Bose-Einstein Condensation with an Entangled Order Parameter

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Ref: Y. Shi and Q. Niu, Phys. Rev. Lett. 96, 140401 (06)

1st Workshop on cold atom physics and quantum information for young researchers，Taiyuan, 2007.7.2

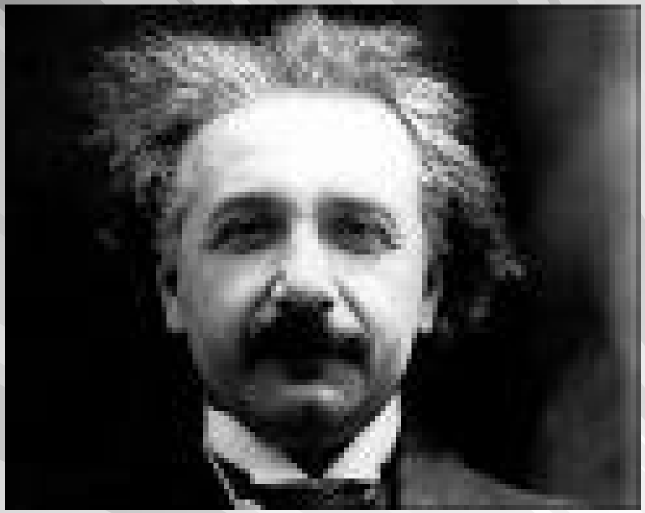
Background

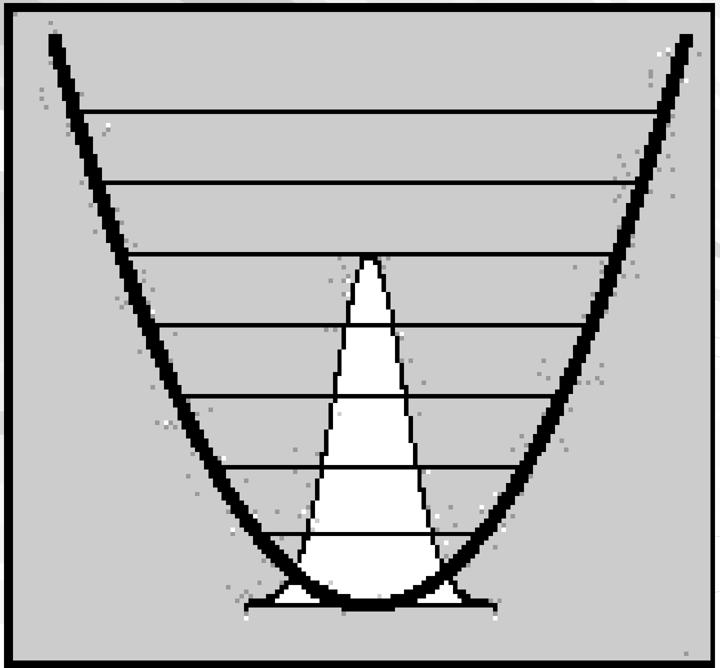
1. BEC（玻色爱因斯坦凝聚）

- Started with a purely theoretical proposal (Einstein 1924):

  "a number of molecules steadily growing with increasing density goes over in the first quantum state …".

- A finite fraction of bosons occupies the single particle ground state.

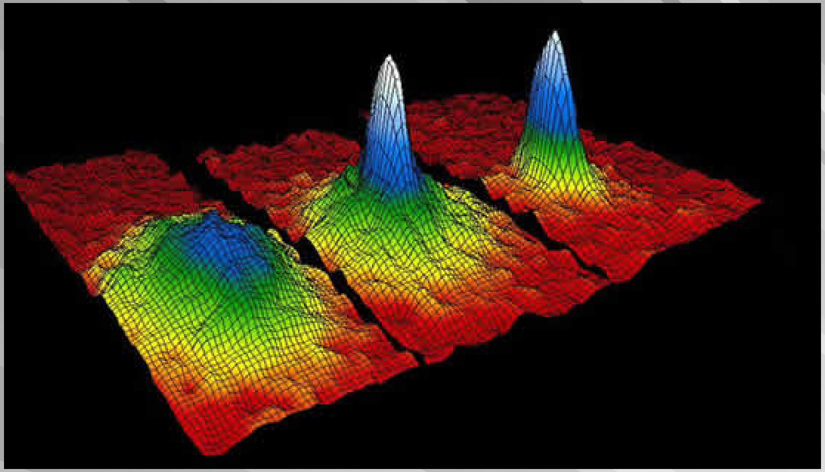




BEC is a thread transcending many fields of physics, and has generated many Nobel Prizes.

- Superfluidity of liquid ^4He . Condensate fraction is 10%.
- BEC of excitons，polaritons and magnons.
- Superconductivity [BEC of cooper pairs. Condensate fraction is $10^{-4} \sim 10^{-2}$ in fermion systems.], including HTSC.
- Superfluidity of liquid ^3He .
- Nucleon pairing.
- Meson condensates inside neutron stars.
- Spontaneous gauge symmetry breaking (originally a description of BEC) is a major component of standard model. Vacuum is a BEC in a generalized sense.
- Supersolid (open).
- BEC of alkali atomic gases.

