Phase transitions in a strongly interacting Bose gas

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4. Coherent mixture of atoms and molecules

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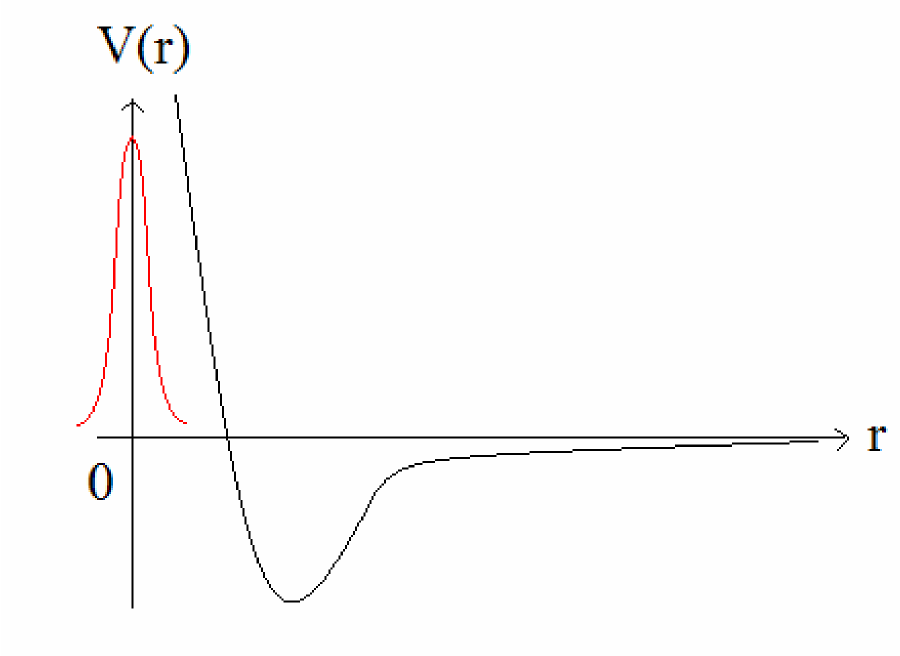
1. Introduction

(1) Tradition theory of dilute Bose gases



S-wave phase shift

$$\delta_0 \to -ka$$

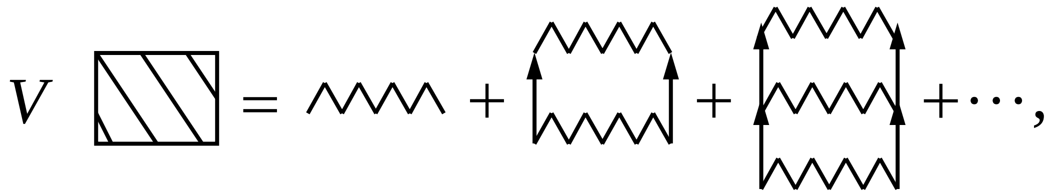
Renormalization of the hard-core potential

$$V(\mathbf{r}) = 4\pi \frac{\hbar^2}{m} a \delta^3(r)$$

(Fermi, Lee & Yang & Huang, Galitskii, Beliaev)

Beliaev's Theory (Homogeneous, T=0):

- The effective potential and the scattering amplitude satisfy similar integral equations.



- Perturbation in gas parameter $\sqrt{8\pi n_0 a^3}$.
- Beliaev damping of the phonon mode $\Gamma_k = \dfrac{3\hbar^2 k^5}{640\pi m n_0}$.

(One loop)

Hugenholtz and Pines Theorem:

The quasi-particle energy is gapless. (Valid in all orders)

Bogoliubov's Theory:

Hamiltonian $H = -\psi^{+}\frac{\hbar^2\nabla^2}{2m}\psi + \frac{g}{2}\psi^{+}\psi^{+}\psi\psi$

Condensation $\langle\psi\rangle = \psi_0$, and fluctuation $\delta\psi \equiv \psi - \psi_0$

Quadratic Hamiltonian

$$H_2 = \sum_{\mathbf{k}}{}'(\varepsilon_{\mathbf{k}} + gn_0)\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}} + \frac{gn_0}{2}\sum_{\mathbf{k}}{}'(\psi_{-\mathbf{k}}^{+}\psi_{\mathbf{k}}^{+} + \psi_{\mathbf{k}}\psi_{-\mathbf{k}})$$

Applying Bogoliubov Transformation $b_{\mathbf{k}} = u_k\psi_{\mathbf{k}} - v_k\psi_{-\mathbf{k}}^{+}$

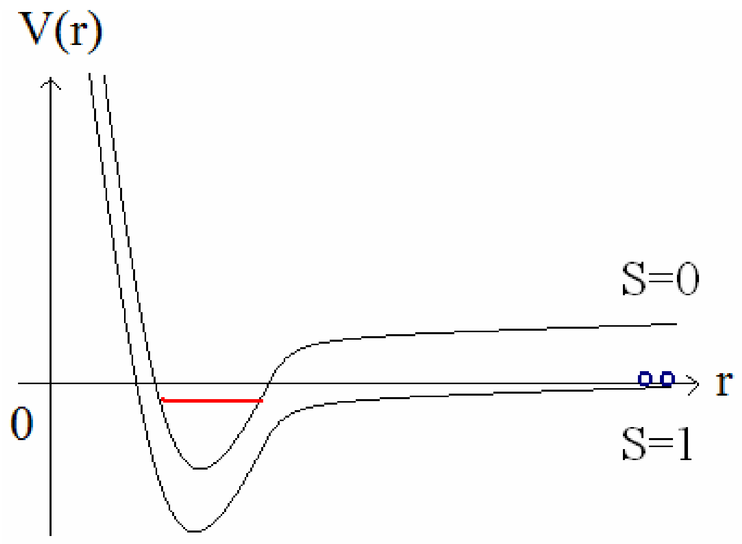
$$H_2 = C + \sum_{\mathbf{k}}{}' E_k b_{\mathbf{k}}^{+} b_{\mathbf{k}}$$

