in a resonant four-wave mixing is critical for the generation of SLPs!

$$\frac{\Omega_{c+}(t)}{\Omega_{c-}(t)} = \sqrt{\frac{\Gamma_{31}}{\Gamma_{41}}}$$



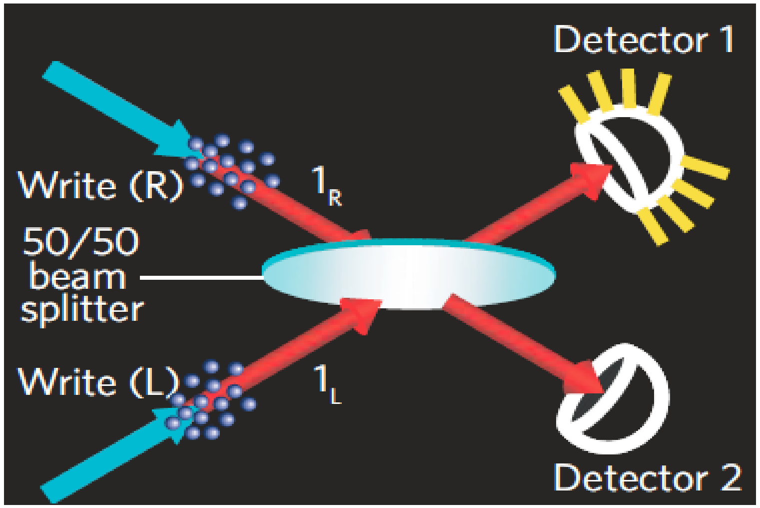


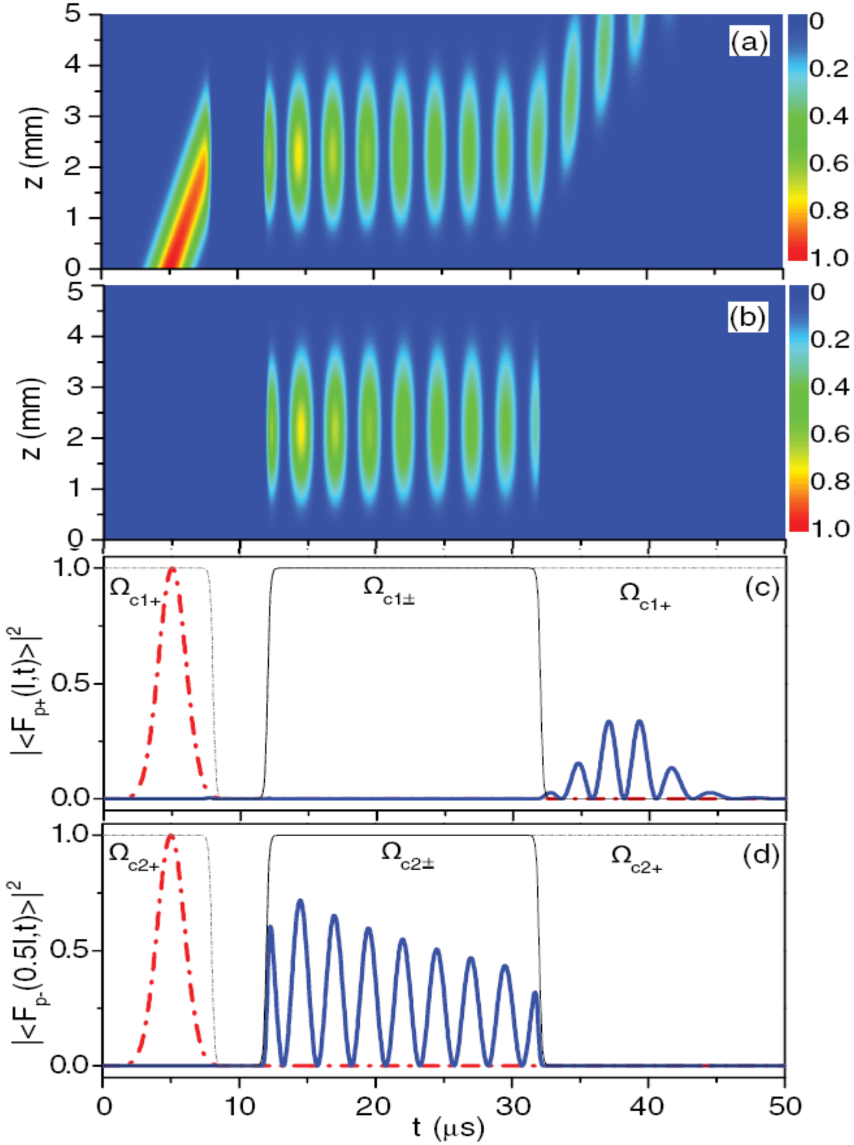
III. Dynamic Generation of Stationary Light Pulses



Optical Routing, Beam Splitter, Beat Generator!

Phys. Rev. A 84, 063812 (2011)





Outlines: Homogeneous Cold Atoms

- I: Background
- II: All-Optically Induced Photonic Band-Gaps
- III: Dynamic Generation of Stationary Light Pulses
- **IV: Conclusions**

IV. Conclusions

- It is possible to induce one or more tunable PBGs in a homogeneous sample of cold atoms by applying standing-wave laser fields!

- The electromagnetically induced PBGs have many applications, e.g. to achieve optical routing and devise dynamic cavities in cold atoms!

- SLPs generated in cold atoms experience very fast decay and diffusion when a standing-wave laser field is applied on one transition!

- The fast decay and diffusion of SLPs can be sufficiently suppressed in a modified coupling scheme: one key of SLPs is the balanced competition in a FWM process rather than the high reflectance of a grating!

Outlines: Ordered Cold Atoms

- I: Background
- II: Optical Enhancement of Radiation Damping
- III: Nonlinear Lasing via Distributed Feedback
- IV: Conclusions

I. Background __ Optomechanics

Physics

Physics **2**, 40 (2009)

Trends

Optomechanics

光力学系统=相干光学系统+微纳机械振子

超灵敏力学传感　　　量子信息处理

Florian Marquardt
Center for Theoretical Physics, Arnold Sommerfeld Center for NanoScience, and Department of Physics, Ludwig-Maximilians-Universität, D-80333 München, Germany

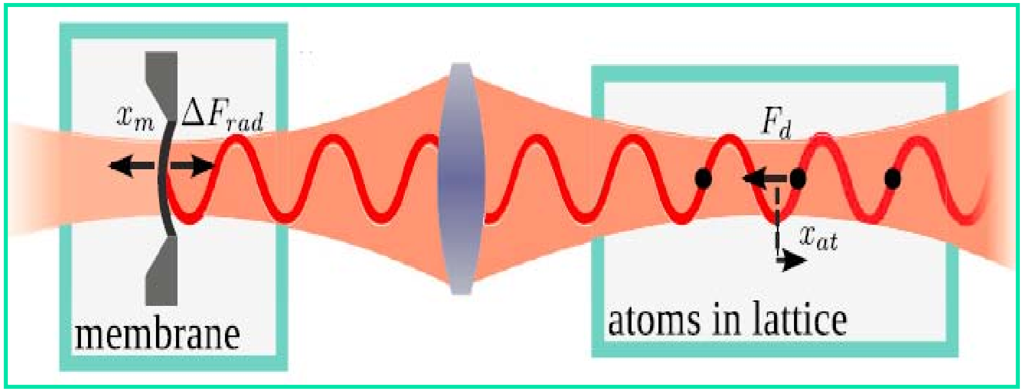
Steve M. Girvin
Department of Physics, Yale University, New Haven, CT 06520-8120, USA

Published May 18, 2009

Coherent optical systems combined with micromechanical devices may enable development of ultrasensitive force sensors and quantum information processing technology, as well as permit observation of quantum behavior in large-scale structures.

Subject Areas: **Quantum Information, Optics, Quantum Mechanics**

Example 1: S. Camerer et al., PRL 107, 223001 (2011)

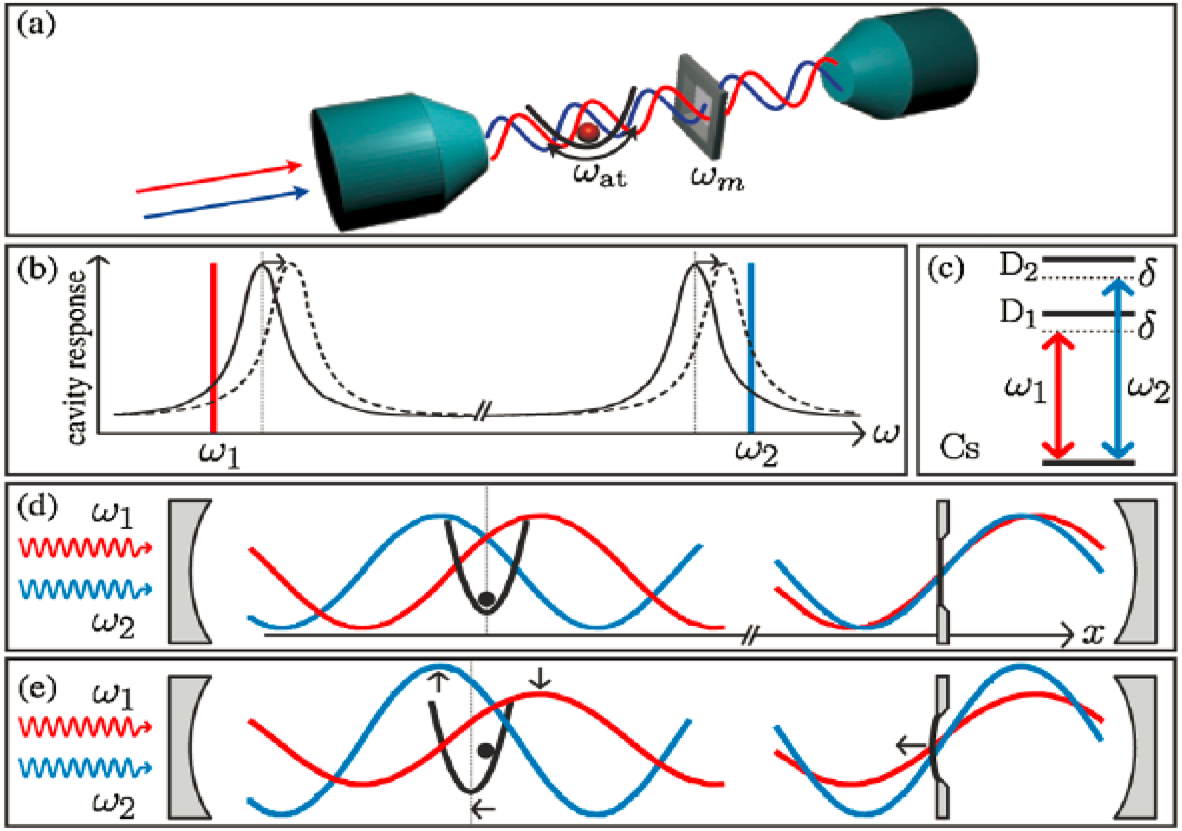


The laser light of an optical lattice mediates the opto-mechanical coupling between membrane vibrations and atomic motion!