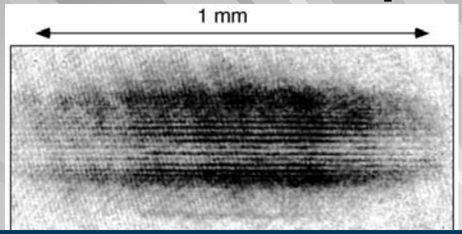
BEC of alkali atomic gases

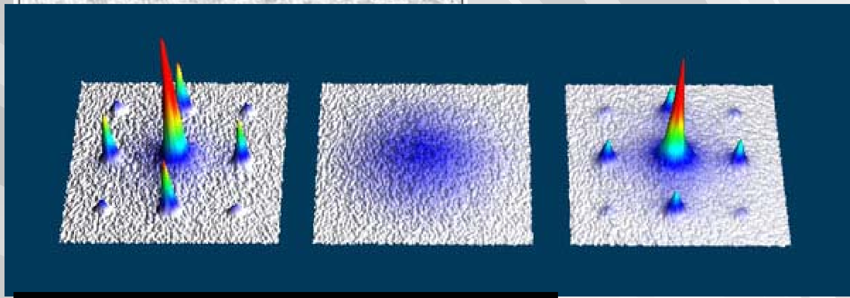


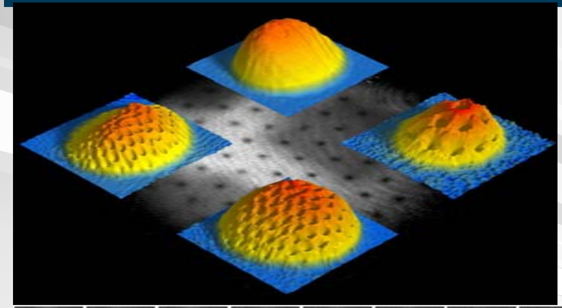
- Based on technology of cooling and trapping atoms (Nobel in 1997).
- First direct observation of BEC. 1995. (Nobel in 2001).
- $T < 170nK.$ Condensate fraction can be almost 1.
- This field has expanded to be a major frontier in physics.

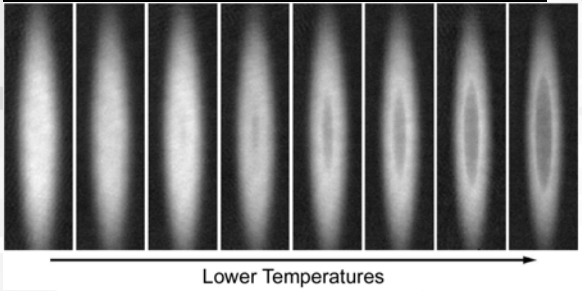
Ultra-cold atomic system as a new playground for condensed matter physics

- The players are atoms, rather than electrons.
- Interference.
- Optical Lattices. Insulator-superfluid Mott transition.
- Rotating condensates. Vortices.
- Fermion condensate, BEC-BCS crossover; Superfluidity of polarized fermi gases; Pairing without superfluidity.









Advantages

- Ultra-low temperature. Can be 450pK. Coldest objects in the Universe! [The universe is 3K].
- Highly controllable.
- Novel many-body quantum states can be created and manipulated in these systems. This is highly interesting for both many-body physics and quantum information science.

2. Quantum entanglement

$$|\Psi\rangle \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

- E.g. $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$
- Discovered by Einstein (with Podolsky and Rosen) and Schroedinger in 1935.
- A correlation stronger than allowed by any classical theory (with probability underlied by some hidden realistic variable). Widely regarded as the most essential difference between quantum mechanics and classical mechanics.

Entanglement entropy

- Entanglement (in a pure state), between the two parts A and B, can be quantified as $S = -Tr\rho_A \log_I \rho_A,$

  $$\rho_A = Tr_B|\Psi\rangle\langle\Psi|, \quad \text{I: dimensionality of } \rho_A,$$

  thanks to quantum information theory.
- Meaning of ρ_A: $\langle\Psi|O_A|\Psi\rangle = Tr(\rho_A O_A)$.
- Maximum entanglement: S=1:

$$S = -\sum_{i=1,I} \frac{1}{I} \log_I \frac{1}{I}.$$

Motivations of Our work

Simplest BEC

$$\psi \approx \phi(\mathbf{r}_1)\cdots\phi(\mathbf{r}_N)$$

$$|\psi\rangle = \frac{1}{\sqrt{N}}(a^\dagger)^N|0\rangle$$

All atoms occupy the same single particle states $\phi(\mathbf{r})$.

Every particle does the same!

Mean field theory.

$\phi(\mathbf{r})$: single particle wavefunction.

It becomes the order parameter [equivalently, sometimes one uses $\sqrt{N}\phi(\mathbf{r})$].

Two-component BEC

A mixture of A-atoms and B-atoms:

$$\psi \approx \phi_a(\mathbf{r}_{a1}) \cdots \phi_a(\mathbf{r}_{aN_a}) \otimes \phi_b(\mathbf{r}_{b1}) \cdots \phi_b(\mathbf{r}_{bN_b})$$

$$|\psi\rangle = \frac{1}{\sqrt{N_a N_b}} (a^\dagger)^{N_a} (b^\dagger)^{N_b} |0\rangle = |\psi\rangle_a \otimes |\psi\rangle_b$$

A-atoms and B-atoms separately condense, with separate order parameters (classically coupled).

No entanglement between the two species.

Similar is a mixture of one species of atoms with two spin states, the numbers of which are conserved respectively.

Spin-1 condensate

$$|\psi\rangle \sim [(a_0^\dagger)^2 - 2a_{-1}^\dagger a_1^\dagger]^{N/2}|0\rangle$$

- Non-mean-field state.

  (symmetry breaking mean-field state: $|\psi\rangle = \frac{1}{\sqrt{N}}(a_\alpha^\dagger)^N|0\rangle$).
- But the particles are all identical. Each particle can flip among the 3 spin states.
- It does not possess the entanglement we want: entanglement between different kinds of particles given.

Generalized cases of BEC

- Molecule BEC: BEC occurs in a molecule bound state.
- BCS: BEC occurs in a paired state of two fermions, with the size of the pair wavefunction larger than the particle distance.

Josephson effect

- Also started with a theoretical proposal.
- In the condensate,

$$\psi \approx \phi(\mathbf{r}_1) \cdots \phi(\mathbf{r}_N)$$

$$\phi(\mathbf{r}) = \phi_\uparrow(\mathbf{r}) + \phi_\downarrow(\mathbf{r}).$$

- The superposition feature of single particle state is manifested by the order parameter.

A step further than Josephson

- Our question: can a condensate wavefunction be entangled between two kinds of particles?
- Josephson effect amplifies single particle superposition to macroscopic scale.

We now amplify quantum entanglement to macroscopic scale !