Quasi-particle energy $E_k = \sqrt{\varepsilon_k(\varepsilon_k + 2gn_0)}$

Quantum depletion $n - n_0 = \frac{1}{V}\sum_{\mathbf{k}}{}'\langle\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}}\rangle = \frac{8}{3}\sqrt{\frac{n_0^3a^3}{\pi}}$

(2) Results about strongly interacting Bose gases

Feshbach resonace

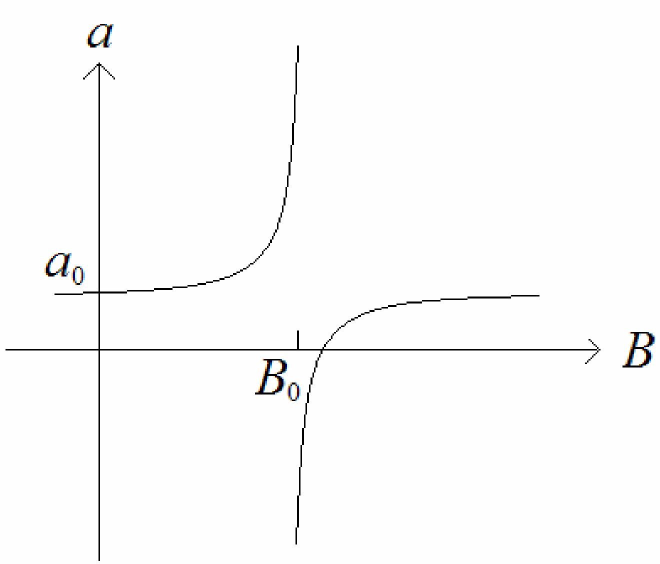


Atom $\mu_a \approx \mu_B$ (triplet)

Diatomic molecule $\mu_m \approx 0$ (singlet)