吉林大学

Example 2: K. Hammerer et al., PRL 103, 063005 (2009)

微腔中薄膜振动和原子质心运动的强耦合



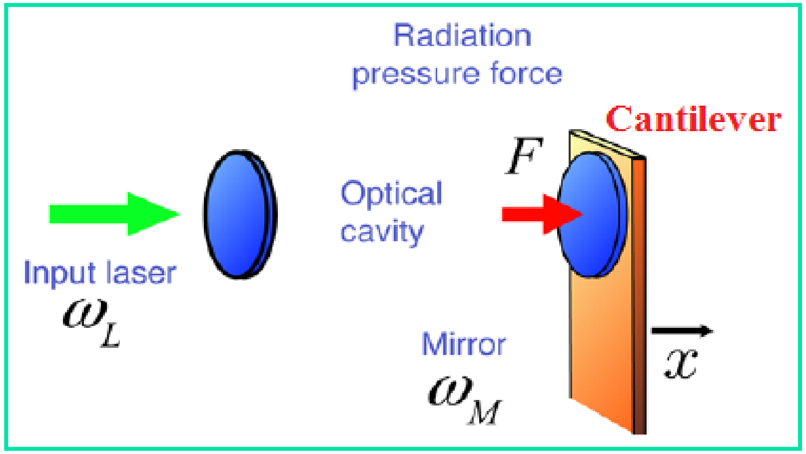
$$H = \omega_m a_m^+ a_m + \omega_{at} a_{at}^+ a_{at} + G(a_{at} + a_{at}^+)(a_m + a_m^+) \qquad G \approx 2\pi \times 45kHz$$

The strong coupling between a single trapped atom and a mechanical oscillator mediated by a quantized light field in a laser driven high-finesse cavity!

Two Typical Effects of Optomechanics:

Radiation Pressure: velocity-independent

Radiation Damping: velocity-dependent

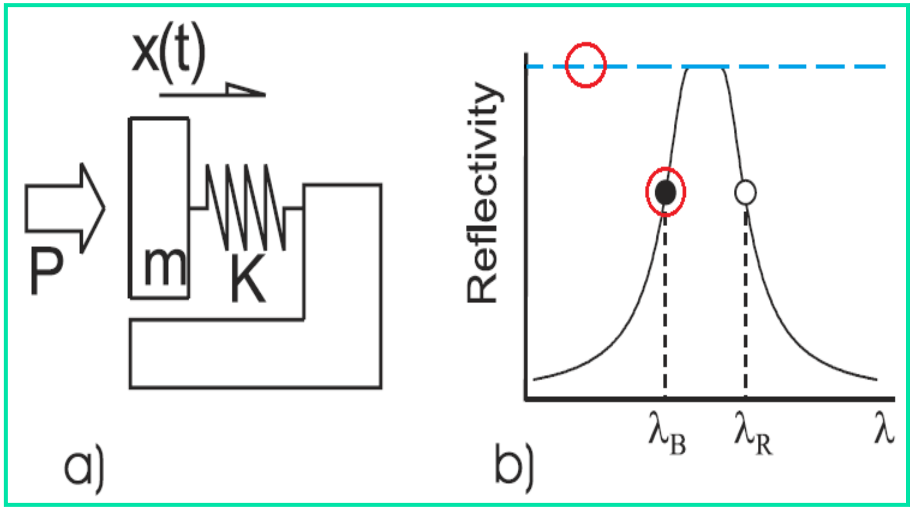


$$\frac{F_{damp}}{F_{press}} \approx 10^{-8}$$ **Ordinary Mirrors**

$$\frac{F_{damp}}{F_{press}} \approx 10^{-5}$$ **Bragg Mirrors (PC)**

$$\frac{F_{damp}}{F_{press}} \approx 10^{-2}$$ **Ordered Cold Atoms**

辐射阻尼：重力波检测 、振子基态冷却



Sov. Phys. JETP 25, 653-655 (1967)

Opt. Commun. 131, 107-113 (1996)

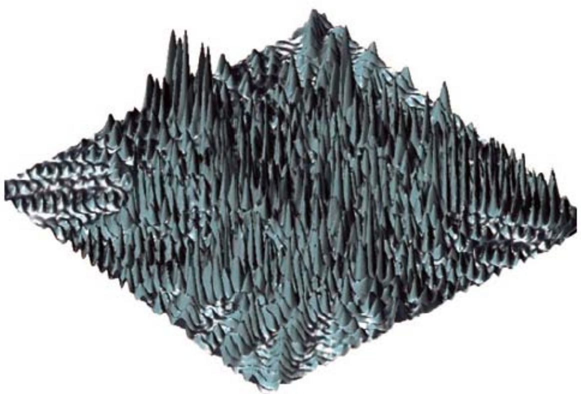
Phys. Rev. Lett. 100, 240801 (2008)

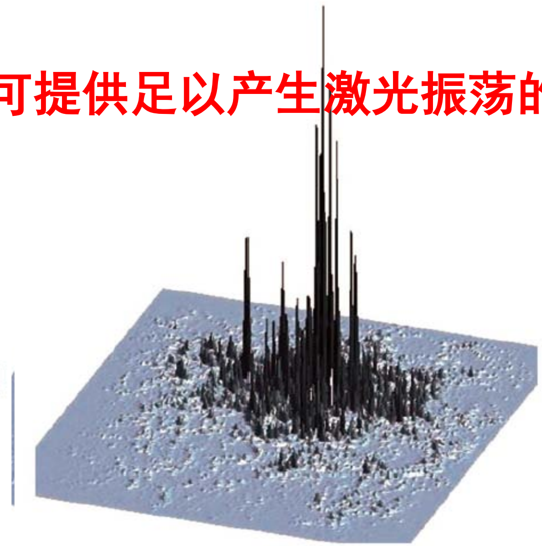
Phys. Rev. Lett. 107, 043602 (2011)

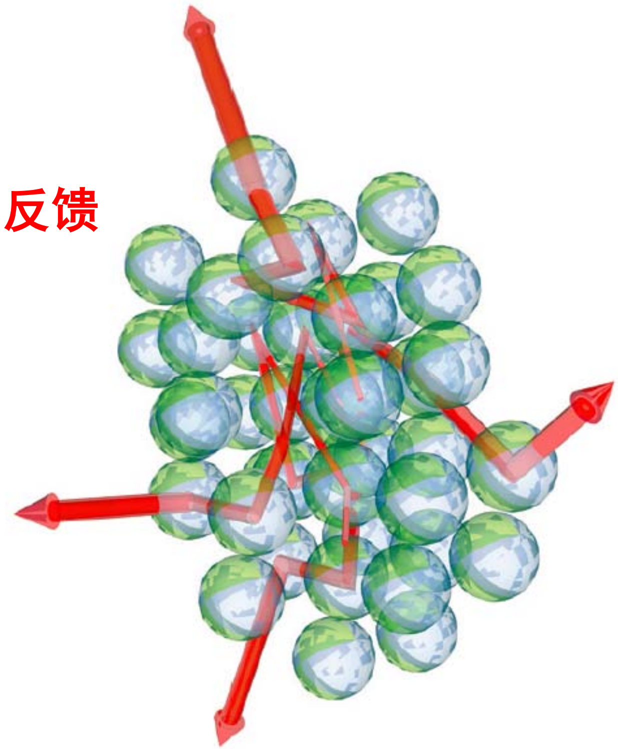
I. Background __ Random Lasing

Nature Physics 4, 359-367 (2008)

随机散射在一定条件下也可提供足以产生激光振荡的反馈







Advantages:

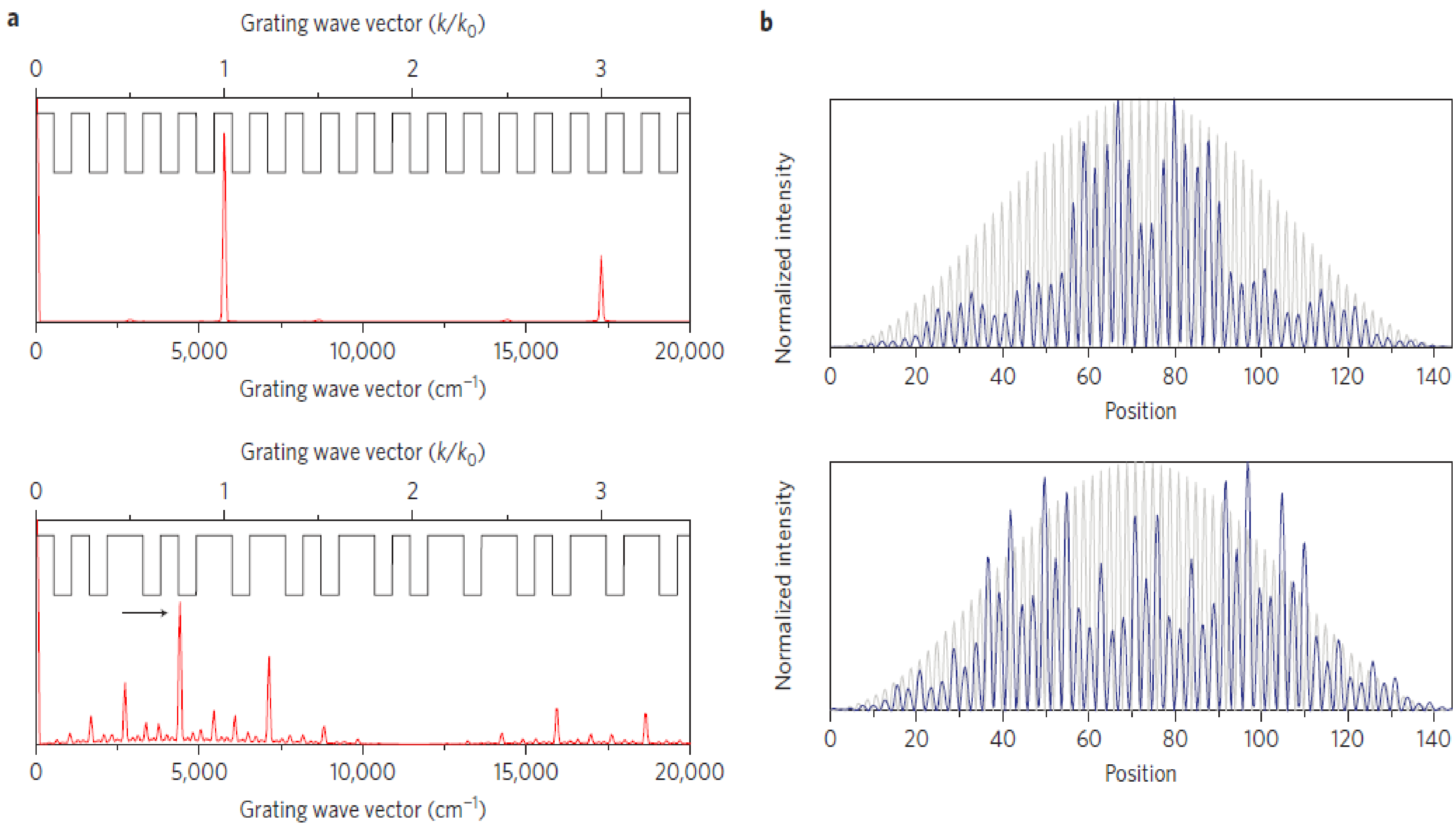
1) Very broad angular distribution, deal for display applications!