Our model

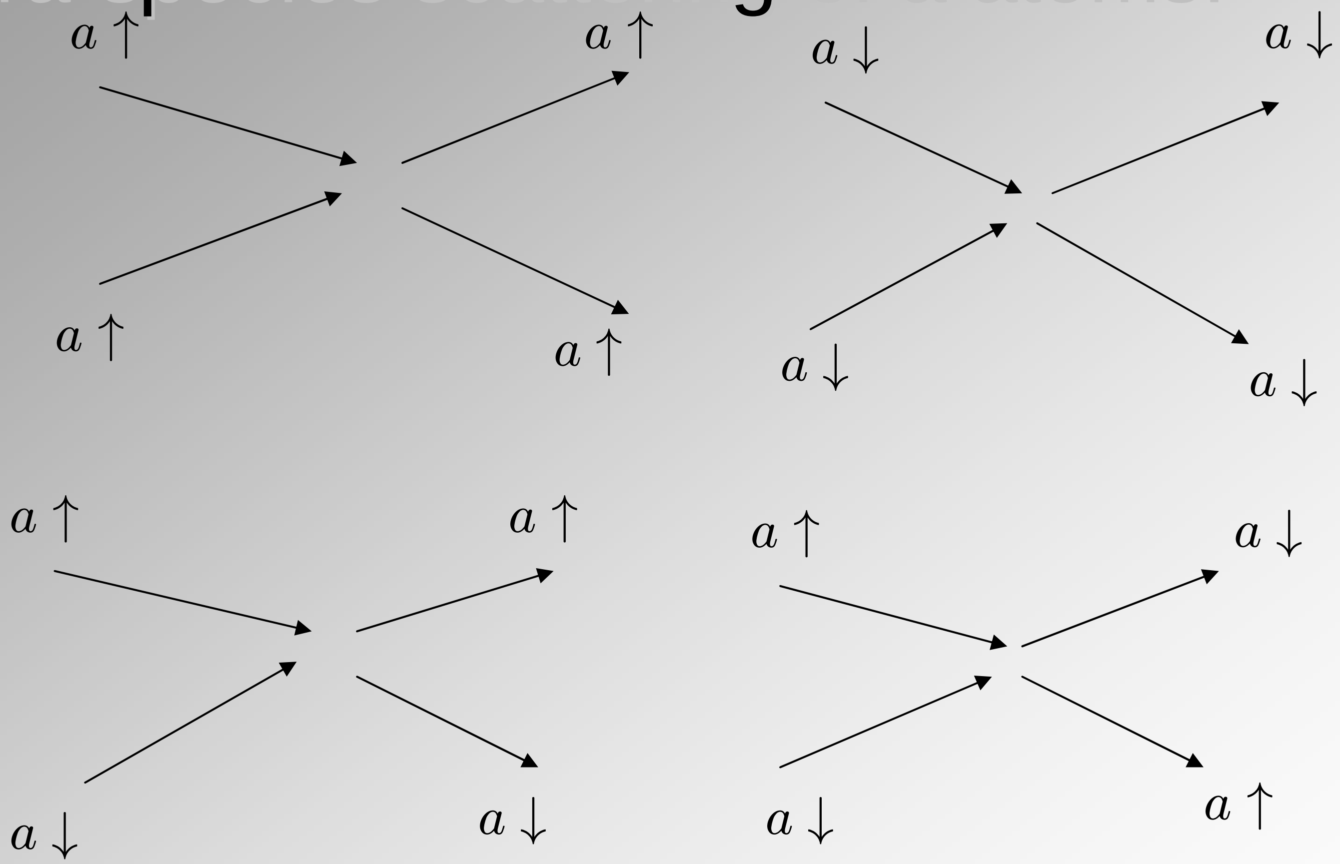
2 species $\times$ 2 internal (spin) states

- Each atom can flip the spin between two states, but cannot transit between the atom species.
- $N_{i\uparrow}$ and $N_{i\downarrow}$ $(i = a, b)$ are not conserved.
  $N_i = N_{i\uparrow} + N_{i\downarrow}$ is conserved.
- Ignore depletion: orbit of each particle is in the lowest wavefunction.
- In a dilute gas, atom-atom interaction is effectively a contact one proportional to scattering length.

Scattering channels,

each described by a scattering length

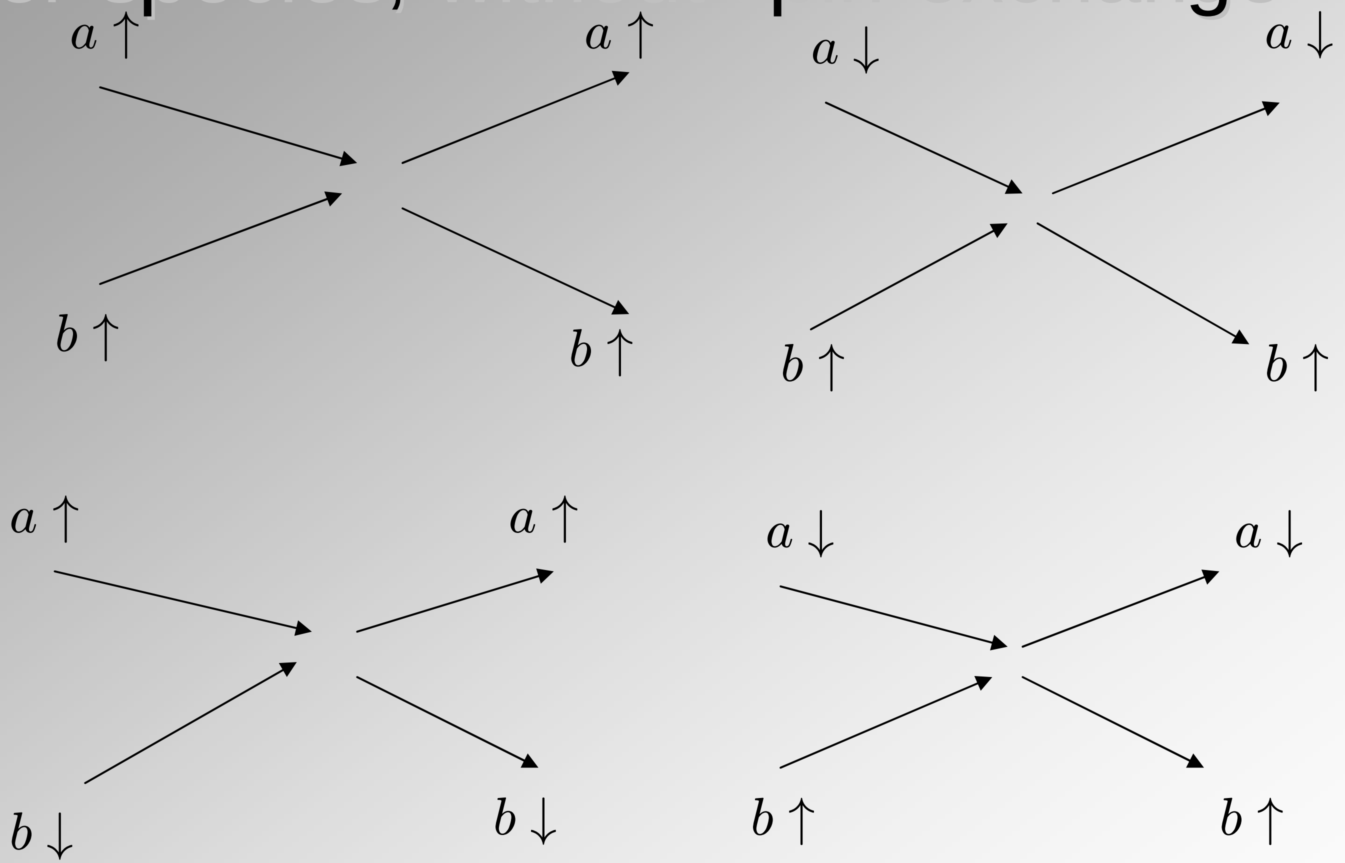
1. Intra-species scattering of a atoms:



2. Similar Intra-species scattering of b atoms.

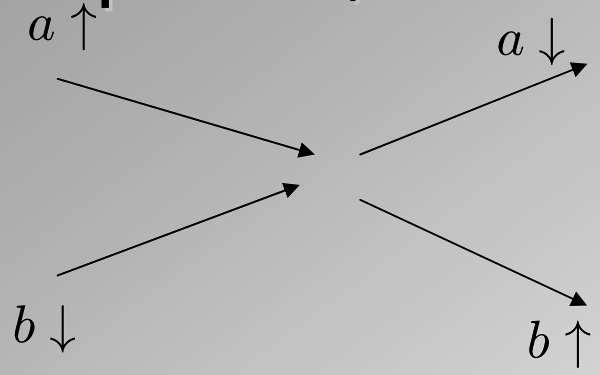
Scattering channels (continued)

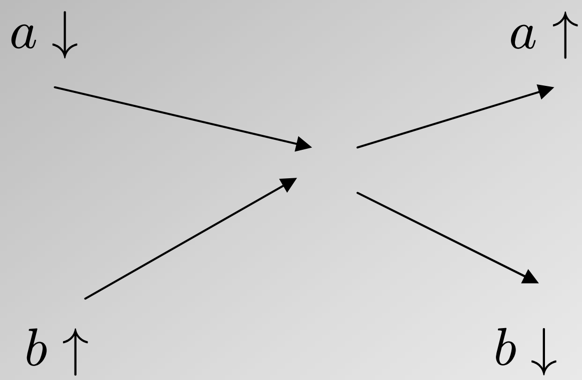
3. Inter-species, without spin-exchange



Scattering channels (continued)

4. Inter-species, with spin-exchange

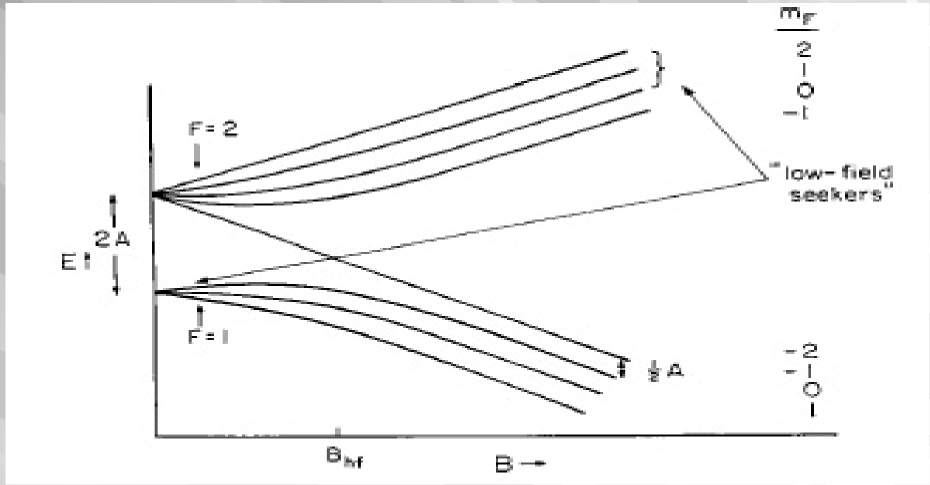




Requirements

- Energy conservation in each scattering.
- Conservation of total z-component spin in each scattering. [This constrains the spin to the two-state subspace].
- They can be satisfied experimentally.

Experimental feasibility



Hyperfine-Zeeman energy

$H = A\mathbf{I}\cdot\mathbf{J} + CJ_z + DI_z$

D is negligible.

In some cases, one can find two internal states with energy difference is independent of atom species.

For example, $|2,\ \S 2\rangle$.

Moreover, when magnetic field is low,

$\Delta E = g_F \Delta m_F \mu_B.$

Hamiltonian

$$\mathcal{H} = \mathcal{H}_a + \mathcal{H}_b + \sum_{\sigma\sigma'} K^{(ab)}_{\sigma\sigma'} N_{a\sigma} N_{b\sigma'} + \frac{K_e}{2}(a^\dagger_\uparrow a_\downarrow b^\dagger_\downarrow b_\uparrow + a^\dagger_\downarrow a_\uparrow b^\dagger_\uparrow b_\downarrow)$$

$K^{(ab)}_{\sigma\sigma}$ is proportional to scattering length of the interspecies scattering conserving each spin.