Zeeman-energy difference

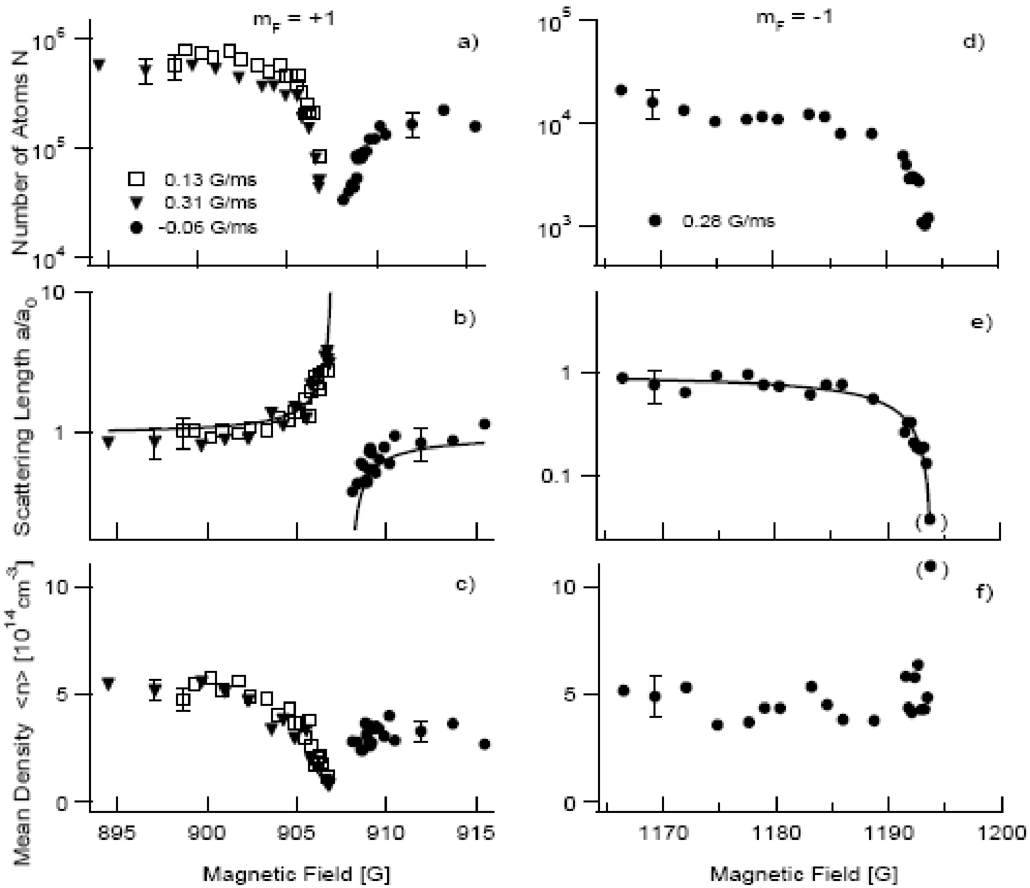
$$\Delta E_Z \approx \mu_B B$$

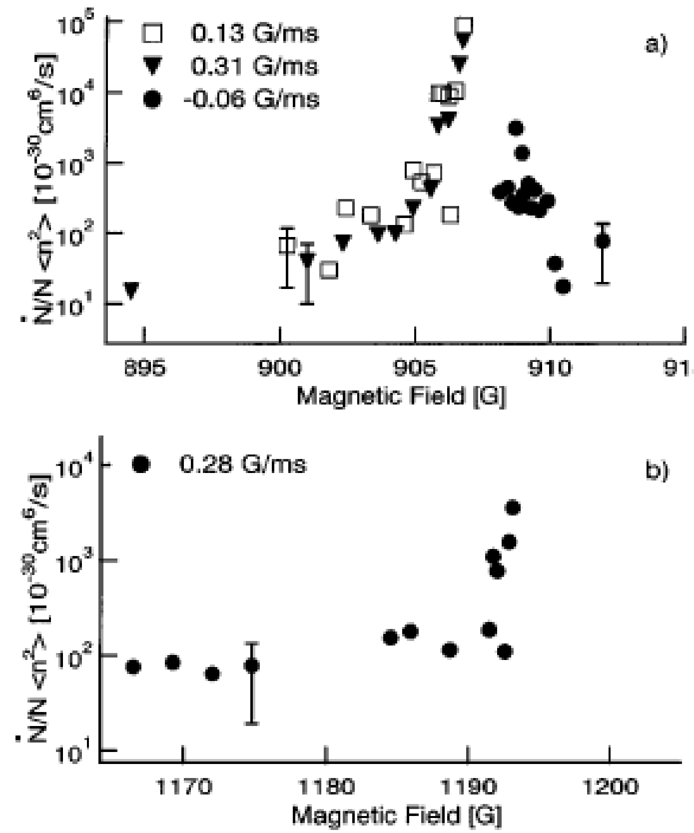


Scattering length can be tuned by magnetic field.

$$a = a_0\left(1 - \frac{\Delta}{B - B_0}\right)$$

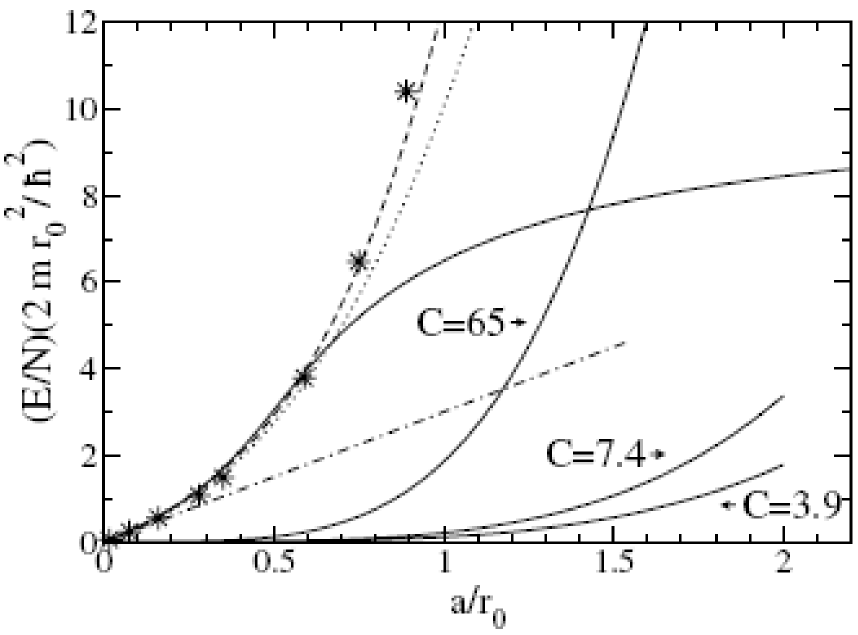
Large atom loss rate near resonance

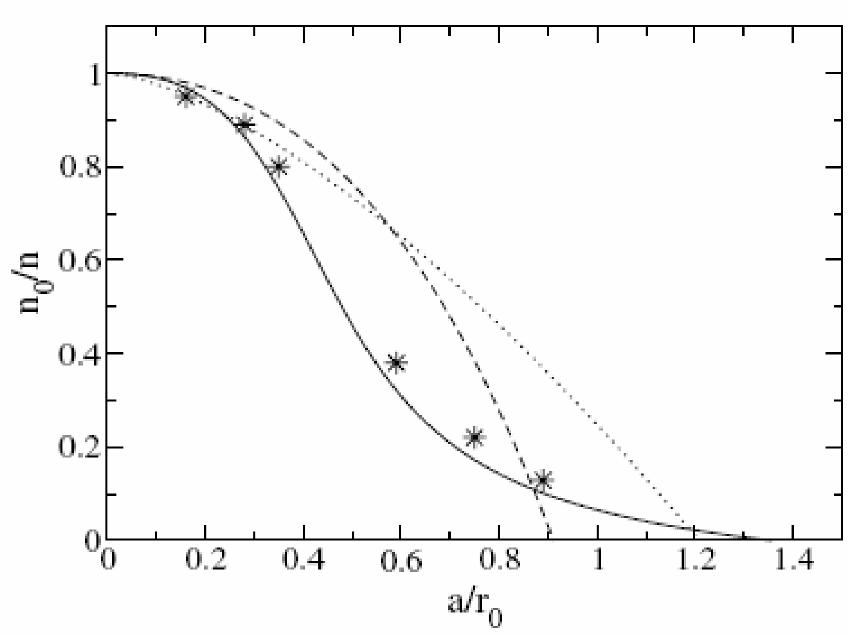




(Ketterle *et al* 98, 99)