2) In the form of suspensions of particles, applied as coatings on surfaces of arbitrary shapes, potential for environment lighting!

3) Spectral sensitivity on environment temperature, potential for remote temperature sensing in hostile environments!

I. Background __ Random Lasing

吉林大学



"Periodic Feedback" … "Quasi-Periodic Feedback" … "Random Feedback"

I. Background __ Random Lasing

吉林大学

Nature Photonics 6, 101-104 (2011)

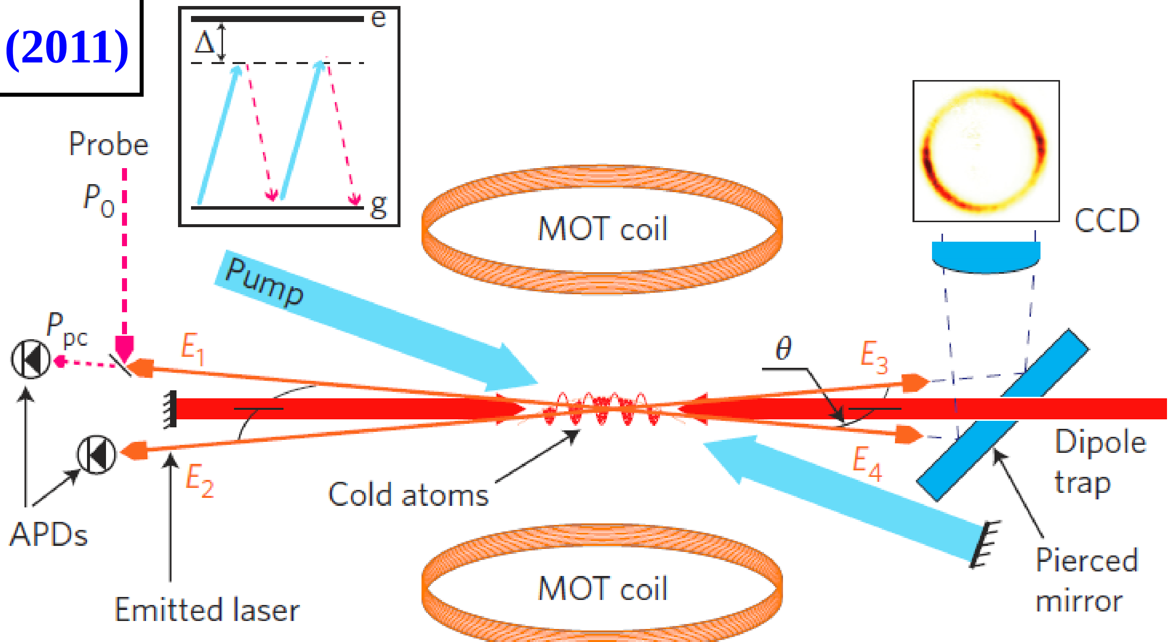
$$\lambda_{dip}\cos\theta = \lambda_0 / \bar{n}$$

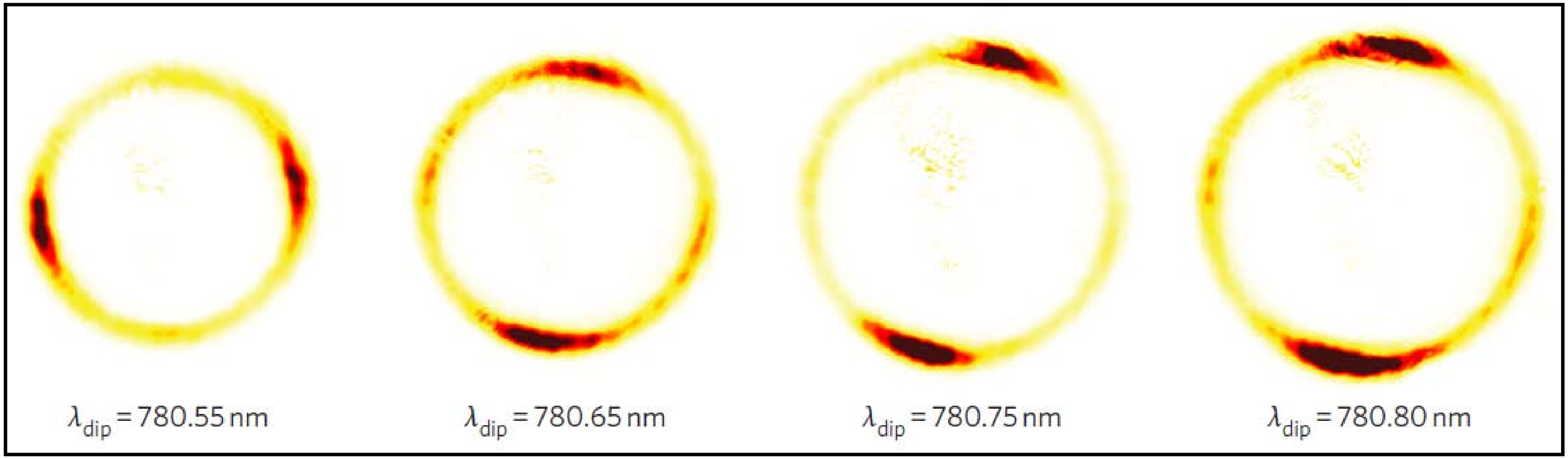
$$N \approx 5\times 10^7$$

$$L \approx 3mm$$

$$T \approx 100\mu K$$

$$\lambda_0 = 780.24nm$$



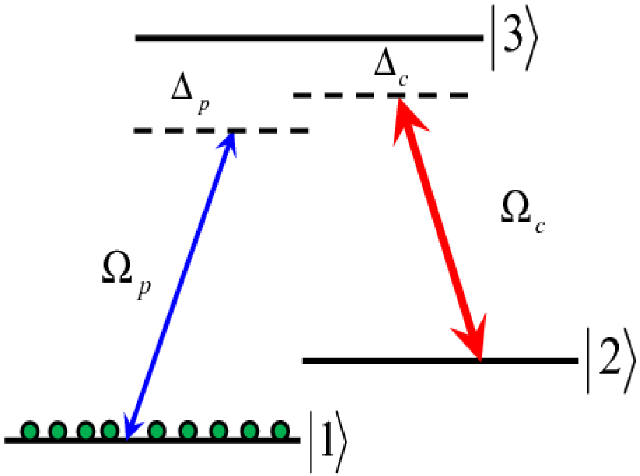


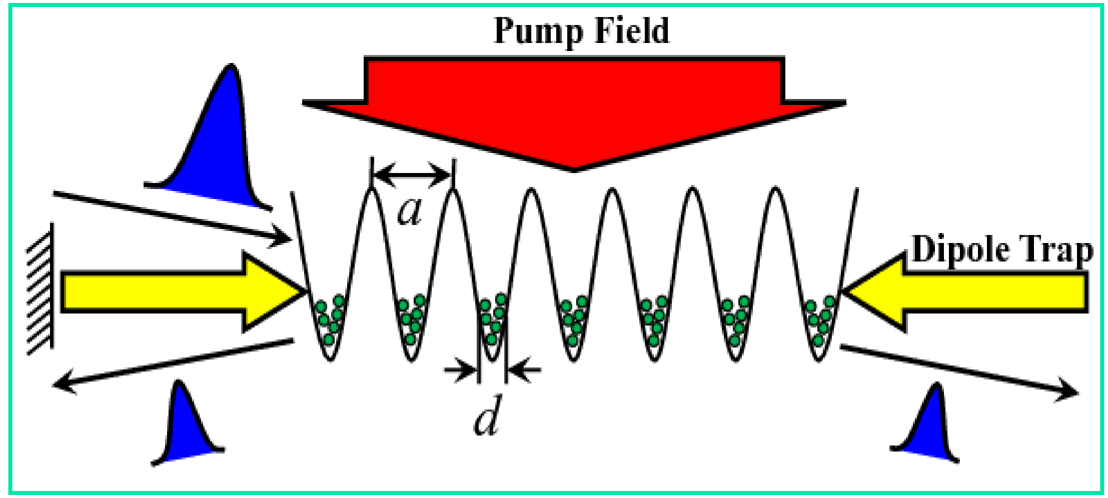
Outlines

- I: Background
- II: Optical Enhancement of Radiation Damping
- III: Nonlinear Lasing via Distributed Feedback
- IV: Conclusions

