Spin mixing dynamics in an atomic spin-1 Bose condensate

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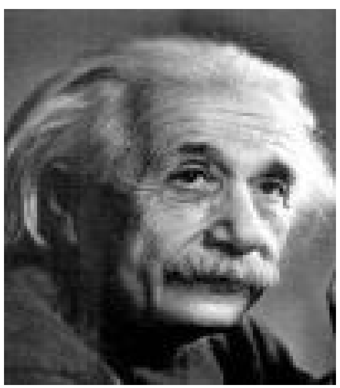
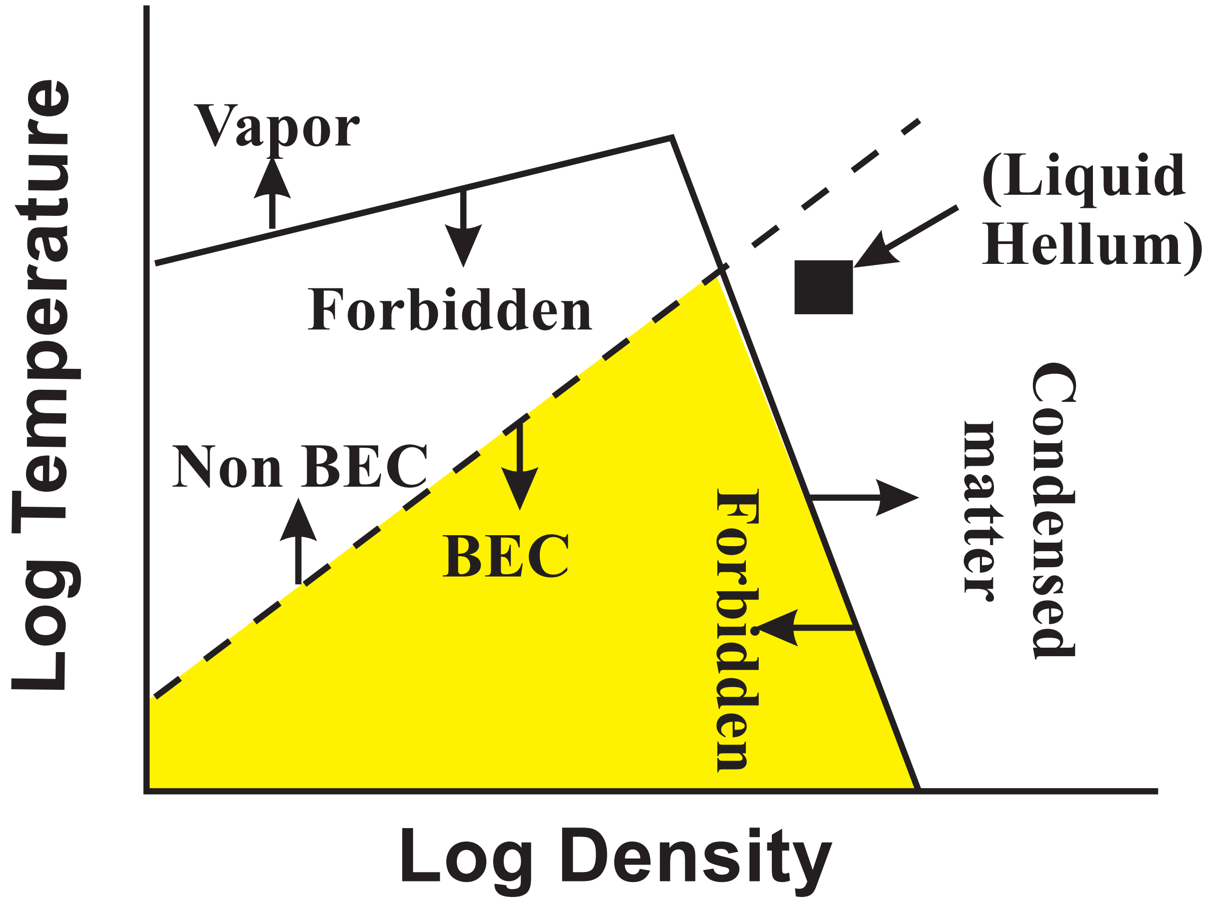
Outline

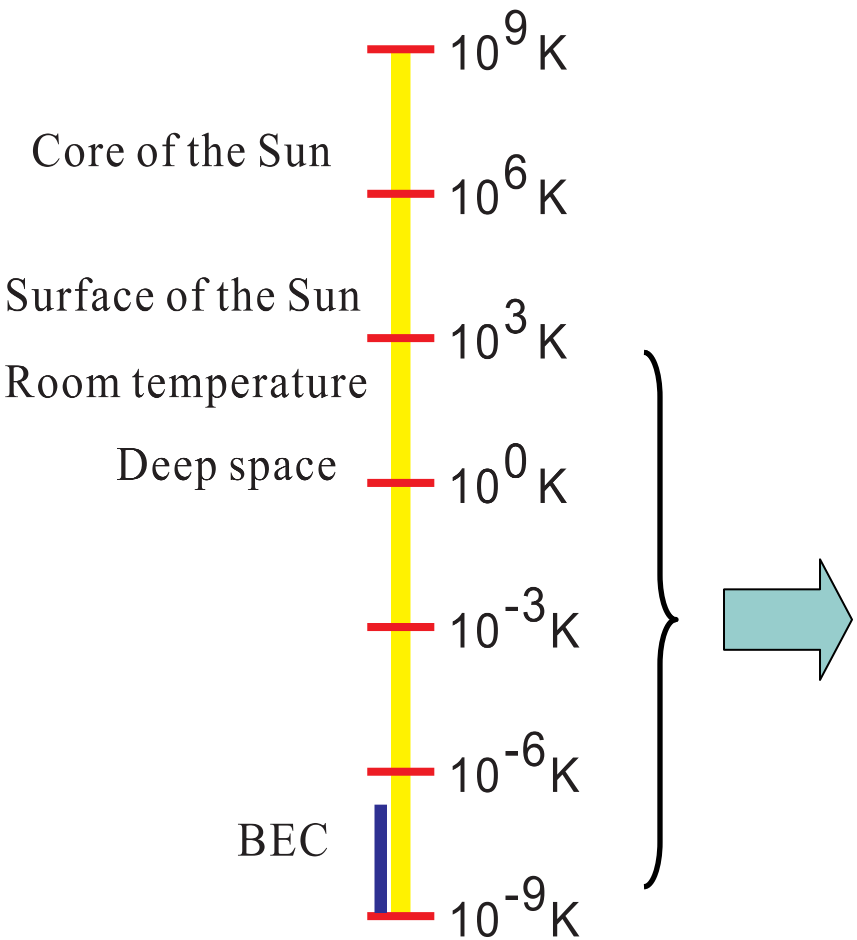
- Bose-Einstein condensation (BEC)
- Interacting spin-1 atomic gases
- Spin mixing dynamics under SMA
- Dynamical instability induced spontaneous spin domains
- Summary