LOCV Result

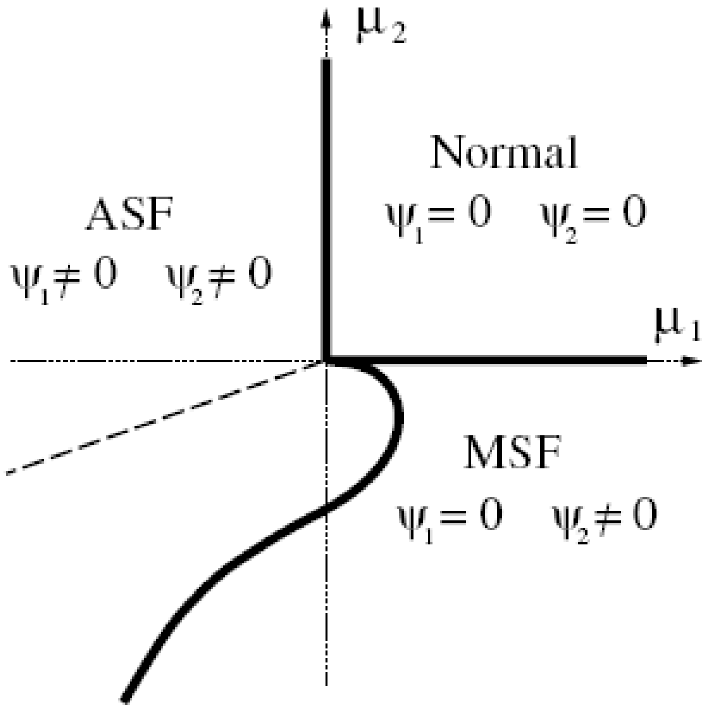




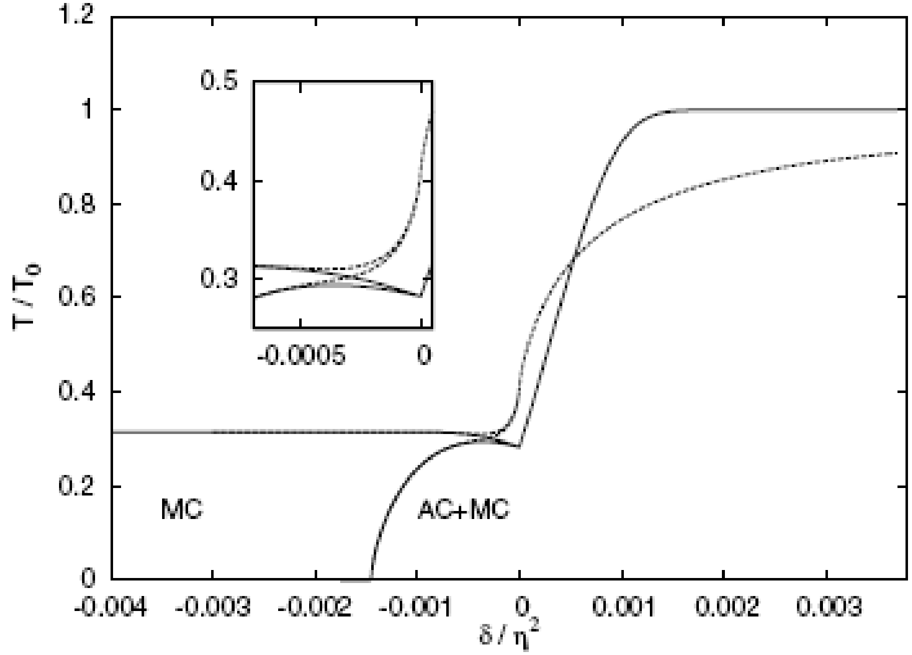
At resonance $$E/N = \frac{\lambda}{2} = \left(\frac{3}{2}\right)^{2/3} \frac{\hbar^2 (kd)^2}{2mr_0^2} = 13.33\hbar^2\,\frac{n^{2/3}}{m}\,.$$

(Pethick *et al* 02)

Transition between atomic and molecular condensations



(Radzihovsky *et al* 04)