K_e is proportional to scattering length of the interspecies scattering exchanging the spins.

K_e term causes a-b entanglement in the ground state!

Interesting properties of our model

- Spinful BEC can also be realized in magnetic traps.
- Motivates experiments on multichannel scattering between different species of atoms.
- It positively answers our question above. It realizes, in the ground state, a new quantum state of matter： entanglement between BECs. .

Meaning of “Entanglement between BECs”

- Like a pure two-particle entangled state, where each particle is not in any pure spin state, there is no simple BEC of either species; there is only a global simple BEC.
- BEC occurs in an entangled inter-species pair state. Somewhat analogous to BEC of Cooper pairing.
- The two-particle entangled wavefunction is the order parameter.
- A new kind of BEC characterized by a novel kind of entanglement.

Some details of solution

Spin representation

$$\mathbf{S}_a = \sum_{\sigma,\sigma'} a^\dagger_\sigma \mathbf{s}_{\sigma\sigma'} a_{\sigma'}, \ \mathbf{S}_b = \sum_{\sigma,\sigma'} b^\dagger_\sigma \mathbf{s}_{\sigma\sigma'} b_{\sigma'}$$

- The Hamiltonian becomes that of two giant spins $S_a = N_a/2$ and $S_b = N_b/2$

$$\frac{\mathcal{H}}{J_z} = \frac{K_e}{J_z}(S_{ax}S_{bx} + S_{ay}S_{by}) + S_{az}S_{bz} + B_a S_{az} + B_b S_{bz} + C_a S_{az}^2 + C_b S_{bz}^2 + \frac{E_0}{J_z}$$

- Coefficients are functions of K’s.

Conserved Quantities

- N_a and N_b, hence S_a and S_b

$$N_i = N_{i\uparrow} + N_{i\downarrow}$$

- Total $S_z = (N_{a\uparrow} - N_{a\downarrow} + N_{b\uparrow} - N_{b\downarrow})/2$

Isotropic point

$$\mathcal{H} = J_z \mathbf{S}_a \cdot \mathbf{S}_b$$

Ground states:

$$|G_{S_z}\rangle = A(a_\uparrow^\dagger)^{n_\uparrow}(a_\downarrow^\dagger)^{n_\downarrow}(a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger)^{N_b}|0\rangle$$

$$n_\uparrow = N_a/2 - N_b/2 + S_z,\, n_\downarrow = N_a/2 - N_b/2 - S_z$$

Degenerate, but unique for a given S_z .

For $N_a = N_b = N$:

$$|G_0\rangle = (\sqrt{N+1}N!)^{-1}(a_\uparrow^\dagger b_\downarrow^\dagger - a_\downarrow^\dagger b_\uparrow^\dagger)^N|0\rangle$$

Using entanglement to characterize the non-mean field nature

$$|G_0\rangle = (\sqrt{N+1})^{-1}\sum_{m=0}^{N}(-1)^{N-m}|m\rangle_{a\uparrow}|N-m\rangle_{a\downarrow}|N-m\rangle_{b\uparrow}|m\rangle_{b\downarrow}$$

- Consider its occupation entanglement . The subsystems are the single particle basis states involved.

  Method: YS, Phys.Rev.A 67, 024301 (03);
  J.Phys.A 37,6807 (04).

- Entanglement entropy: von Neumann entropy of the reduced density matrix of a subsystem, which measures the entanglement with the rest of the system.

Entanglement between the species

- The basis of i (i=a,b) species is chosen to be $(i\uparrow, i\downarrow)$.
- The occupation number of the basis of each species [always (m,N-m)] is N+1-valued, so the base of the logarithm in entanglement entropy is set to be N+1.
- The state is an equal superposition of states that are orthogonal in both a and b bases, consequently the entanglement between the two species is 1.

$$|G_0\rangle = (\sqrt{N+1})^{-1}\sum_{m=0}^{N}(-1)^{N-m}|m\rangle_{a\uparrow}|N-m\rangle_{a\downarrow}|N-m\rangle_{b\uparrow}|m\rangle_{b\downarrow}$$

Entanglement as a kind of pairing

- $(a^\dagger_\uparrow b^\dagger_\downarrow - a^\dagger_\downarrow b^\dagger_\uparrow)^N|0\rangle = [\sqrt{2}\int d^3r_a d^3r_b \psi^\dagger_a(\mathbf{r}_a)\psi^\dagger_b(\mathbf{r}_b)\phi(\mathbf{r}_a,\mathbf{r}_b)]^N|0\rangle$

$$\psi_a(\mathbf{r}) = \sum_\sigma a_\sigma \phi_{a\sigma}(\mathbf{r}_a)|\sigma\rangle_a, \quad \psi_b(\mathbf{r}) = \sum_\sigma b_\sigma \phi_{b\sigma}(\mathbf{r}_b)|\sigma\rangle_b$$

$$\boxed{\phi(\mathbf{r}_a,\mathbf{r}_b) = \frac{1}{\sqrt{2}}[\phi_{a\uparrow}(\mathbf{r}_a)|\uparrow\rangle_a\phi_{b\downarrow}(\mathbf{r}_b)|\downarrow\rangle_b - \phi_{a\downarrow}(\mathbf{r}_a)|\downarrow\rangle_a\phi_{b\uparrow}(\mathbf{r}_b)|\uparrow\rangle_b]}$$

- Real space wavefunction:

$$\Psi = \mathcal{N}S[\phi(\mathbf{r}_{a1},\mathbf{r}_{b1})\cdots\phi(\mathbf{r}_{aN},\mathbf{r}_{bn})].$$

$\mathcal{N}$: normalization. S: symmetrization.

- $|G_0\rangle$ is a condensation of interspecies pairs in the same two-particle entangled state.
- $\phi(\mathbf{r}_a,\mathbf{r}_b)$ is the entangled order parameter.
- Analogous to Cooper pairing!