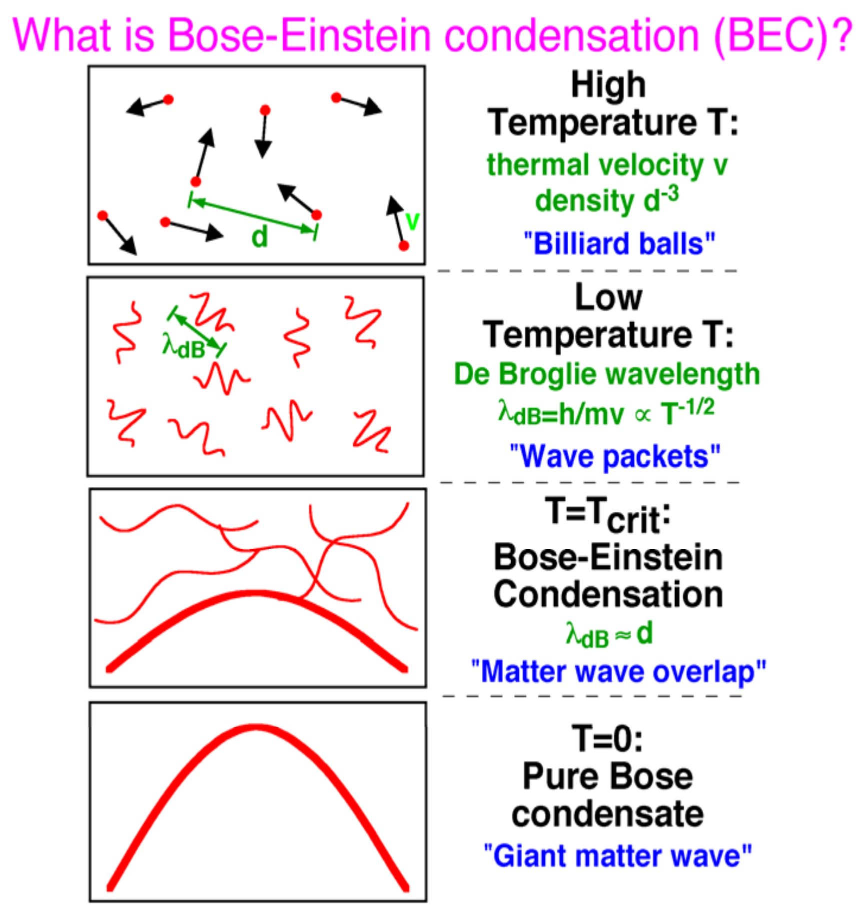
BEC in phase diagram





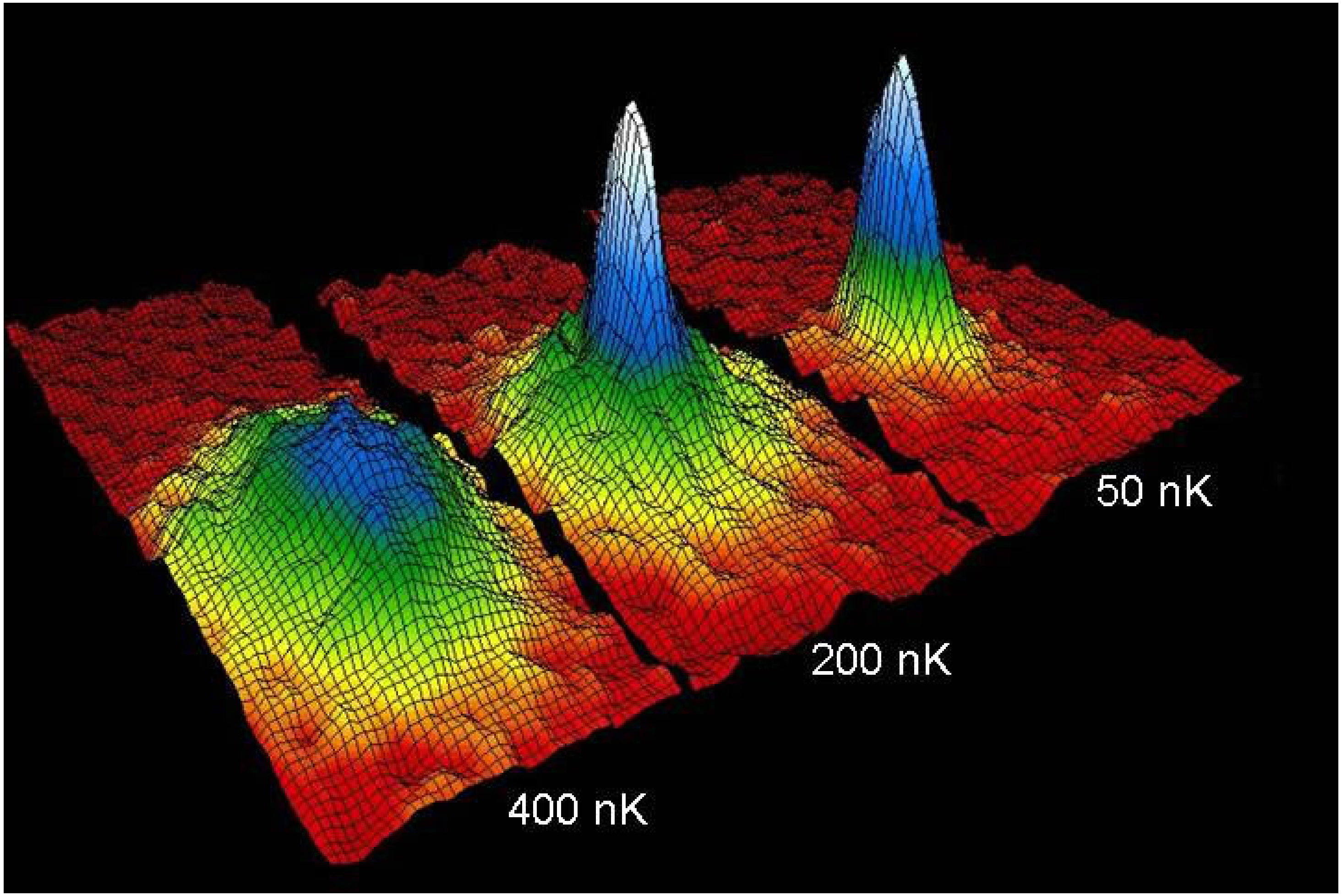
Roadmap to BEC





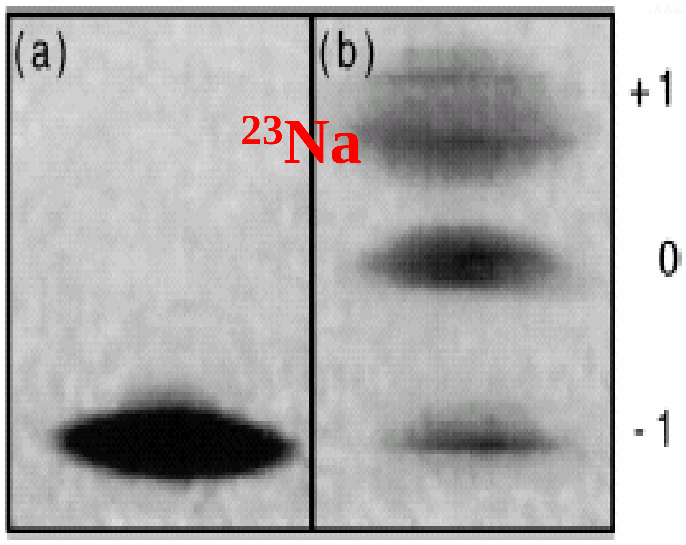
Nobel lecture, Ketterle

Signal of BEC

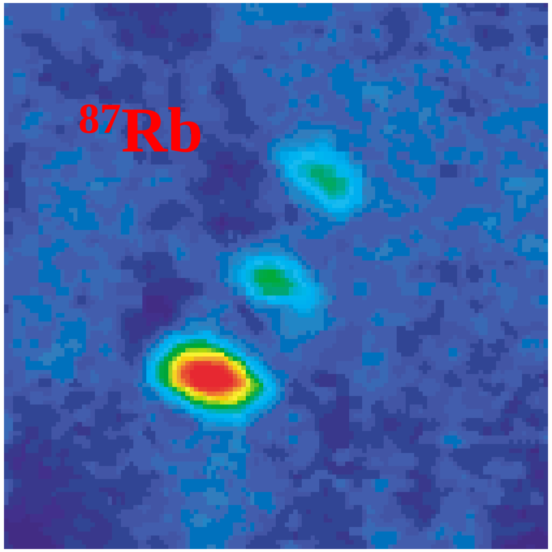


JILA

Spin-1 BEC experiments



PRL **80**, 2027 (1998).



PRL **87**, 010404 (2001).

- MIT, W. Ketterle (1998)