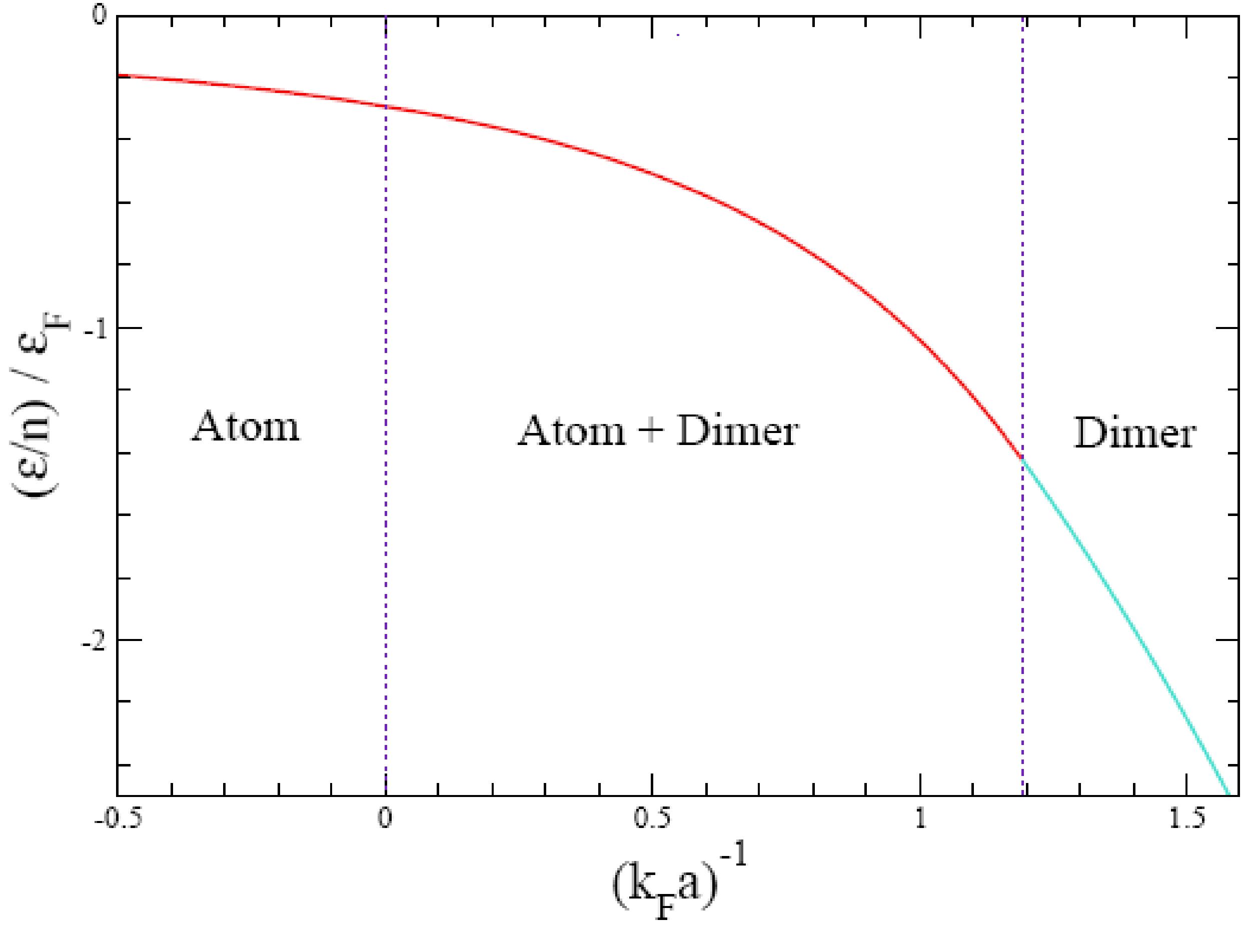


(Stoof *et al* 04)

Negative compressibility of atomic condensation (a<0)

(Mueller *et al* 05)

Braaten's Theory (07)



2. Instability of Atomic condensation (*a*>0)

Hamiltonian $H = -\psi^{+}\frac{\hbar^2\nabla^2}{2m}\psi + \frac{g}{2}\psi^{+}\psi^{+}\psi\psi, \quad g \equiv 4\pi\frac{\hbar^2 a}{m}$

Condensation $\langle\psi\rangle = \psi_0$, and fluctuation $\delta\psi \equiv \psi - \psi_0$

Quadratic Hamiltonian

$$H_2 = \sum_{\mathbf{k}}{}'(\varepsilon_{\mathbf{k}} + gn_0)\psi^{+}_{\mathbf{k}}\psi_{\mathbf{k}} + \frac{gn_0}{2}\sum_{\mathbf{k}}{}'(\psi^{+}_{-\mathbf{k}}\psi^{+}_{\mathbf{k}} + \psi_{\mathbf{k}}\psi_{-\mathbf{k}})$$

Bogoliubov Transformation $b_{\mathbf{k}} = u_k\psi_{\mathbf{k}} - v_k\psi^{+}_{-\mathbf{k}}$

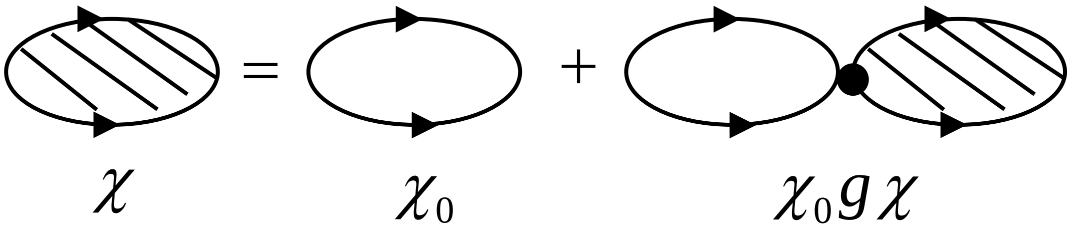
$$H_2 = C + \sum_{\mathbf{k}}{}' E_k b^{+}_{\mathbf{k}} b_{\mathbf{k}}$$

Quasi-particle energy $E_k = \sqrt{\varepsilon_k(\varepsilon_k + 2gn_0)}$

Quantum depletion $n - n_0 = \frac{1}{V}\sum_{\mathbf{k}}{}'\langle\psi^{+}_{\mathbf{k}}\psi_{\mathbf{k}}\rangle = \frac{8}{3}\sqrt{\frac{n_0^3 a^3}{\pi}}$

The spectrum of molecular excitations are given by the poles of two-particle correlation functions.