Entangled pairing lowers the energy

A simple example:

$$h(\mathbf{r}_a) + h(\mathbf{r}_b) + U_1(\mathbf{r}_a - \mathbf{r}_b) + U_2(\mathbf{r}_a - \mathbf{r}_b)(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow|)$$

$$U_2 > 0$$

$$\phi_a(\mathbf{r}_a)\phi_b(\mathbf{r}_b)(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

has lower energy than

$$\phi_a(\mathbf{r}_a)\phi_b(\mathbf{r}_b)|\sigma\rangle|\sigma'\rangle$$

How the entanglement survives the coupling anisotropy and the nonvanishing of

$$B_a,\ B_b,\ C_a\ ,\ C_b$$

$$\frac{\mathcal{H}}{J_z} = \frac{K_e}{J_z}(S_{ax}S_{bx} + S_{ay}S_{by}) + S_{az}S_{bz} + B_a S_{az} + B_b S_{bz} + C_a S_{az}^2 + C_b S_{bz}^2 + \frac{E_0}{J_z}$$

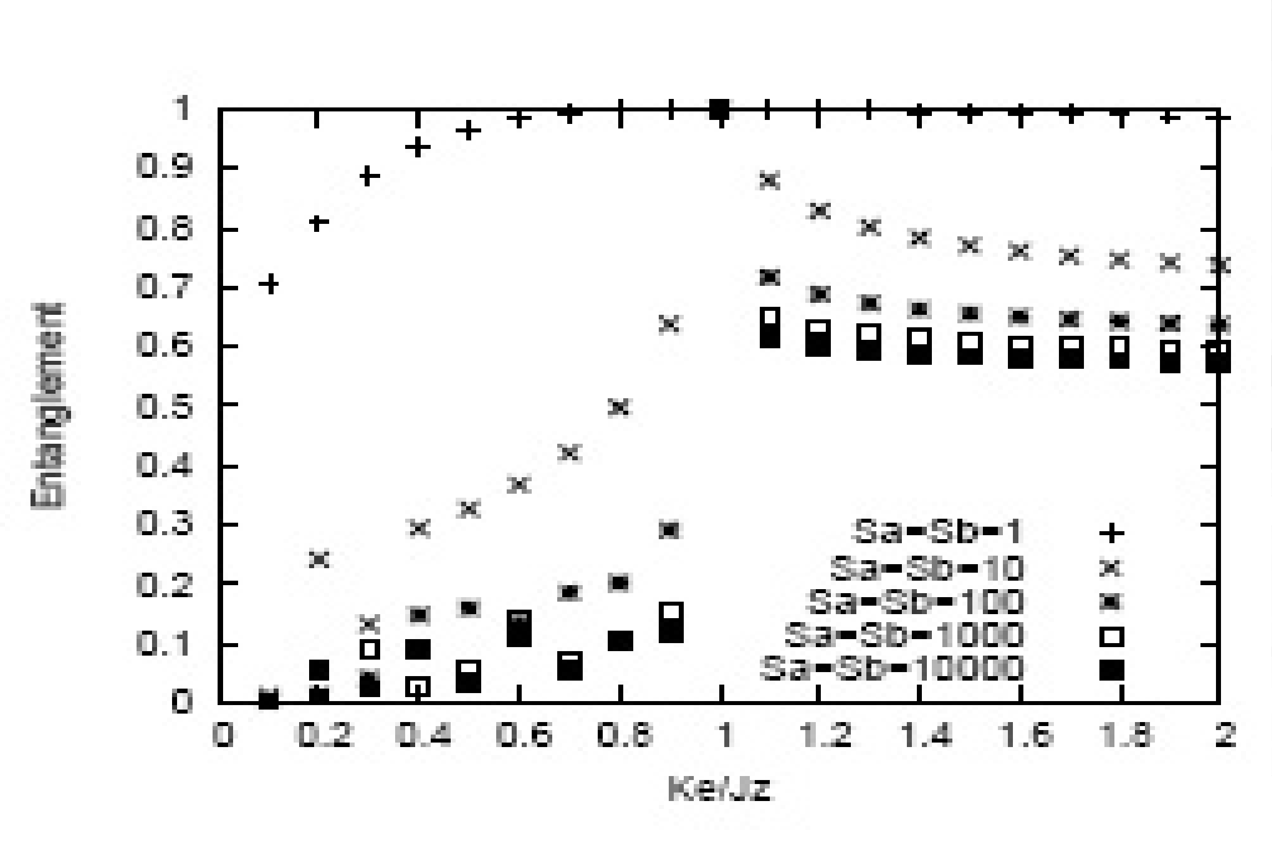
- Use the Lanczos method to find the ground state numerically.

It is found that in a wide parameter regime, the ground state is of non-mean-field with significant entanglement.

Persistence of entanglement in a wide parameter regime (1)