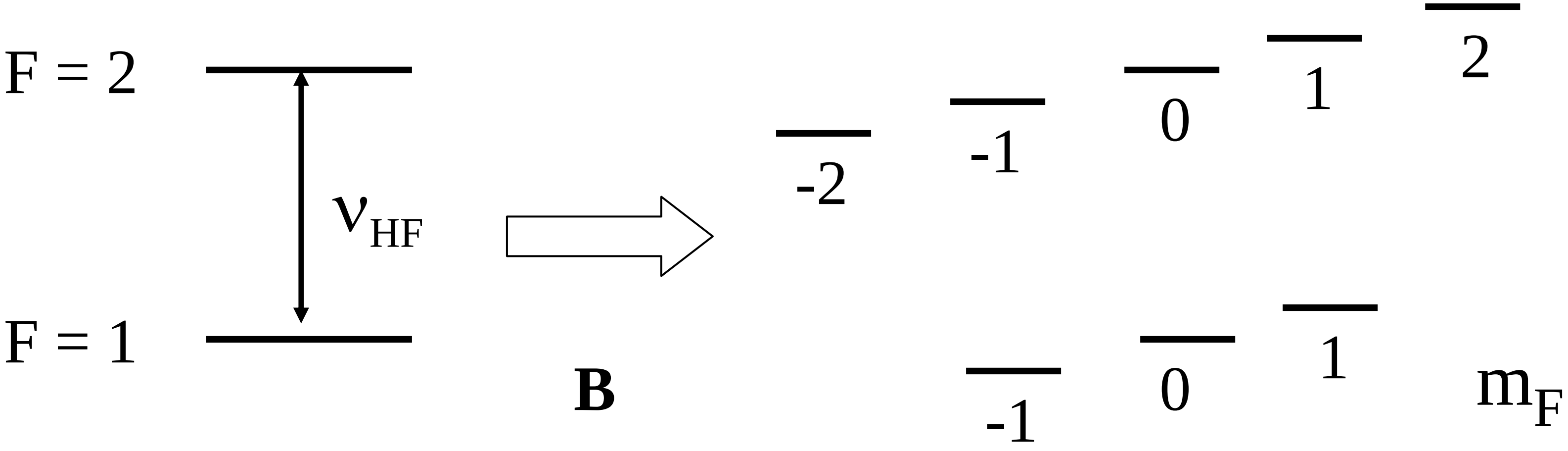
- GaTech, M. S. Chapman (2001)
- U. Hamburg, K. Sengstock
- Gakushuin Univ., T. Hirano
- Berkeley, D. M. Stamper-Kurn

Hyperfine structure

$$L = 0$$

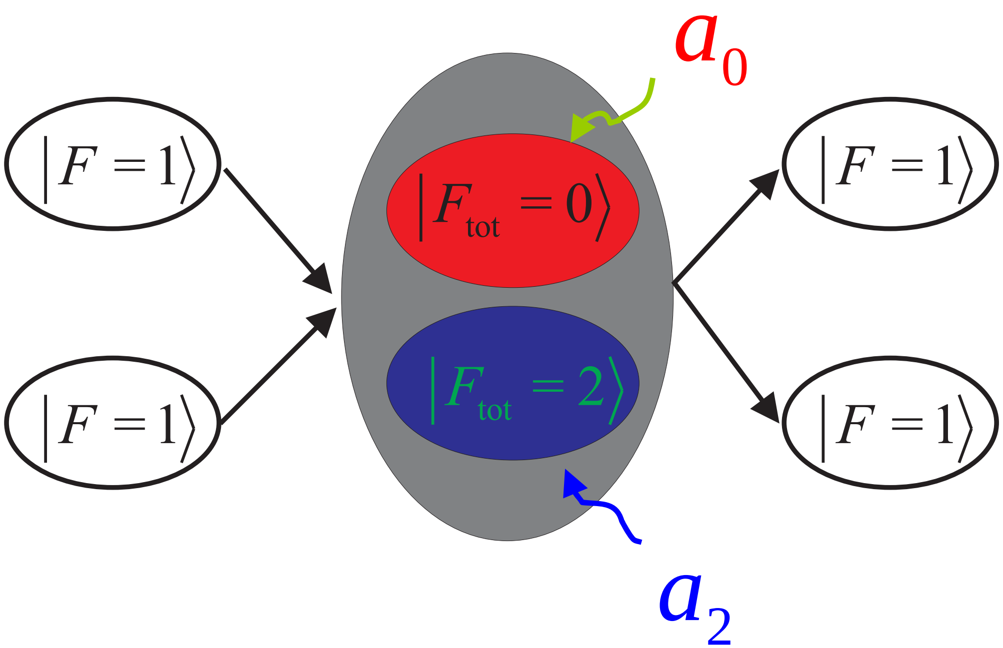
$$S = 1/2 \quad \Rightarrow \quad F = L + S + I = \begin{cases} 2 \\ 1 \end{cases}$$

$$I = 3/2$$



The atomic ground state is |F=1>, if kT << $h\nu_{HF}$.