In RPA approximation $\chi = (1 - g\chi_0)^{-1}\chi_0$

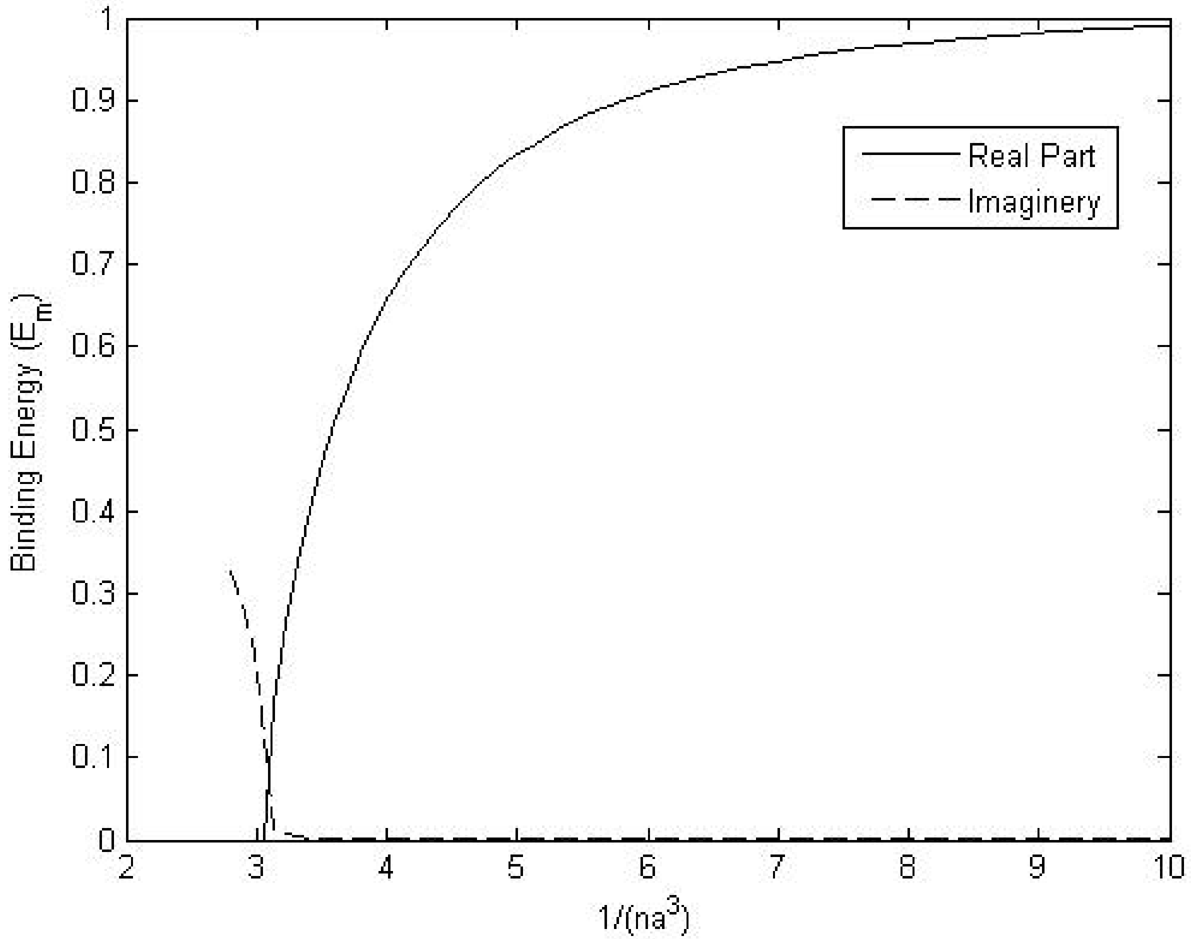


Correlation function $\chi_{ij}(\mathbf{k},\omega) \equiv \left\langle T\{d_i(\mathbf{k},\omega)d_j(-\mathbf{k},-\omega)\}\right\rangle$

$$d_1(\mathbf{k}) \equiv \sum_{\mathbf{k}'}\psi_{\mathbf{k}'}\psi_{\mathbf{k}-\mathbf{k}'}, \quad d_2(\mathbf{k}) \equiv d_1^+(-\mathbf{k}), \quad d_3(\mathbf{k}) \equiv \sum_{\mathbf{k}'}\psi^+_{-\mathbf{k}'}\psi_{\mathbf{k}-\mathbf{k}'}$$

Dispersion Equation $\det\left|\chi(\mathbf{k},\omega)\right| = 0$

The molecule binding energy



$$E_m \equiv \frac{\hbar^2}{ma^2}$$

When $na^3 > 0.035$, the excitation energy is purely **imaginary** and the atomic condensation state is **unstable**.

3. The molecular condensation state

Hamiltonian $H = -\psi^{+}\frac{\hbar^2\nabla^2}{2m}\psi + \frac{g}{2}\psi^{+}\psi^{+}\psi\psi, \quad g \equiv 4\pi\frac{\hbar^2 a}{m}$

Pair condensation $\Delta \equiv \frac{g}{V}\sum_{\mathbf{k}}\langle\psi_{\mathbf{k}}\psi_{-\mathbf{k}}\rangle$

Mean-field Hamiltonian

$$H_2 = \sum_{\mathbf{k}}(\varepsilon_{\mathbf{k}} - \mu)\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k}}(\Delta\psi_{-\mathbf{k}}^{+}\psi_{\mathbf{k}}^{+} + \Delta^{*}\psi_{\mathbf{k}}\psi_{-\mathbf{k}})$$

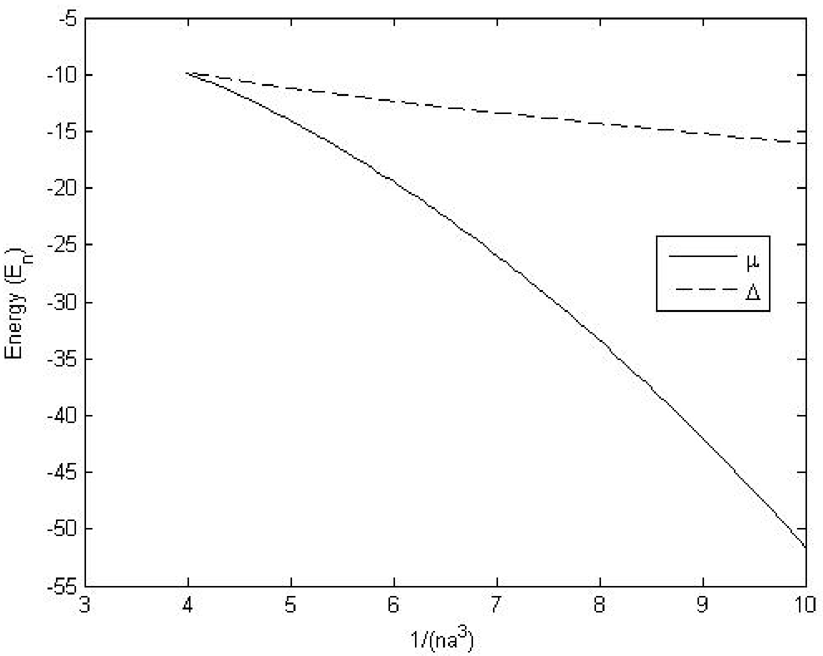
Bogoliubov Transformation $b_{\mathbf{k}} = u_k\psi_{\mathbf{k}} - v_k\psi_{-\mathbf{k}}^{+}$