- Coupling anisotropy K_e/J_z , $B_a = B_b = C_a = C_b = 0$

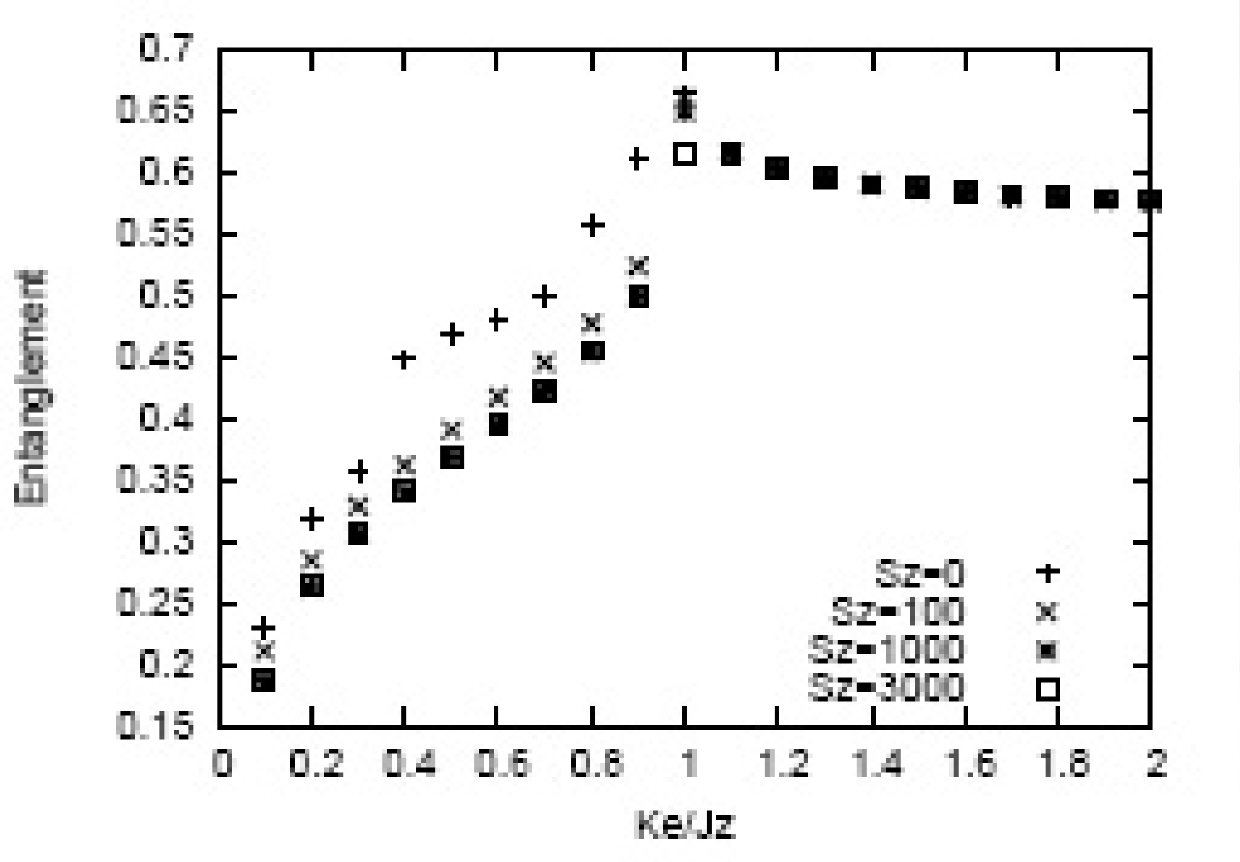
  $S_z = 0$



Persistence of entanglement in a wide parameter regime (2)

- Coupling anisotropy K_e/J_z $\quad B_a = B_b = C_a = C_b = 0$

$$S_a = 12000,\ S_b = 10000$$

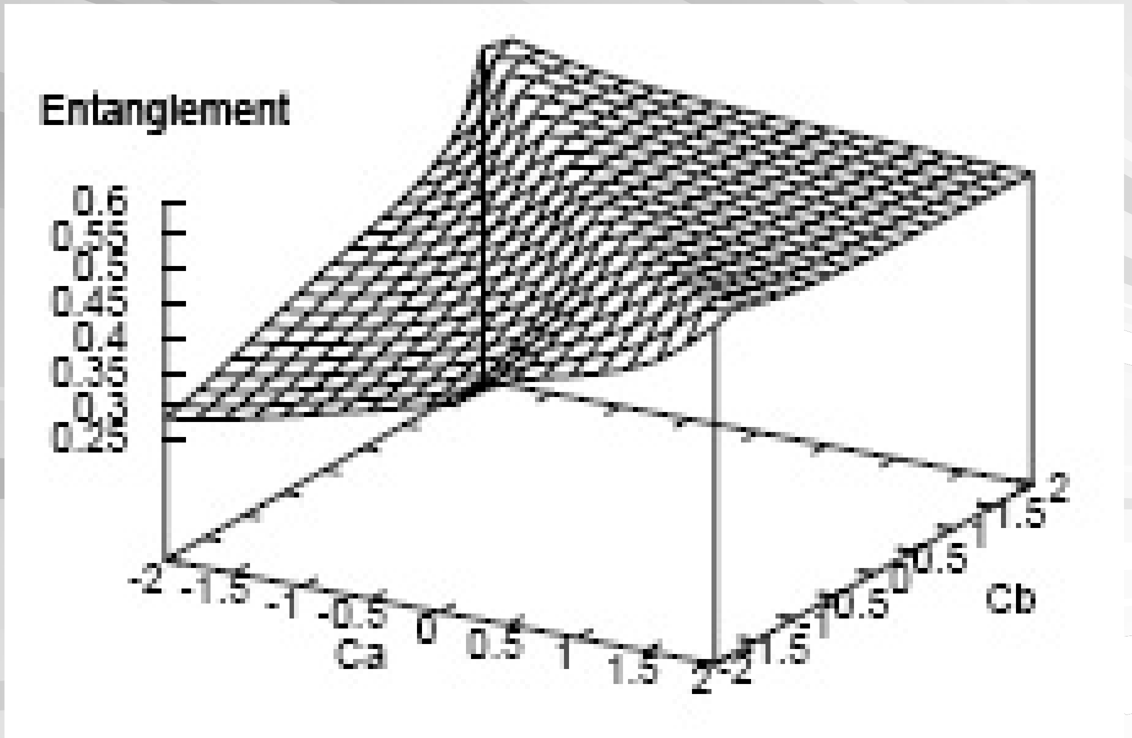


Persistence of entanglement in a wide parameter regime (3)