II. Optical Enhancement of Radiation Damping

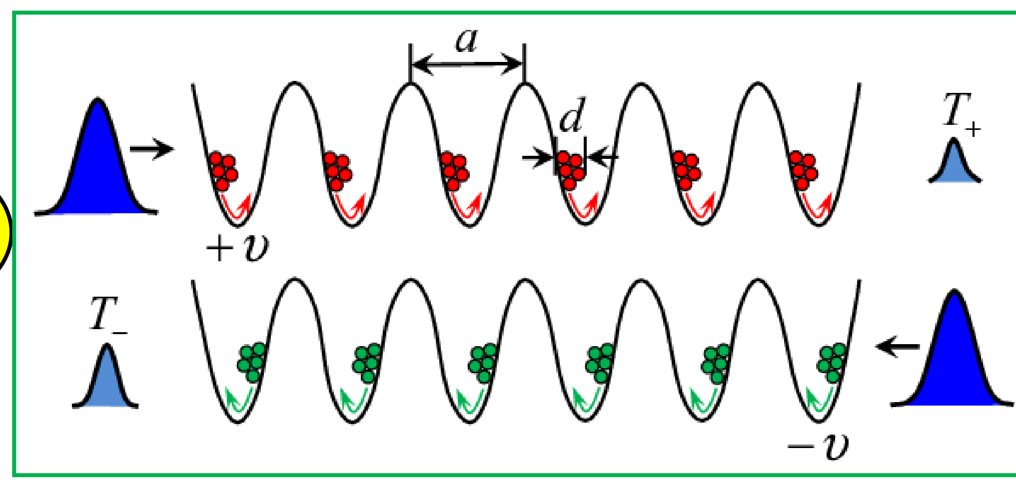
Schematics of a Possible Experiment:





光晶格中不同格点处的超冷原子进行集体振荡

Accessible in experiment at a velocity of ~1m/s for a moving atomic lattice?

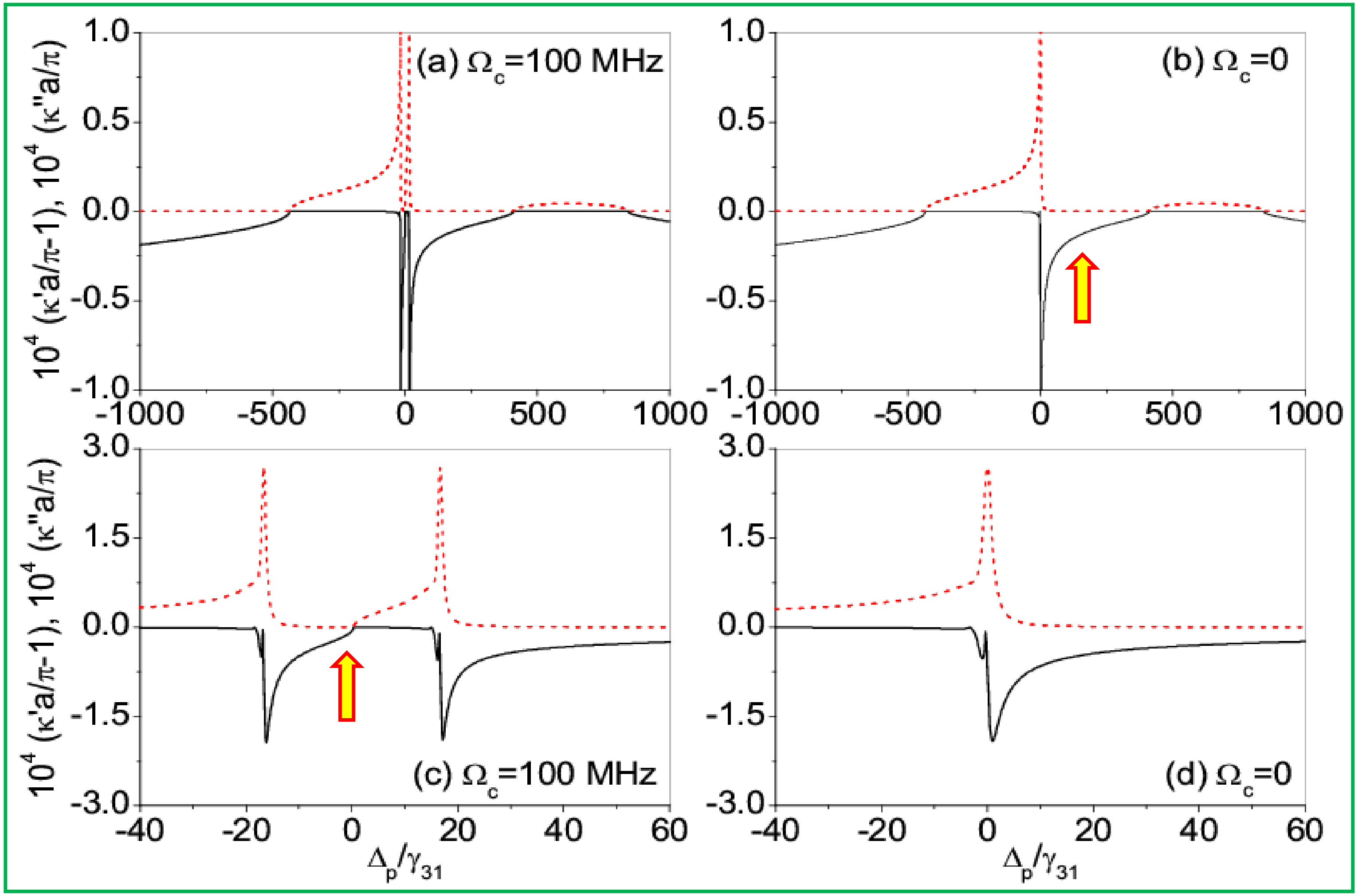


II. Optical Enhancement of Radiation Damping

吉林大学

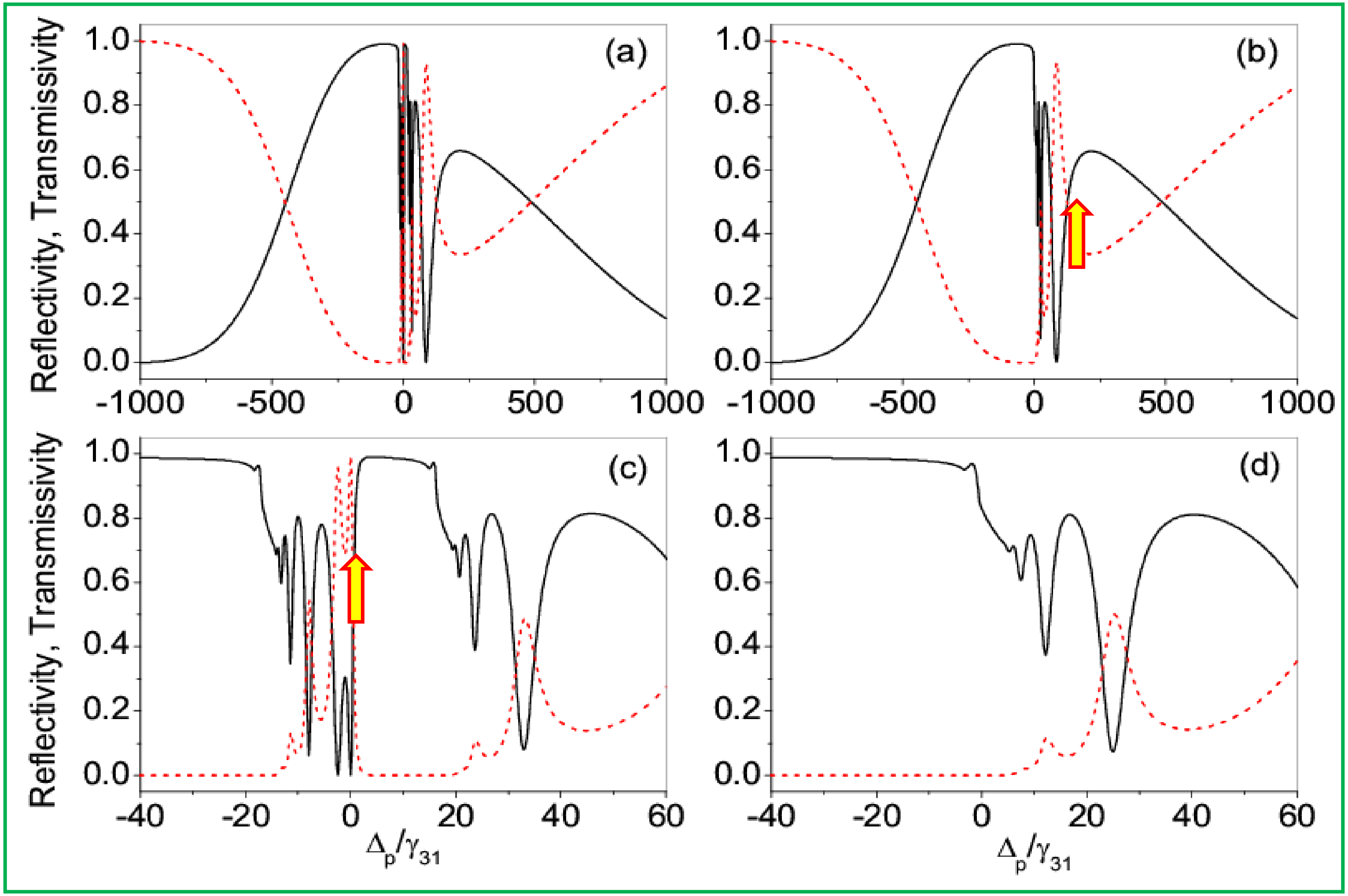
Without the pump, there are two stop-bands of several GHz in width!

With the pump, we find a third stop-band of several tens of MHz in width, which can be optically controlled on demand!



II. Optical Enhancement of Radiation Damping

Reflectivity and transmissivity of a sample of length L=2.0 cm with the pump on in (a,c) and the pump off in (b,d). The central narrower stop-band has a much sharper slop in reflectivity and transmissivity!