$$H_2 = C + \sum_{\mathbf{k}} E_k b_{\mathbf{k}}^{+} b_{\mathbf{k}}$$

Quasi-particle energy $E_k = \sqrt{(\varepsilon_k - \mu)^2 - |\Delta|^2}$

Number Equation $N = \sum_{\mathbf{k}}\langle\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}}\rangle$

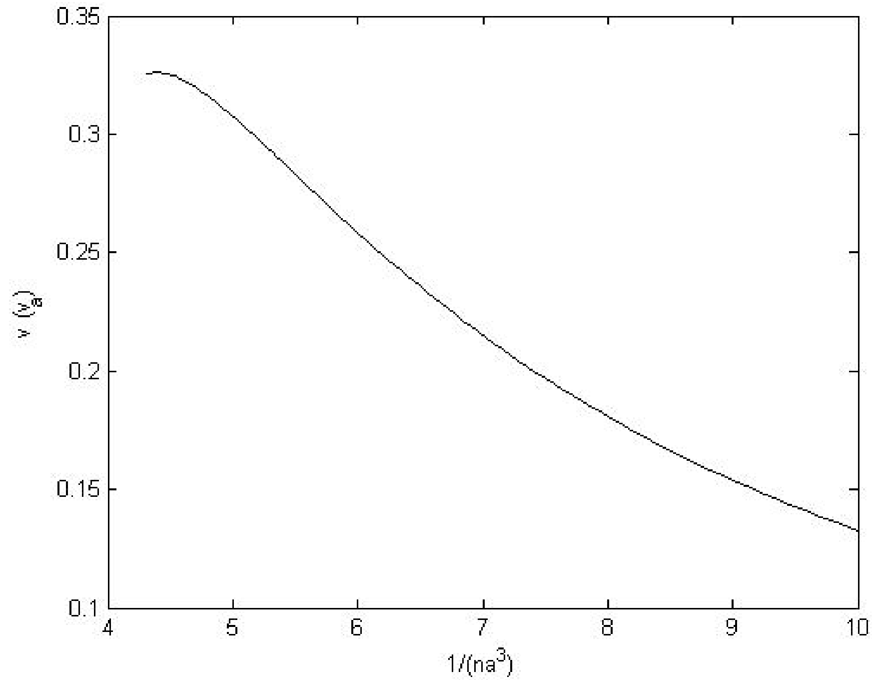
The gap



$$E_n \equiv \frac{\hbar^2 n^{2/3}}{m}$$

The solution only exists when $1/(n^{1/3}a)>4$.

The molecule velocity



$$v_a \equiv \frac{\hbar}{ma}$$

Molecule energy near $k=0$,

$$\Omega_k \approx vk$$

4. Coherent mixture of atoms and molecules

Hamiltonian $H = -\psi^{+}\frac{\hbar^2\nabla^2}{2m}\psi + \frac{g}{2}\psi^{+}\psi^{+}\psi\psi, \quad g \equiv 4\pi\frac{\hbar^2 a}{m}$

Atom and molecule condensation $\Delta \equiv \frac{g}{V}[\psi_0^2 + \sum_{\mathbf{k}}{}'\langle\psi_{\mathbf{k}}\psi_{-\mathbf{k}}\rangle]$

Mean-field Hamiltonian

$$H_2 = \sum_{\mathbf{k}}(\varepsilon_{\mathbf{k}} - \mu)\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k}}(\Delta\psi_{-\mathbf{k}}^{+}\psi_{\mathbf{k}}^{+} + \Delta^{*}\psi_{\mathbf{k}}\psi_{-\mathbf{k}})$$

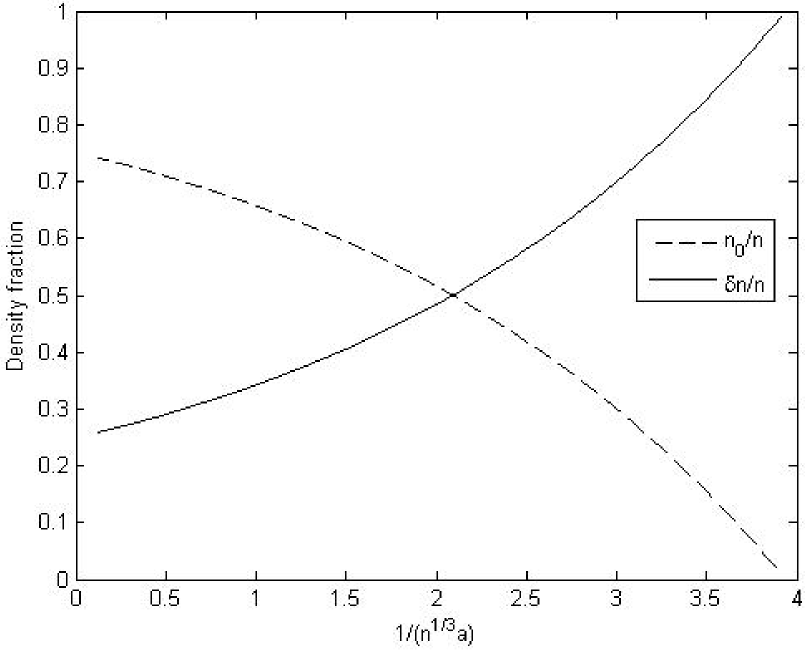
Gapless condition $|\Delta| = |\mu|$

Bogoliubov Transformation $b_{\mathbf{k}} = u_k\psi_{\mathbf{k}} - v_k\psi_{-\mathbf{k}}^{+}$

$$H_2 = C + \sum_{\mathbf{k}} E_k b_{\mathbf{k}}^{+} b_{\mathbf{k}}, \quad E_k = \sqrt{\varepsilon_k(\varepsilon_k + 2|\Delta|)}$$