Spin-1 atoms collisions



$$V_F = \frac{4\pi\hbar^2}{m}\Big(a_0 |F_{tot}=0\rangle\langle F_{tot}=0| + a_2 |F_{tot}=2\rangle\langle F_{tot}=2|\Big)$$

- Dilute atomic gas
- Low temperature
- 2-body collision dominates
- *s*-wave scattering length
- Symmetric in spatial degree of freedom
- Symmetric in spin degree of freedom (No a_1)

Spin interaction

$$\left|F_1 F_2 F m_F\right\rangle \rightarrow \left|F_1 F_2 m_1 m_2\right\rangle$$

CG coefficient

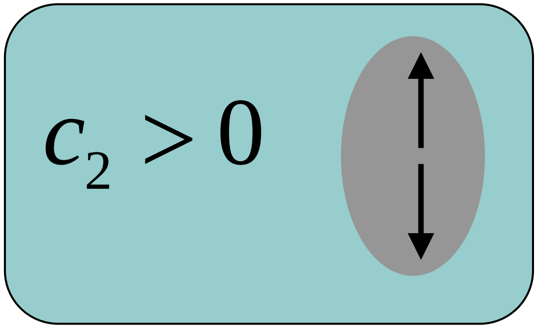
$$V_F = c_0 \boldsymbol{I} + c_2 \boldsymbol{F} \bullet \boldsymbol{F}$$

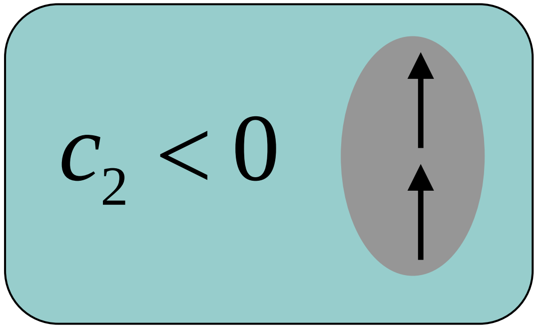
Spin independent

Spin dependent

$$c_0 = \frac{4\pi\hbar^2}{3m}(a_0 + 2a_2)$$

$$c_2 = \frac{4\pi\hbar^2}{3m}(a_2 - a_0)$$





PRL **81**, 742 (1998)

Hamiltonian

$$\mathrm{H}=\int d\vec{\mathrm{r}}\,\psi_i^{?}\left(-\frac{\hbar^2}{2\mathrm{m}}\nabla \;\; +V_{\mathrm{ext}}(\vec{r})\right)\psi_i+\frac{c_0}{2}\psi_i^{\text{唵}}\psi_j\psi_j\psi_i$$

$$+\frac{c_2}{2}\psi_i^{\text{唵}}\psi_k\left(F_\eta\right)_{ij}\left(F_\eta\right)_{kl}\psi_j\psi_l,$$

where

$$F_x=\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix},\quad F_y=\frac{i}{\sqrt{2}}\begin{pmatrix}0&-1&0\\1&0&-1\\0&1&0\end{pmatrix},\quad F_z=\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}.$$

$$[H,N]=0,\qquad [H,M]=0,\qquad M=\langle F_z\rangle$$

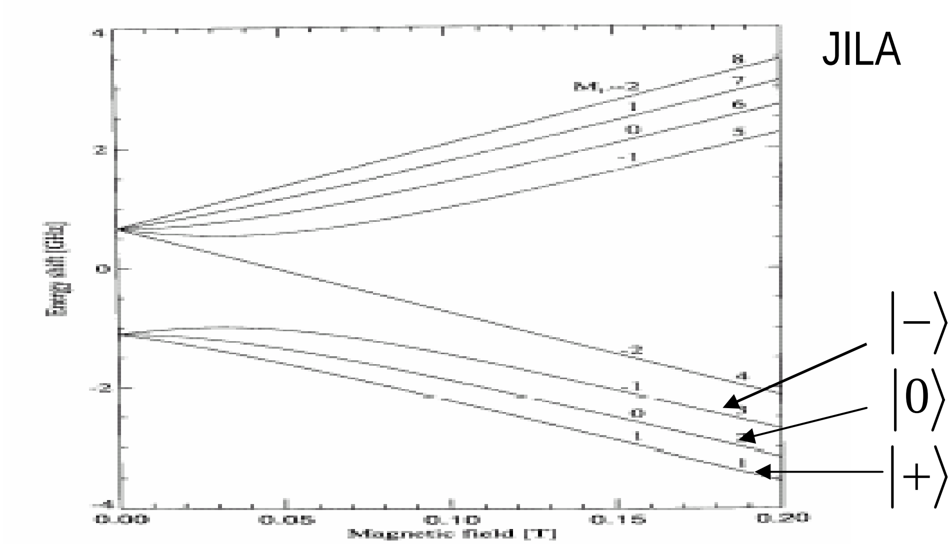
PRL **81**, 742 (1998), J. Phys. Soc. Jpn. **67**, 1822 (1998), PRL **81**, 5257 (1998).

Zeeman effect

$$H=\int d\vec{r}\,\psi_i^{?}\left(-\frac{\hbar^2}{2m}\nabla \;+V_{\text{ext}}(\vec{r})\right)\psi_i+\psi_i H_{ZM}\psi_i$$

$$+\frac{c_0}{2}\psi_i^{\text{唵}}\psi_j^{\text{唵}}\psi_j\psi_i+\frac{c_2}{2}\psi_i^{\text{唵}}\psi_k^{\text{唵}}\left(F_\eta\right)_{ij}\left(F_\eta\right)_{kl}\psi_j\psi_l,$$

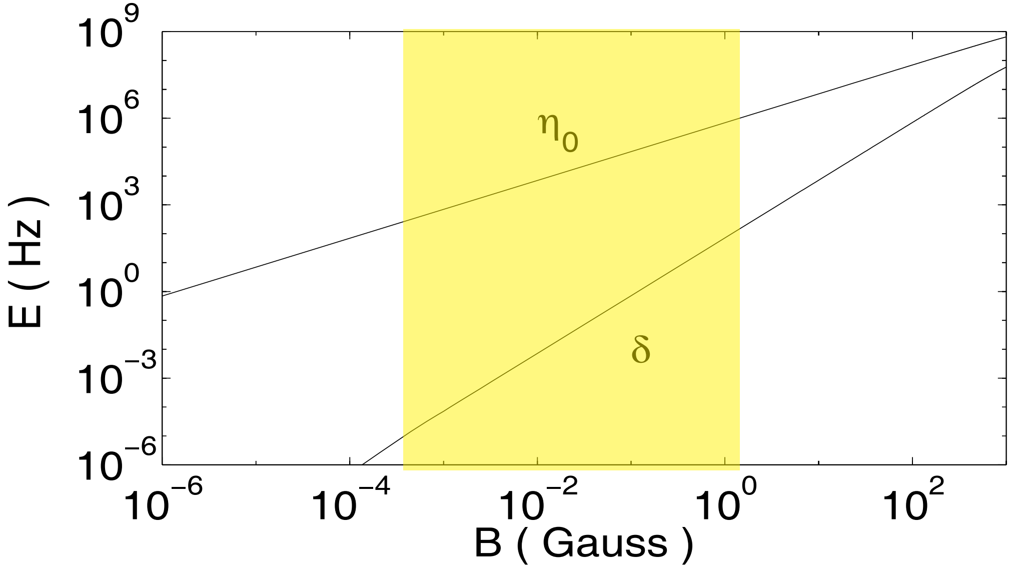
$$H_{ZM}(B)=\begin{pmatrix} E_+ & 0 & 0 \\ 0 & E_0 & 0 \\ 0 & 0 & E_- \end{pmatrix},$$