吉林大学

In the frame of a moving atom, the average transfer rate of the four-momentum ($P^{\mu=0,x,y,z}$) can be expressed as

$$\left\langle \frac{dP^{\mu}}{dt} \right\rangle = \frac{P}{c} \begin{pmatrix} [1-R(\omega_p)-T(\omega_p)] \\ [1+R(\omega_p)-T(\omega_p)]\vec{x} \end{pmatrix}$$

Maxwell Stress Tensor in Electrodynamics of Continuous Media

In the lab frame, it turns upon the Lorentz transformation into

$$\left\langle \frac{dP^{\mu}}{dt} \right\rangle' = \frac{\eta^2 P'}{\sqrt{c^2-\upsilon^2}} \begin{pmatrix} \wp^0(\eta\omega_p') \\ \wp^x(\eta\omega_p')\vec{x} \end{pmatrix}$$

Very Complicated Velocity Dependence

$$\wp^0 = [1-R(\eta\omega_p')-T(\eta\omega_p')] + \frac{\upsilon}{c}[1+R(\eta\omega_p')-T(\eta\omega_p')]$$

$$\wp^x = [1+R(\eta\omega_p')-T(\eta\omega_p')] + \frac{\upsilon}{c}[1-R(\eta\omega_p')-T(\eta\omega_p')]$$

$$\eta = \sqrt{(1-\upsilon/c)/(1+\upsilon/c)}$$

II. Optical Enhancement of Radiation Damping

吉林大学

In the typical case of $\upsilon << c$, we have $\eta \approx 1-\upsilon / c$ and thus

$$R(\eta\omega_p^{'}) = R(\omega_p^{'}) - \omega_p^{'}(\upsilon / c)[\partial R(\omega_p^{'}) / \partial \omega_p^{'}]$$

$$T(\eta\omega_p^{'}) = T(\omega_p^{'}) - \omega_p^{'}(\upsilon / c)[\partial T(\omega_p^{'}) / \partial \omega_p^{'}]$$

Then the four-momentum transfer rate becomes

$$\left\langle \frac{dP^{\mu}}{dt} \right\rangle^{'} = \frac{P'}{c}\begin{pmatrix} [F_{(0)}^{0} - (\upsilon / c)F_{(1)}^{0}] \\ [F_{(0)}^{x} - (\upsilon / c)F_{(1)}^{x}]\vec{x} \end{pmatrix}$$

To distinguish the velocity-dependent pressure force and velocity-independent damping force!

$$F_{(0)}^{0} = 1 - R(\omega_p^{'}) - T(\omega_p^{'})]$$

$$F_{(1)}^{0} = 1 - 3R(\omega_p^{'}) - T(\omega_p^{'}) - \omega_p^{'}[\frac{\partial R(\omega_p^{'})}{\partial \omega_p^{'}} + \frac{\partial T(\omega_p^{'})}{\partial \omega_p^{'}}]$$

$$F_{(0)}^{x} = 1 + R(\omega_p^{'}) - T(\omega_p^{'})]$$

$$F_{(1)}^{x} = 1 + 3R(\omega_p^{'}) - T(\omega_p^{'}) + \omega_p^{'}[\frac{\partial R(\omega_p^{'})}{\partial \omega_p^{'}} - \frac{\partial T(\omega_p^{'})}{\partial \omega_p^{'}}]$$

For a lossless and non-dispersive medium, we have

$$\left\langle \frac{dP^x}{dt} \right\rangle = \frac{2RP}{c} - \frac{\upsilon}{c}\frac{4RP}{c}$$

where the radiation pressure (former, velocity independent) is much larger than the radiation damping force (latter, velocity-sensitive).

To have $F_{(0)}^x \approx (\upsilon / c) F_{(1)}^x$ for practical velocities and attain a large enough radiation damping, we should find a highly dispersive medium with

$$\frac{\partial R(\omega_p^{'})}{\partial \omega_p^{'}} - \frac{\partial T(\omega_p^{'})}{\partial \omega_p^{'}} \approx \frac{c}{\upsilon}\omega_p^{'}$$

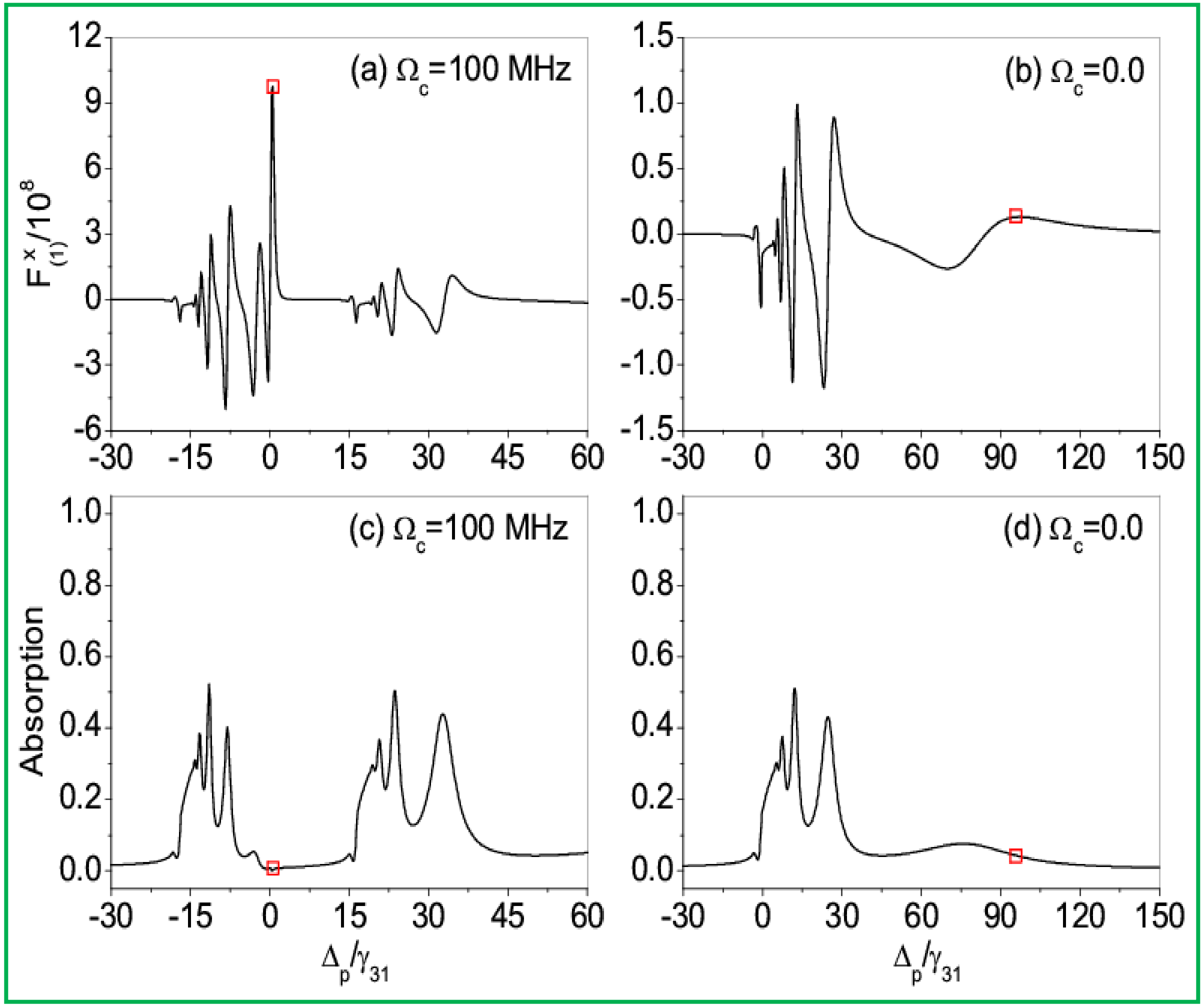
yet accompanied by negligible absorption ($1 > T + R > 0.99$)!

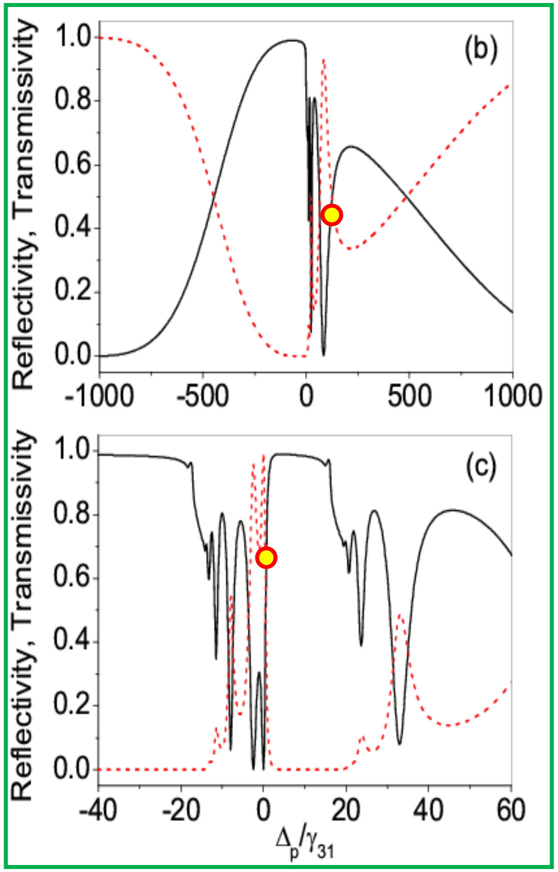
II. Optical Enhancement of Radiation Damping

吉林大学

When the pump is off, we have $F^{x}_{(1)} = +1.3\times10^{7}$ **and** $A = 7.6\%$ **at** $\Delta_{p0} = 97\gamma_{31}$ **!**

When the pump is on, we have $F^{x}_{(1)} = +9.7\times10^{8}$ **and** $A = 0.3\%$ **at** $\Delta_{p0} = 0.5\gamma_{31}$ **!**





If the probe field is a Gaussian light pulse instead of a plane light wave, we can check how the momentum transfer depends on the oscillating velocity of an ordered atomic structure:

$$\left\langle \frac{dP^{\mu}}{dt} \right\rangle' = \frac{\eta^2 P'}{\sqrt{c^2 - \upsilon^2}} \begin{pmatrix} \wp^0(\eta\omega_p') \\ \wp^x(\eta\omega_p')\vec{x} \end{pmatrix}$$

$$\frac{\Delta P^{\mu'}}{\overline{N}\hbar k} = \frac{\eta}{\sqrt{1-\upsilon^2/c^2}} \int_{-\infty}^{+\infty} d\omega_p' \left| f(\omega_p') \right|^2 \begin{pmatrix} \wp^0(\eta\omega_p') \\ \wp^x(\eta\omega_p')\vec{x} \end{pmatrix}$$

$$f(\omega_p') = \frac{1}{\sqrt{\pi}\delta_{p0}} \exp[-(\omega_p' - \omega_{p0})^2 / \delta_{p0}^2]$$