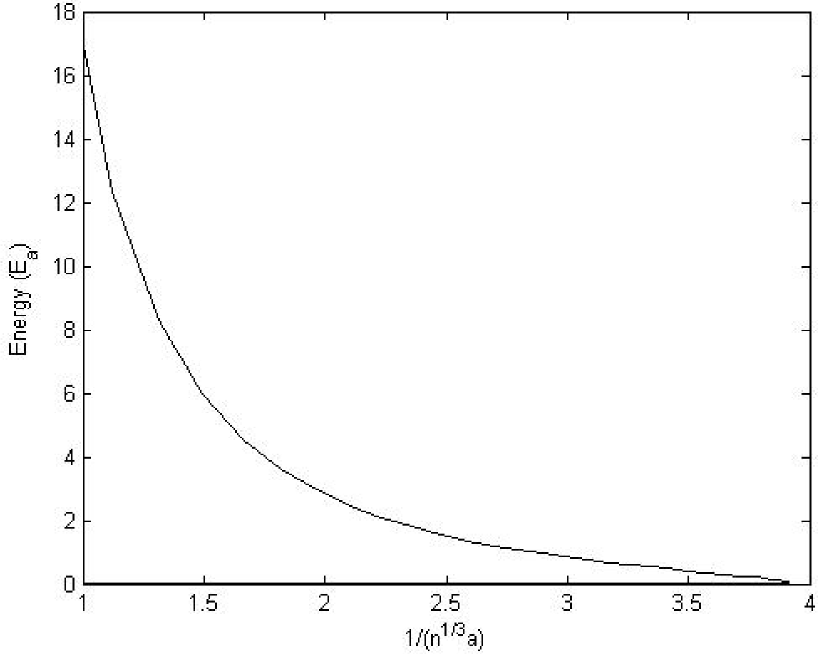
Number Equation $N = N_0 + \sum_{\mathbf{k}}{}'\langle\psi_{\mathbf{k}}^{+}\psi_{\mathbf{k}}\rangle$

Density fraction



At resonance, the condensate density is ¾ of total density.

Molecule Energy



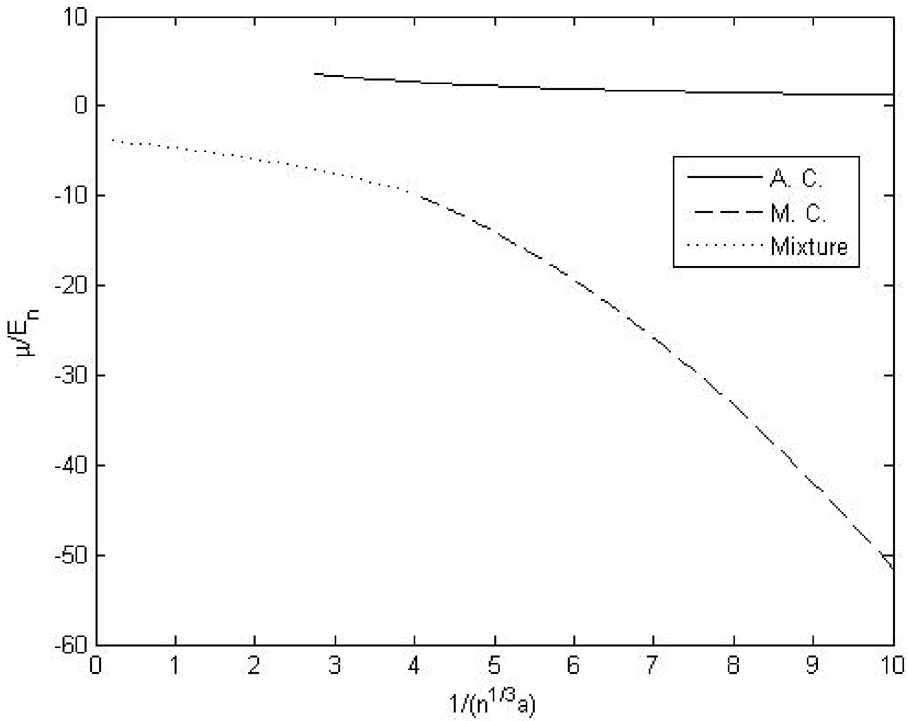
$$E_a \equiv \frac{\hbar^2}{ma^2}$$

Molecular excitation opens a gap.

The signs of ψ_0^2 and $\langle \psi_{\mathbf{k}} \psi_{-\mathbf{k}} \rangle$ are opposite.

5. Conclusion

We have studied three possible phases of a repulsive Bose gas. The atomic condensation state is metastable when $na^3<0.035$. There is a transition between the molecular condensation and the mixture states when $na^3=0.015$. The atomic excitation is gapless in the AC and mixture states. The molecular excitation is gapless in the MC state.



$$E_n \equiv \frac{\hbar^2 n^{2/3}}{m}$$