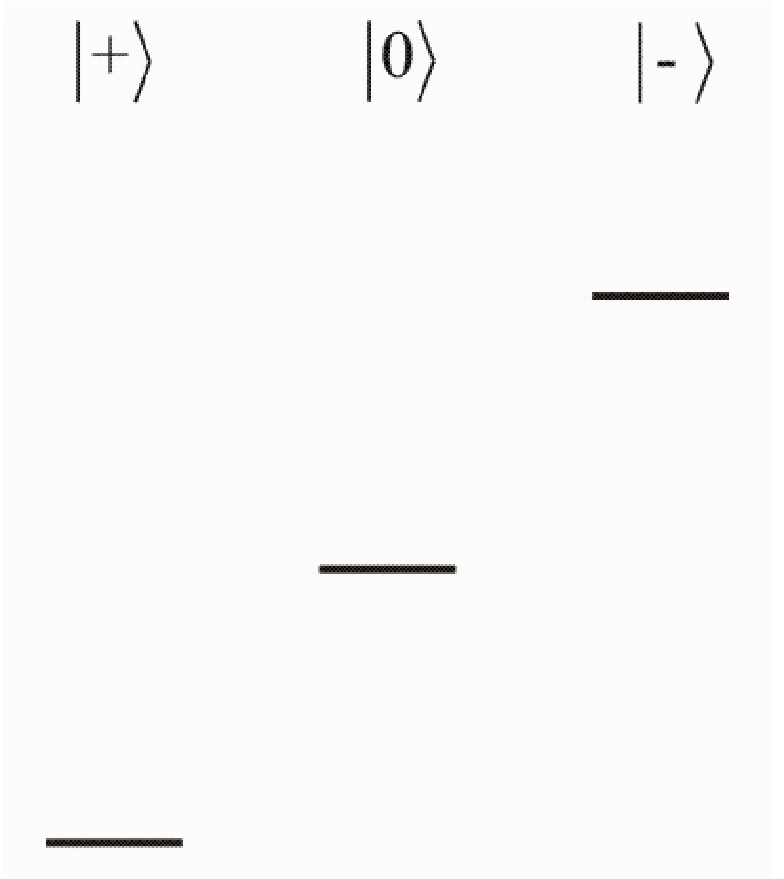


Zeeman effects

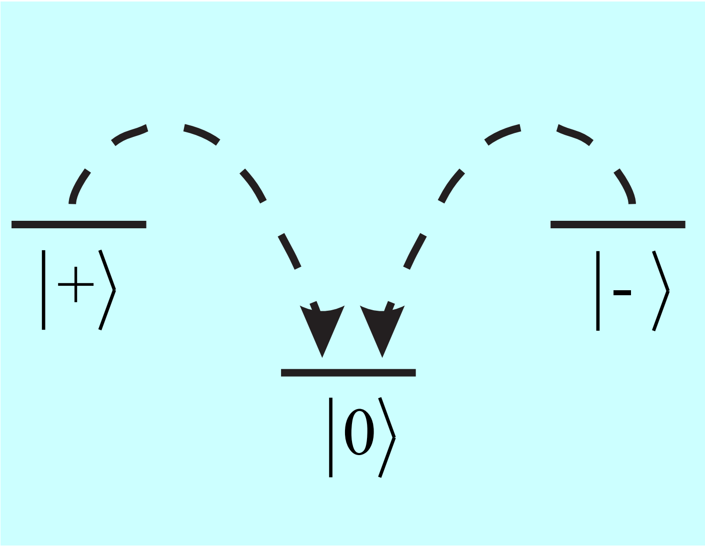
$$\text{Linear}:\ \eta_0 = \frac{E_- - E_+}{2}$$

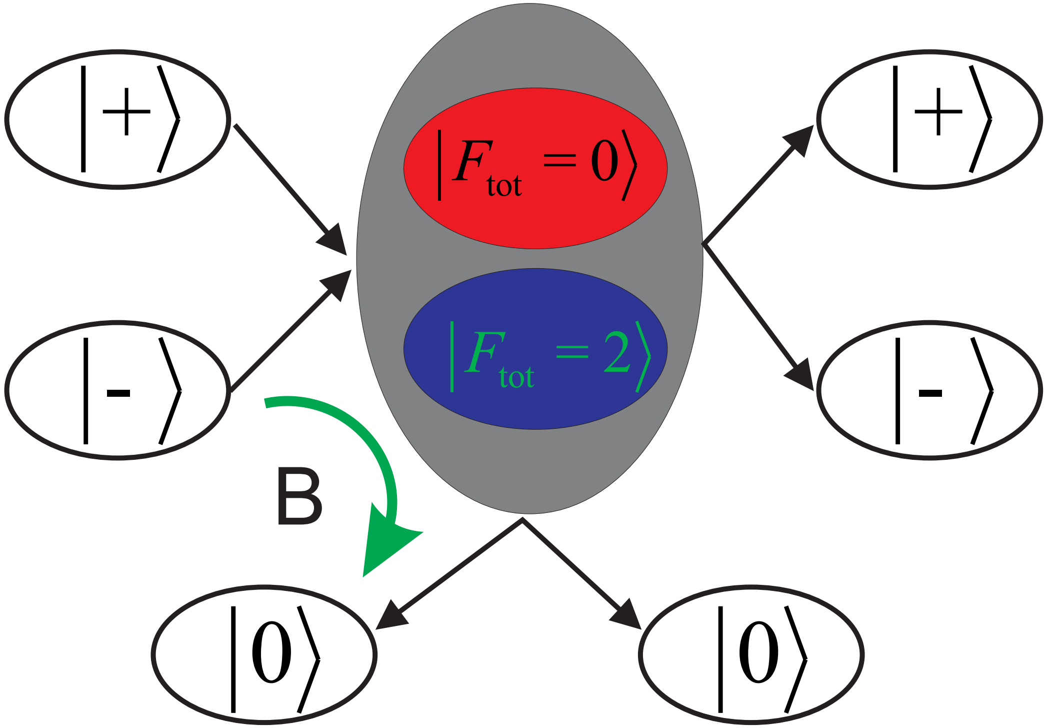
$$\text{Quadratic}:\ \delta = \frac{E_+ + E_- - 2E_0}{2}$$





Collisions in B field





B field prefers more |0> component.