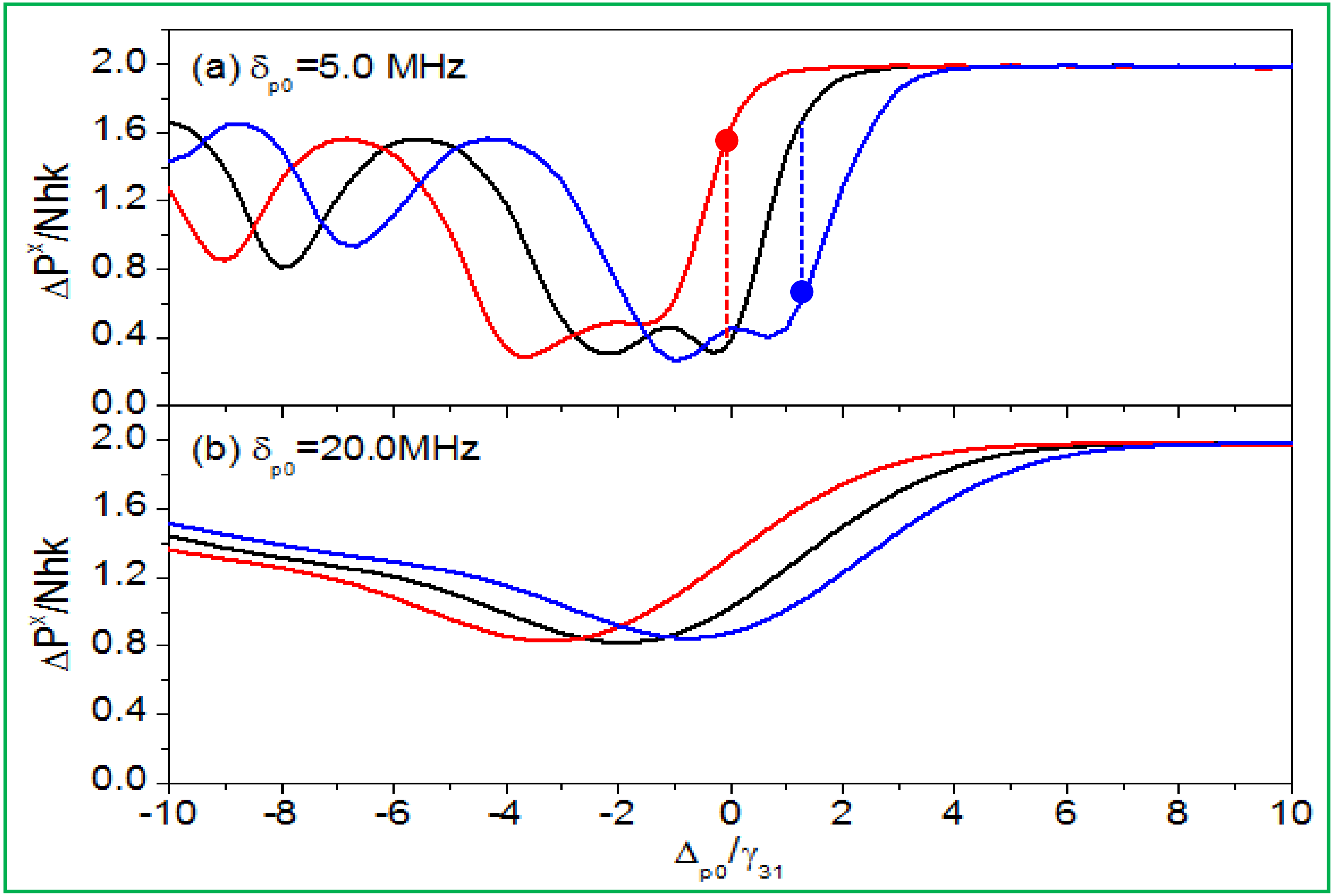
which can be used as a direct evidence to determine whether the radiation damping is comparable with the radiation pressure?

II. Optical Enhancement of Radiation Damping

吉林大学

At $\Delta_{p0} = 0.0\gamma_{31}$ **, we have** $\Delta P^x / \overline{N}\hbar k = 0.38 \quad (1.62)$ **for** $\upsilon = 0.0 \quad (-6.0)$ **m/s!**

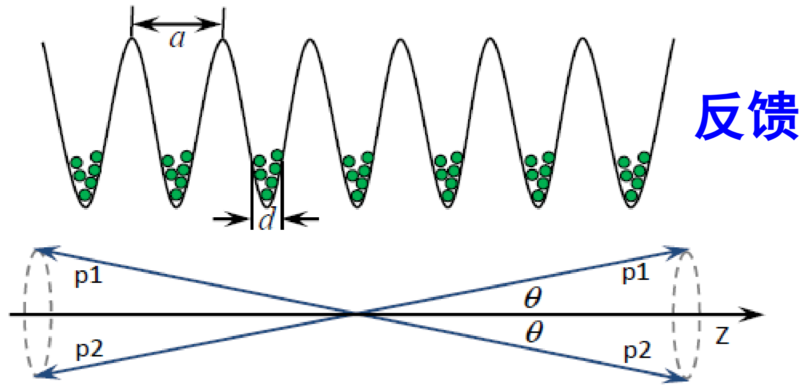
At $\Delta_{p0} = 1.33\gamma_{31}$ **, we have** $\Delta P^x / \overline{N}\hbar k = 1.70 \quad (0.66)$ **for** $\upsilon = 0.0 \quad (+6.0)$ **m/s!**

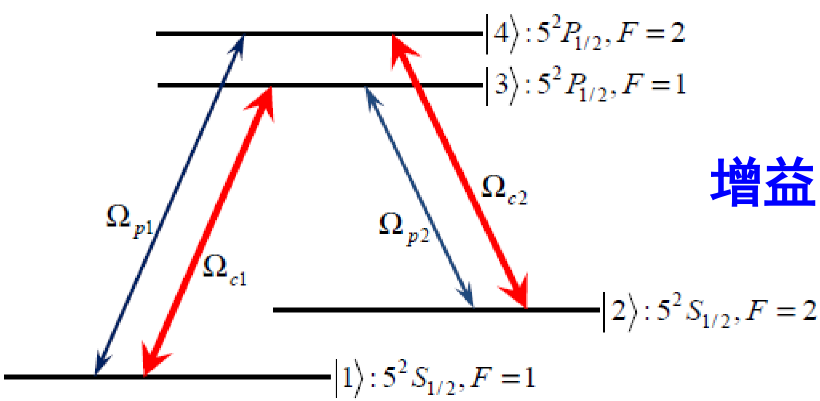


Outlines: Ordered Cold Atoms

- I: Background
- II: Optical Enhancement of Radiation Damping
- **III: Nonlinear Lasing via Distributed Feedback**
- IV: Conclusions

III. Nonlinear Lasing via Distributed Feedback





First-Order Solutions

$$\chi_{p1} = \chi'_{p1} + i\chi''_{p1} = \frac{N d_{14}^2}{\varepsilon_0 \hbar}\,\frac{\rho_{14}(t \to \infty)}{\Omega_{p1}}$$

$$\chi_{p2} = \chi'_{p2} + i\chi''_{p2} = \frac{N d_{23}^2}{\varepsilon_0 \hbar}\,\frac{\rho_{23}(t \to \infty)}{\Omega_{p2}}$$

$$\rho_{11} = \frac{\Gamma_{41}|\Omega_{c2}|^2[C_{13} - |\Omega_{c1}|^2]}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

$$\rho_{22} = \frac{\Gamma_{32}|\Omega_{c1}|^2[C_{24} - |\Omega_{c1}|^2]}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

$$\rho_{33} = \frac{\Gamma_{41}|\Omega_{c1}|^2|\Omega_{c2}|^2}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

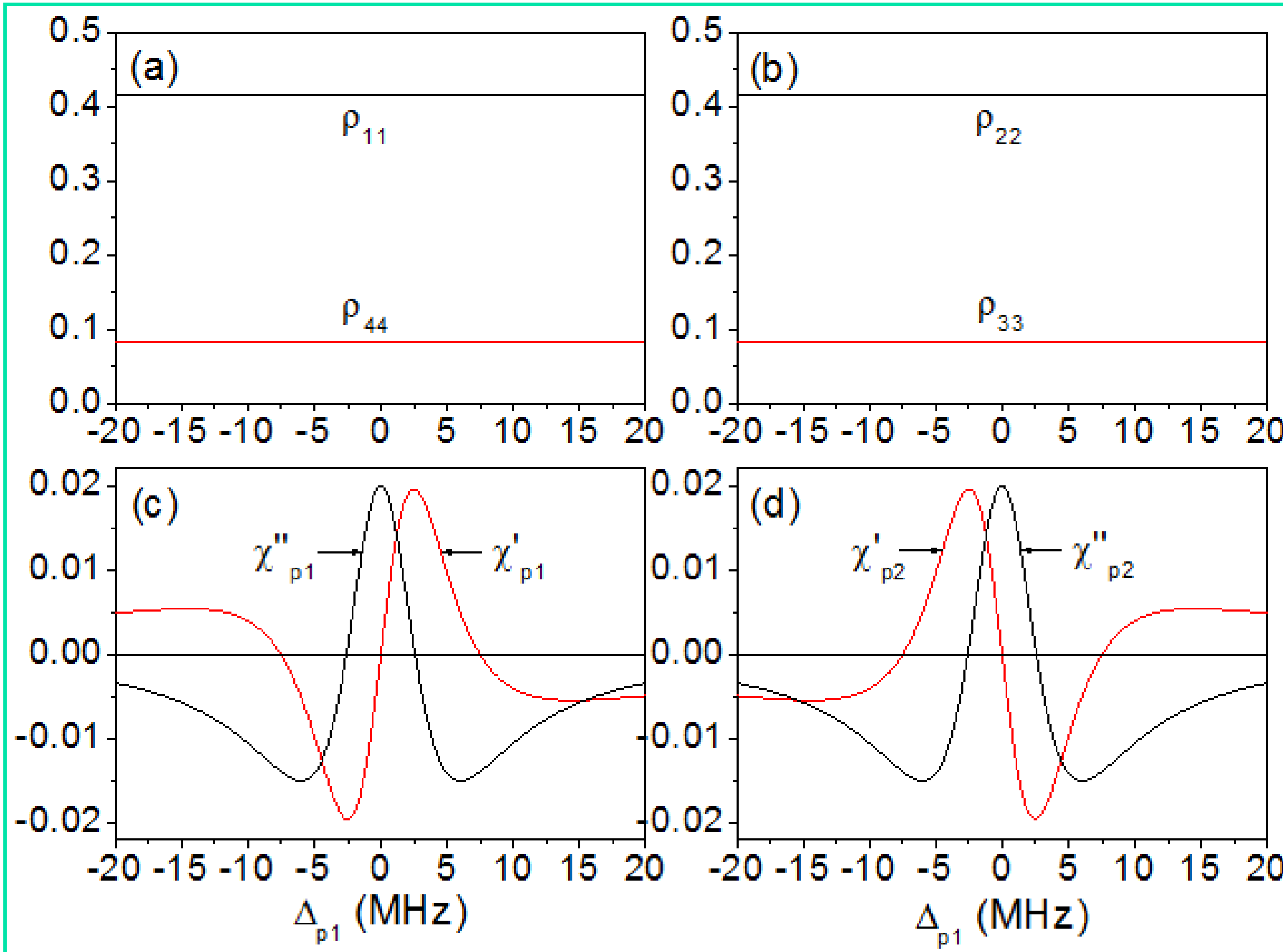
$$\rho_{44} = \frac{\Gamma_{32}|\Omega_{c1}|^2|\Omega_{c2}|^2}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