- C_a and C_b nonzero, $B_a = B_b = 0$, under typical values

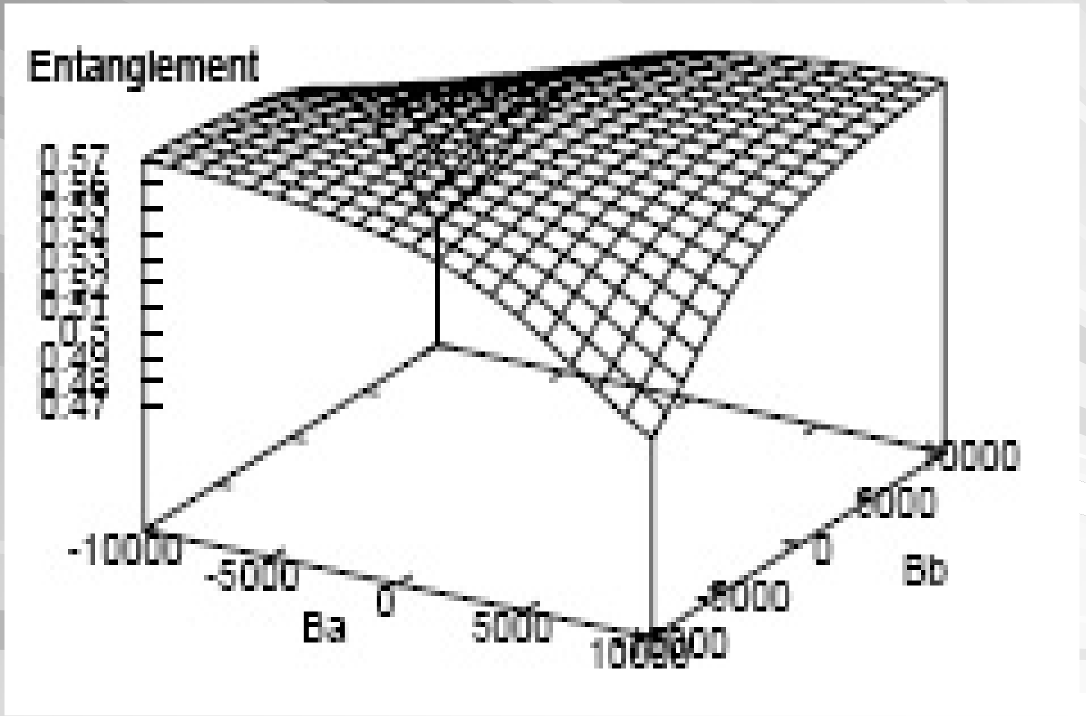
  $S_a = 12000$, $S_b = 10000$, $S_z = 1000$, $K_e/J_z = 1.2$

  J_z, C_aJ_z and C_bJ_z are of the same order of magnitude



Persistence of entanglement in a wide parameter regime (4)

Typically choose $C_a = 0.2$ and $C_b = 0.4$



Detection of the entanglement (1)

- Fluctuations of $N_{i\sigma}$

$$\sqrt{\langle N_{a\sigma}^2 \rangle - \langle N_{a\sigma} \rangle^2} / \langle N_{a\sigma} \rangle \neq 0$$

- Can be obtained from density fluctuation, which is self-averaging, and can be studied in a single optical image

$$\rho_{i\sigma}(\mathbf{r}_i) = N_{i\sigma} |\phi_{i\sigma}(\mathbf{r}_i)|^2$$

$$\sqrt{\langle \rho_{i\sigma}(\mathbf{r}_i)^2 \rangle - \langle \rho_{i\sigma}(\mathbf{r}_i) \rangle^2} / \langle \rho_{i\sigma}(\mathbf{r}_i) \rangle = \sqrt{\langle N_{i\sigma}^2 \rangle - \langle N_{i\sigma} \rangle^2} / \langle N_{i\sigma} \rangle$$

Detection of the entanglement (2)

- Nonvanishing of the connected correlations

$$C_{\sigma,\sigma'} = \langle N_{a\sigma} N_{b\sigma'} \rangle - \langle N_{a\sigma} \rangle \langle N_{b\sigma'} \rangle$$

$$g(\mathbf{r}_a, \sigma; \mathbf{r}_b, \sigma') = \langle \rho_{a\sigma}(\mathbf{r}_a) \rho_{b\sigma'}(\mathbf{r}_b) \rangle - \langle \rho_{a\sigma}(\mathbf{r}_a) \rangle \langle \rho_{b\sigma'}(\mathbf{r}_b) \rangle$$

$$g(\mathbf{r}_a, \sigma; \mathbf{r}_b, \sigma') / \langle \rho_{a\sigma}(\mathbf{r}_a) \rangle \langle \rho_{b\sigma'}(\mathbf{r}_b) \rangle = C_{\sigma,\sigma'} / \langle N_{a\sigma} \rangle \langle N_{b\sigma'} \rangle$$

Detection of entanglement (3)

- Measuring spin of an A-atom,

$$P_\sigma = \langle a^\dagger_\sigma a_\sigma \rangle / \sum_{\sigma'} \langle a^\dagger_{\sigma'} a_{\sigma'} \rangle$$

- Joint measurement of the spins of an A-atom and a B-atom which leave the trap

$$P_{\sigma,\sigma'} = \langle b^\dagger_{\sigma'} a^\dagger_\sigma a_\sigma b_{\sigma'} \rangle / \sum_{\sigma_a,\sigma_b} \langle b^\dagger_{\sigma_b} a^\dagger_{\sigma_a} a_{\sigma_a} b_{\sigma_b} \rangle.$$

Detection of entanglement (3) (continued)

- Mean-field (non-entangled) state:

$$(\sqrt{N_1!N_2!N_3!N_4!})^{-1} a^{\dagger}_{\hat{\mathbf{n}}}{}^{N_1} a^{\dagger}_{-\hat{\mathbf{n}}}{}^{N_2} b^{\dagger}_{\hat{\mathbf{m}}}{}^{N_3} b^{\dagger}_{-\hat{\mathbf{m}}}{}^{N_4} |0\rangle,$$

$$P_{\sigma_a,\sigma_b} = P_{\sigma_a} P_{\sigma_b}$$

- Non-mean-field (entangled) BEC:

$$P_{\sigma_a,\sigma_b} \neq P_{\sigma_a} P_{\sigma_b}$$

Summary

We proposed a non-mean-field ground state of BEC, occurring in an interspecies two-particle entangled state. A new state of quantum matter.

- The (macroscopic) order parameter is entangled. We extend the notion of entanglement from usual quantum mechanics to order parameter of a many-particle system. This is a new concept.
- A nice extension of Josephson physics.
- Motivates experimental study of interspecies multichannel scattering.

Thank you for your attention!

多谢关注！