Mean field approximation

$$\hat{\Psi} = \left\langle \Psi \right\rangle + \delta\Psi$$

Order parameter $\Phi = \left\langle \hat{\Psi} \right\rangle$: Condensate mean field wave function

Classical field description

$$\mathrm{H}_{\mathrm{MF}} = \int \mathrm{d}\vec{\mathrm{r}} \sum_i \Phi_i^* \left(-\frac{\hbar^2}{2\mathrm{m}} \nabla^2 + V_{\mathrm{ext}}(\vec{r}) \right) \Phi_i + \Phi_i^* H_{ZM} \Phi_i + \frac{c_0}{2} \left(\sum_i \left| \Phi_i \right|^2 \right)^2 + \frac{c_2}{2} \sum_{i,j,k,l} \Phi_i^* \left(F_\eta \right)_{ij} \Phi_j \Phi_k^* \left(F_\eta \right)_{kl} \Phi_l ,$$

n^2 f^2

Single mode approx.

$c_0 \gg |c_2|$ AND BEC size < spin healing length

$$\begin{pmatrix} \Phi_+(\vec{r}) \\ \Phi_0(\vec{r}) \\ \Phi_-(\vec{r}) \end{pmatrix} = \Phi_{SMA}(\vec{r}) \begin{pmatrix} \xi_+ \\ \xi_0 \\ \xi_- \end{pmatrix}$$

- Separation of spatial and spin degree of freedom
- Three components share the same spatial wave function Φ_{SMA}

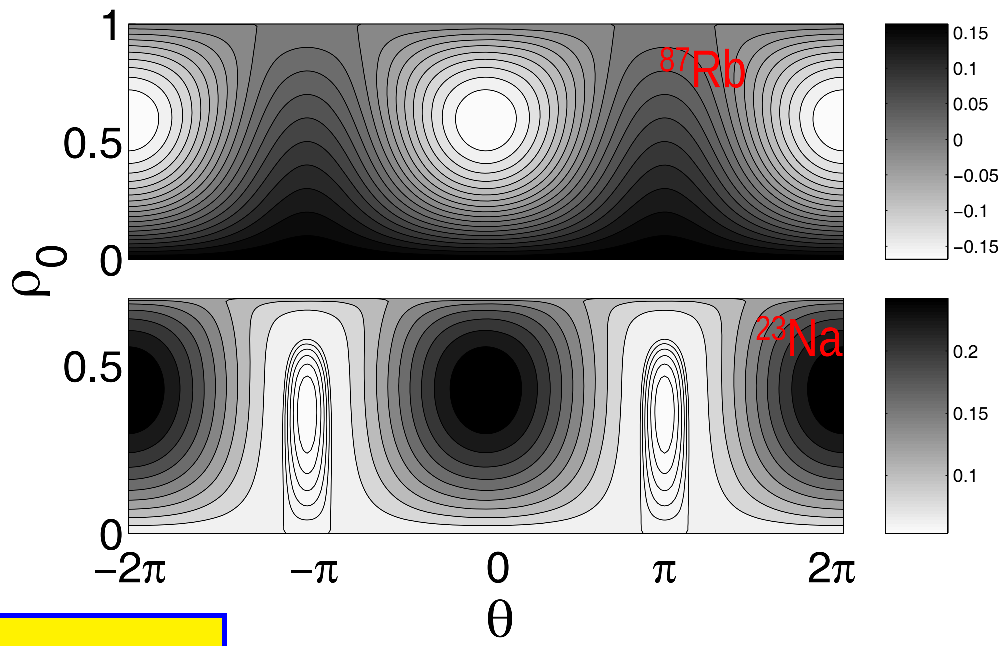
Two simple cases:

1. Small spin-1 condensate
2. Homogeneous spin-1 condensate (valid at short times)

SMA orbits in B fields

$$\begin{pmatrix} \xi_+ \\ \xi_0 \\ \xi_- \end{pmatrix} = \begin{pmatrix} \sqrt{\rho_+} e^{i\theta_+} \\ \sqrt{\rho_0} e^{i\theta_0} \\ \sqrt{\rho_-} e^{i\theta_-} \end{pmatrix}$$

$$\theta = \theta_+ + \theta_- - 2\theta_0$$



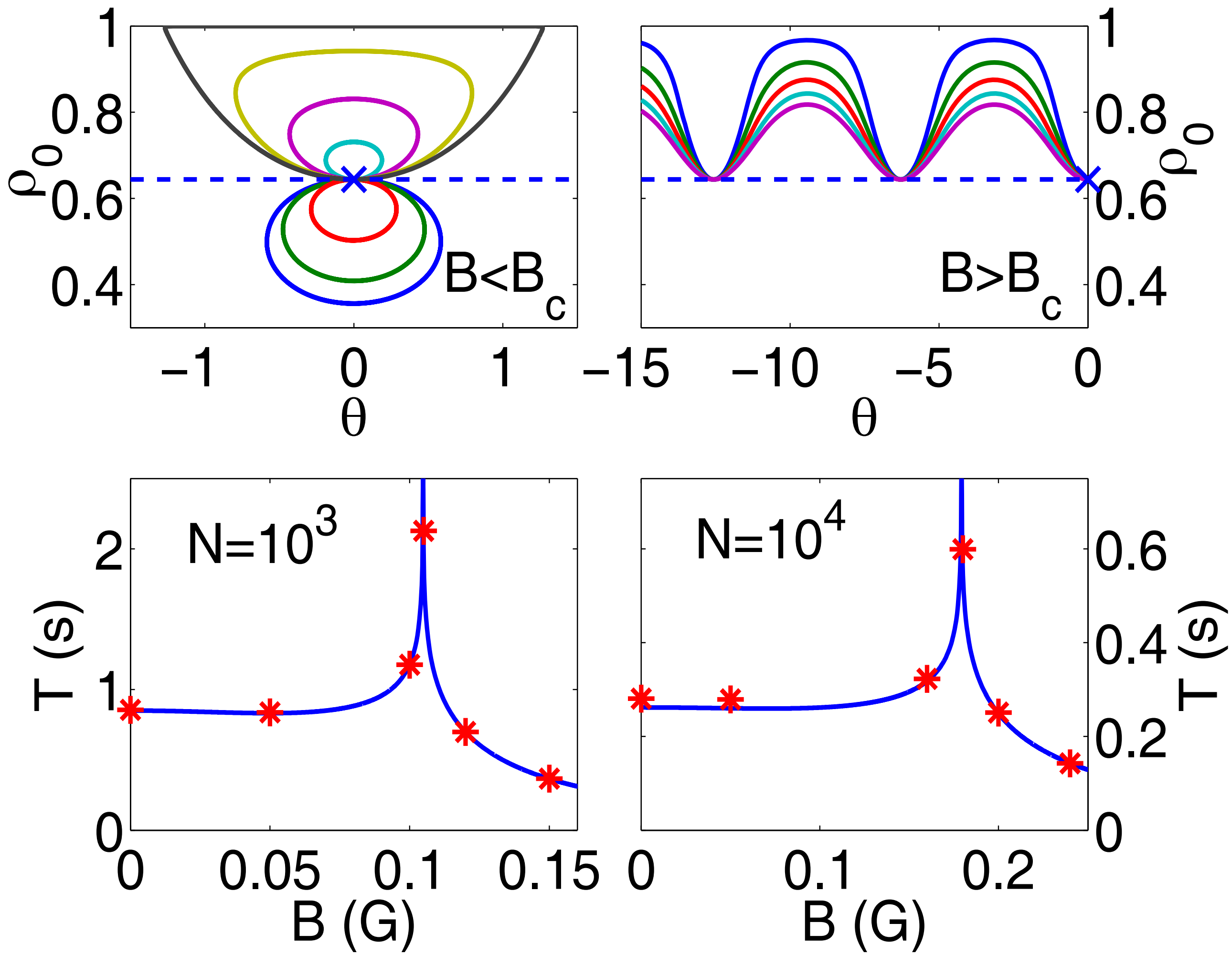
$$\dot{\rho}_0 = \frac{2c}{\hbar} \rho_0 \sqrt{(1-\rho_0)^2 - m^2} \sin\theta$$