Zero-Order Solutions

$$\rho_{13} = \frac{-i\Gamma_{41}(\gamma_{13} - i\Delta_{c1})\Omega_{c1}^{*}|\Omega_{c2}|^2}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

$$\rho_{24} = \frac{-i\Gamma_{32}(\gamma_{24} - i\Delta_{c2})\Omega_{c2}^{*}|\Omega_{c1}|^2}{\Gamma_{41}|\Omega_{c2}|^2 C_{13} + \Gamma_{32}|\Omega_{c1}|^2 C_{24}}$$

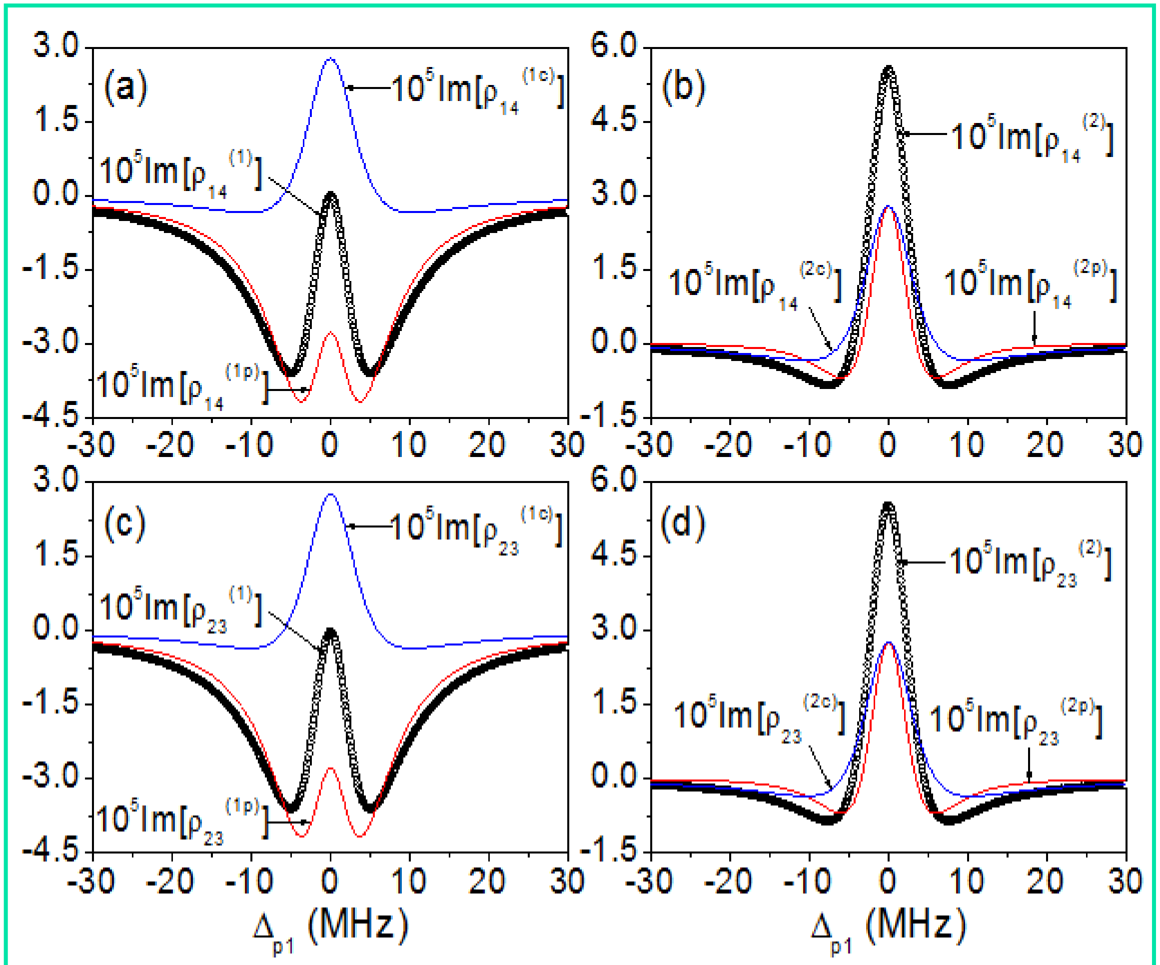
III. Nonlinear Lasing via Distributed Feedback



III. Nonlinear Lasing via Distributed Feedback

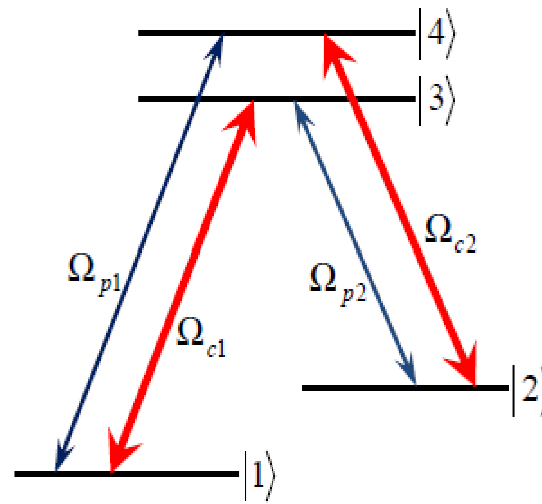
Direct coupling term: $\rho_{14}^{(1)} \propto \Omega_{p1}^{*}[......(\rho_{44}-\rho_{11})+......\rho_{31}+......\rho_{42}]$

Crossed coupling term: $\rho_{14}^{(2)} \propto \Omega_{p2}[......(\rho_{33}-\rho_{22})+......\rho_{24}+......\rho_{13}]$



线性的直接耦合项只提供了一个透明背景

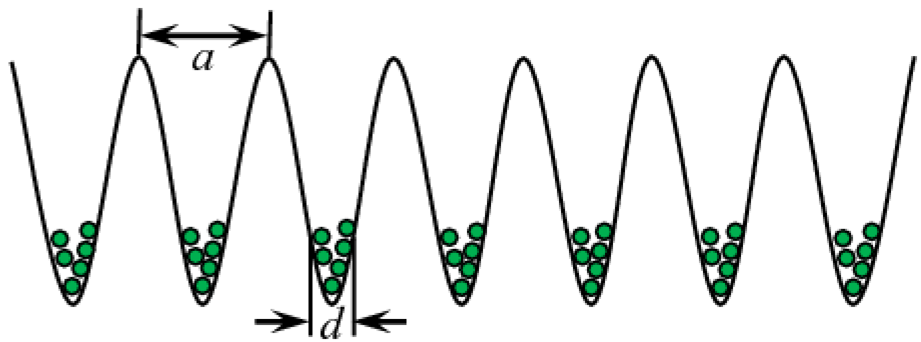
非线性的间接耦合项才是探测增益的根源



III. Nonlinear Lasing via Distributed Feedback

$$\chi(z)=\chi_p\sum_{l=-\infty}^{+\infty} rect\left[\frac{z+la}{d}\right]=\chi_p\sum_{m=-\infty}^{+\infty}C_m e^{i2\pi mz/a}\approx\chi_p\left[\frac{d}{a}+\frac{d}{a}e^{-i2\pi z/a}+\frac{d}{a}e^{-i2\pi z/a}\right]$$