$$\dot{\theta} = \frac{2c}{\hbar} \left[(1-2\rho_0) + \frac{(1-\rho_0)(1-2\rho_0) - m^2}{\sqrt{(1-\rho_0)^2 - m^2}} \cos\theta \right] + \frac{\delta}{2\hbar}$$

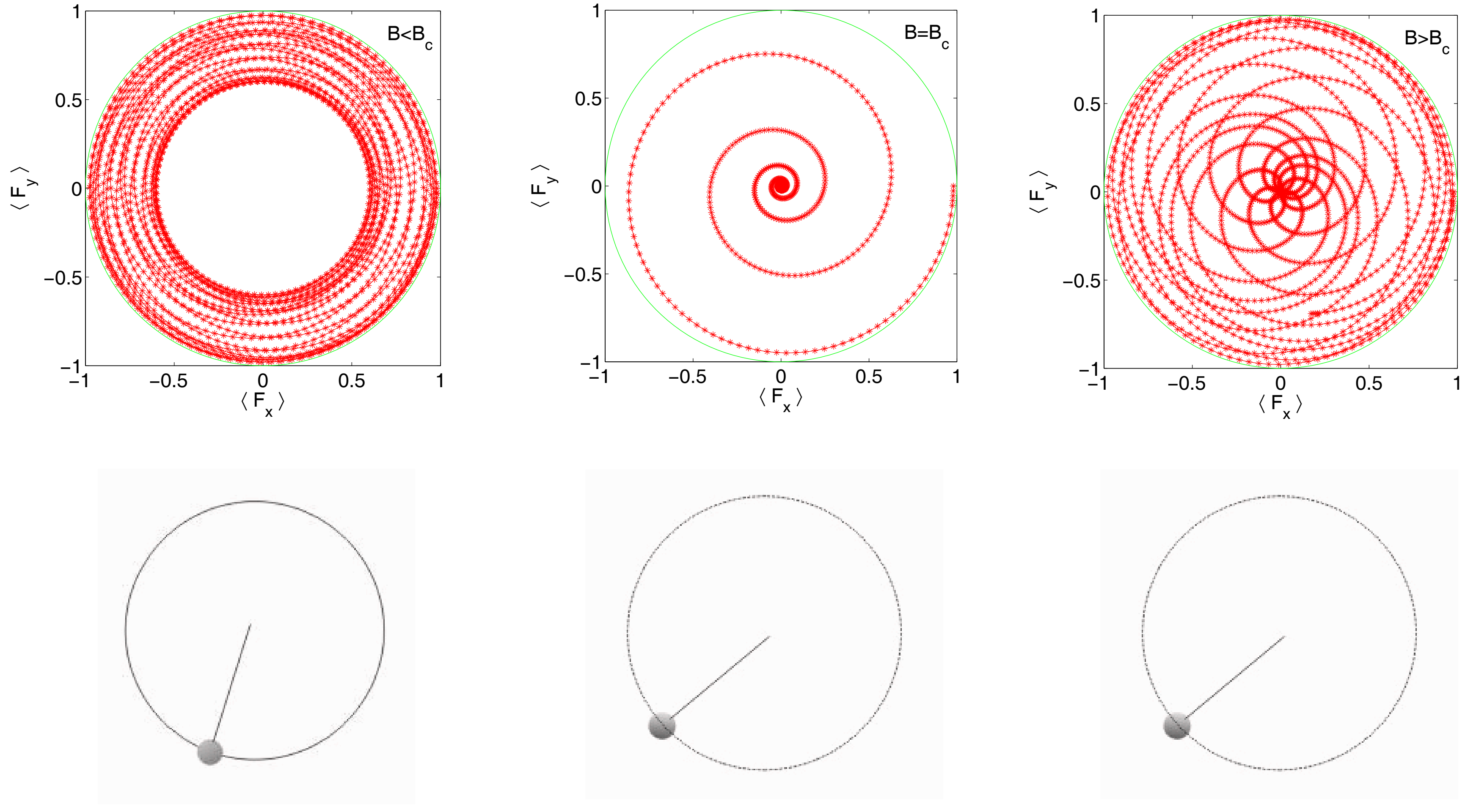
$$\mathcal{E} = c\rho_0 \left[(1-\rho_0) + \sqrt{(1-\rho_0)^2 - m^2} \cos\theta \right] + \delta(1-\rho_0)$$

PRA 72, 013602 (2005).