在获得非线性相干增益的基础上，分析介质的分布反馈是否将会导致激光振荡？



$$\chi(z)\approx\chi_0+2\chi_1\cos(2\pi z/a)=\chi_0+2\chi_1\cos(kz)$$

$$E(z)=E_p(z)e^{i\kappa z}=\left[\sum_{n=-\infty}^{+\infty}E_n e^{i2nkz}\right]e^{i\kappa z}\approx E_0e^{i\kappa z}+E_{-1}e^{i(\kappa-2k)z}$$

$$\frac{\partial^2}{\partial z^2}E(z)+k_p^2[1+\chi(z)]E(z)=0$$

耦合模方程的**双模近似理论**可以很好解答这一问题！

III. Nonlinear Lasing via Distributed Feedback

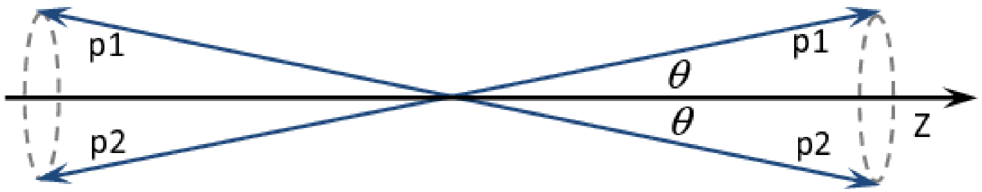
吉林大学

$$\begin{pmatrix} k_p^2(1+\chi_0)-\kappa^2 & k_p^2\chi_1 \\ k_p^2\chi_1 & k_p^2(1+\chi_0)-(\kappa-2k)^2 \end{pmatrix}\begin{pmatrix} E_0 \\ E_{-1} \end{pmatrix}=0$$

$$\kappa_\pm = k \pm \frac{1}{2k}\sqrt{[k_p^2(1+\chi_0)-k^2]^2-k_p^4\chi_1^2}$$

介质反射率和透射率相等且呈指数增长标志着激光振荡的出现

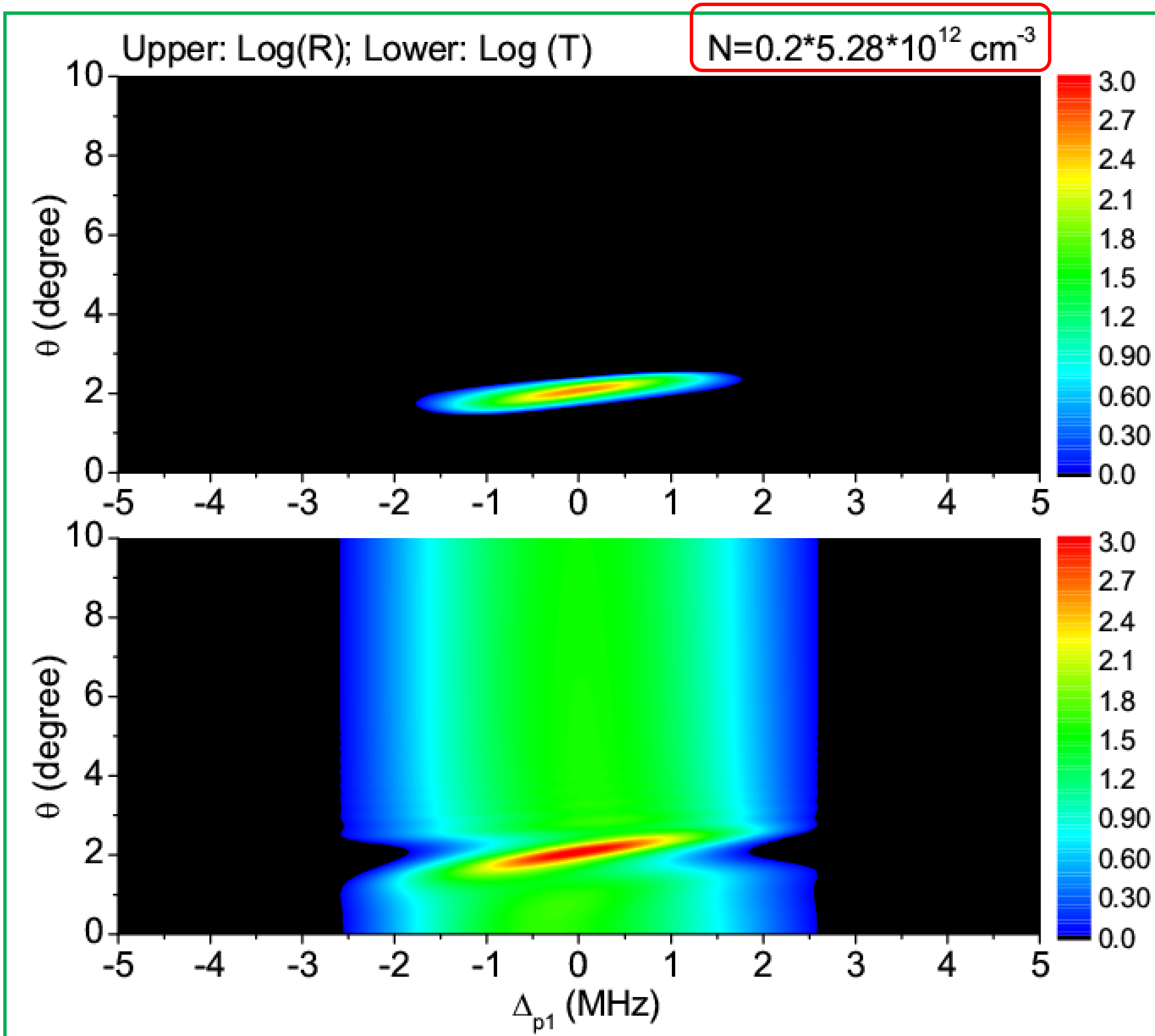
$$k_p = \cos\theta \cdot 2\pi/\lambda_p$$



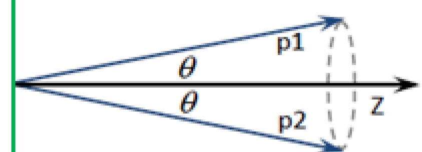
$$T(\Delta_p,\theta)=\left|\frac{2A_-(1+X_+)e^{+iqL}-2B_-(1+X_-)e^{-iqL}}{A_-B_+e^{+iqL}-A_+B_-e^{-iqL}}-1\right|^2$$

$$R(\Delta_p,\theta)=\left|\frac{2A_-(1+X_+)-2B_-(1+X_-)}{A_-B_+e^{+iqL}-A_+B_-e^{-iqL}}\right|^2$$

III. Nonlinear Lasing via Distributed Feedback



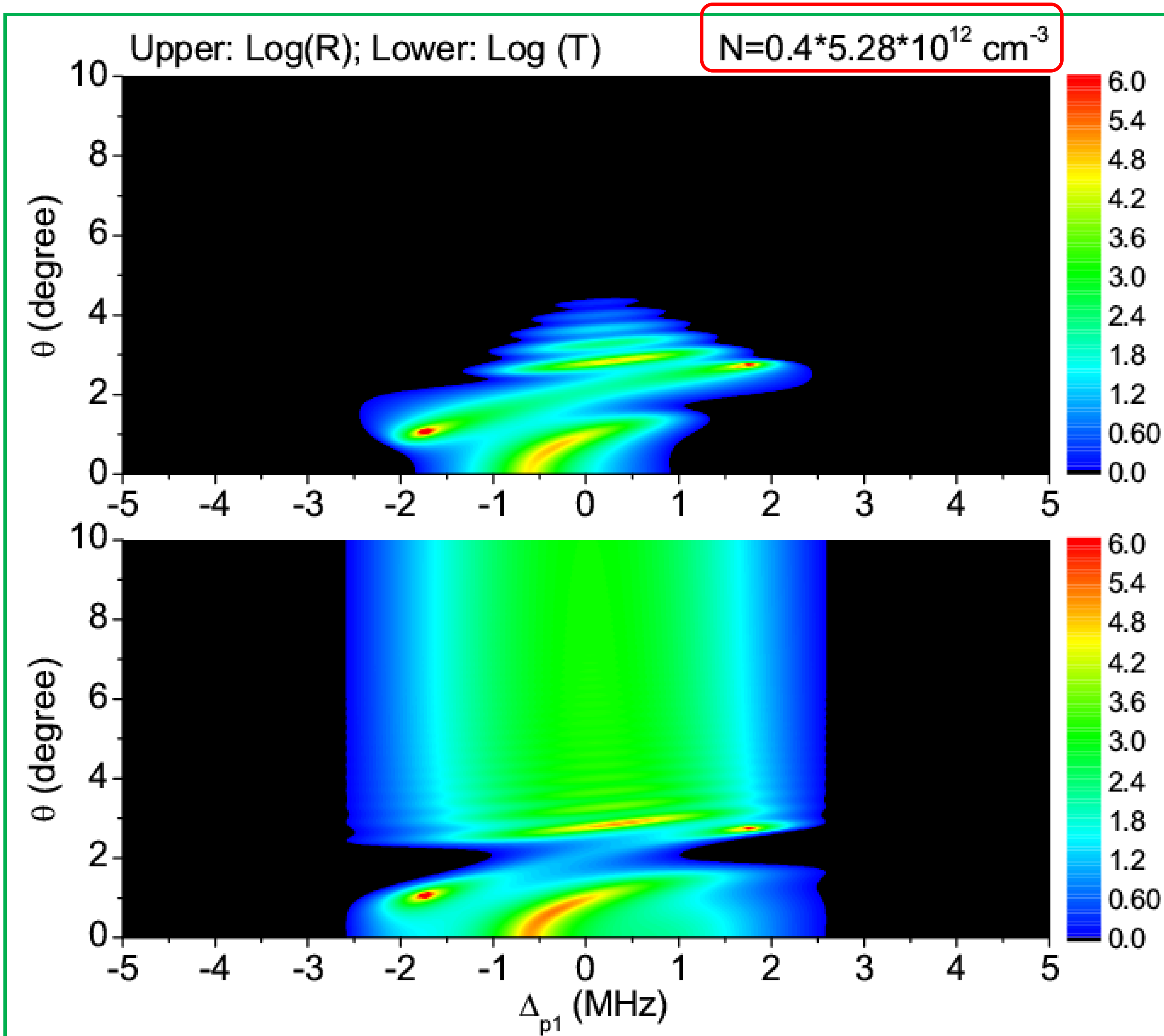
原子密度比较小时，增益系数刚刚超过增益阈值，所以仅有零阶模式的激光振荡！



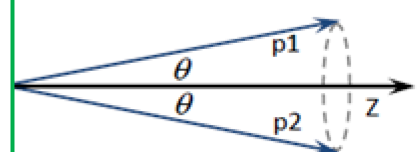
这种分布反馈非线性激光振荡的谱宽度和角宽度极小！

III. Nonlinear Lasing via Distributed Feedback

吉林大学



原子密度比较大时，增益系数远远超过增益阈值，所以有多个激光模式同时振荡！



零阶模式所对应的角度和光谱位置出现光子禁带迹象？

Outlines: Ordered Cold Atoms

- I: Background
- II: Optical Enhancement of Radiation Damping
- III: Nonlinear Lasing via Distributed Feedback
- **IV: Conclusions**

IV. Conclusions

吉林大学

- Radiation damping can be greatly amplified to be comparable with radiation pressure yet with negligible absorption!
- The amplified radiation damping is accessible in experiment for cold atoms coherently moving at the velocity of ~1.0 m/s!
- Nonlinear coherent gain can be simultaneously attained on two probe transitions in a resonant FWM process!
- Distributed feedback in an ordered structure of cold atoms may further result in a two-color lasing of narrow spectrum!

Thank you for your kind attention!

欢迎批评指正！