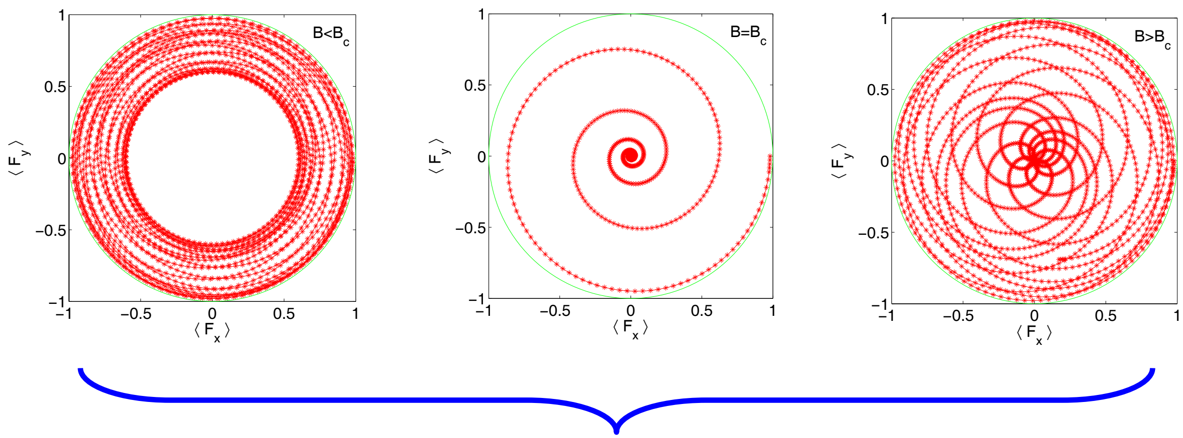
SMA orbits in B fields

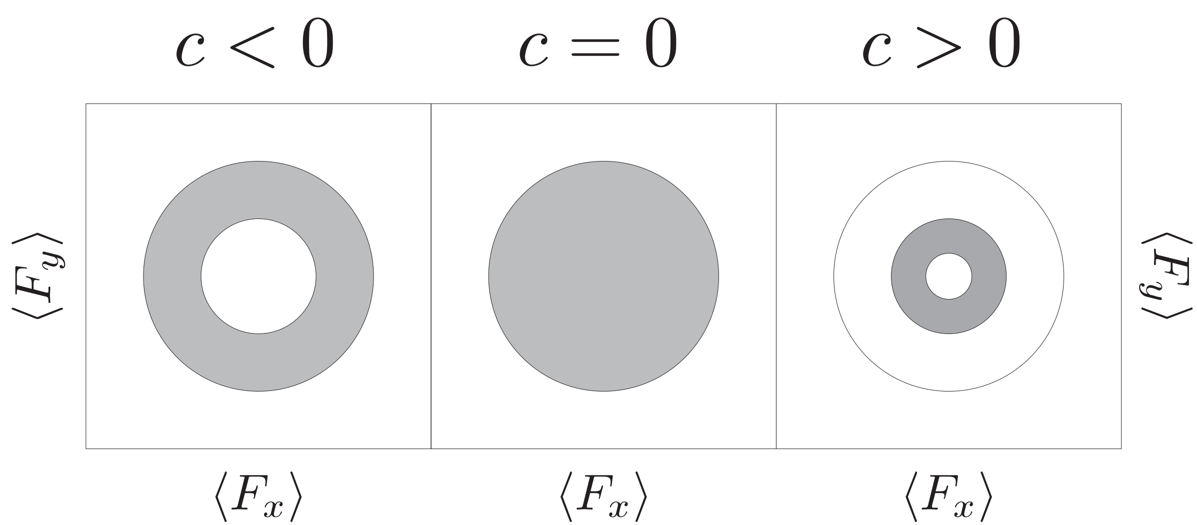


Spin evolution

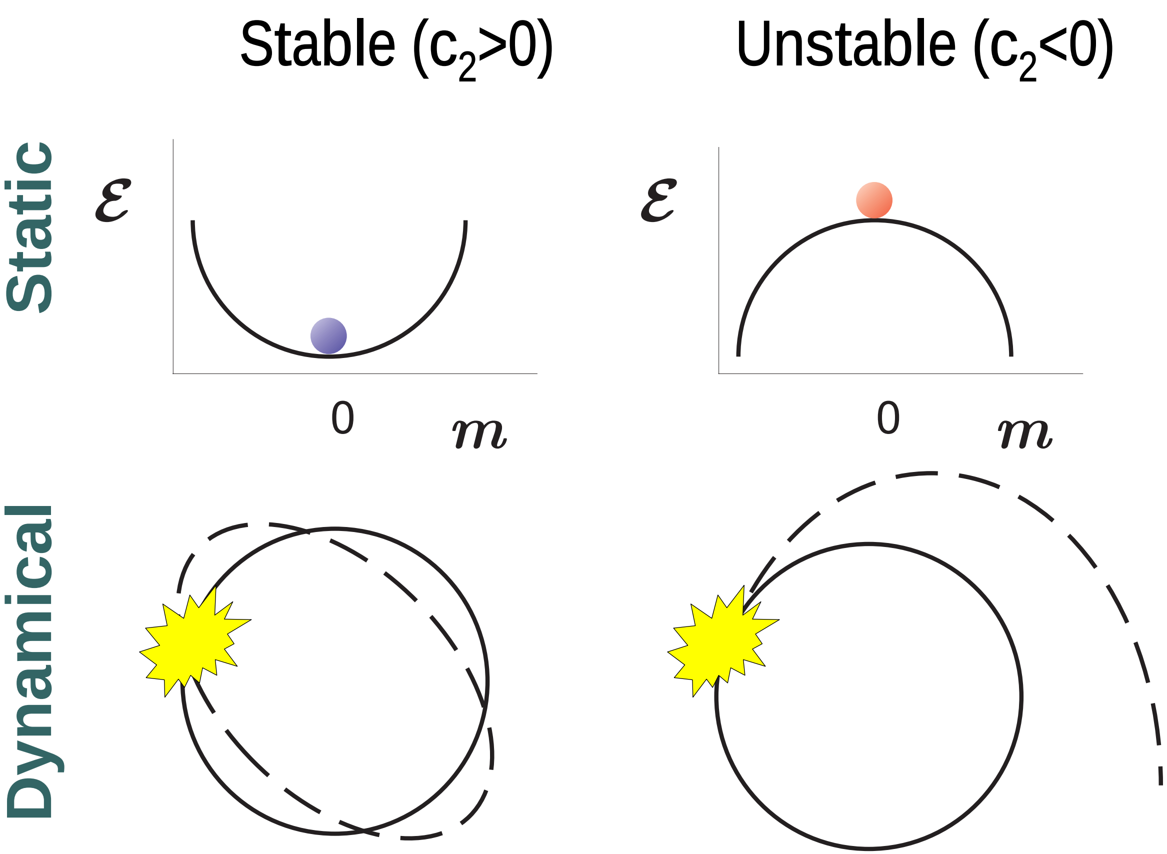


Spin Evolution





Dynamical stability



Homogeneous and B = 0

Methods:

- Effective potential method (analytical and qualitative for uniform system)
- Bogoliubov eigen-mode (analytical and quantitative for uniform system)
- Numerical simulation (trapped system)

Classical version

Interaction picture (moving reference frame)

orbit in lab frame → stationary point in moving frame

$$\mathcal{E} = \frac{1}{2}c_0 n^2 + \frac{1}{2}c_2 m^2 + \underbrace{\frac{1}{2}c_2\rho_0\left[(1-\rho_0) + \sqrt{(1-\rho_0)^2 - m^2}\cos\theta\right]}_{\frac{1}{2}c_2\left(f_x^2 + f_y^2\right)}$$

Lab frame

$$\longrightarrow \quad \boxed{\mathcal{E}_I = \mathcal{E} - \mu n - \eta m - \delta_x f_x - \delta_y f_y}$$

Moving frame

- Stable, semi-positive-definite of Hessian matrix ($c_2 > 0$)

$$D = \nabla_{n,m,f_x,f_y} \nabla_{n,m,f_x,f_y} \mathcal{E}_I$$

- Unstable, otherwise ($c_2 < 0$)

Quantum version

Energy in mov. frame, $\mathcal{E}_I = \mathcal{E}_I(\Phi) = \mathcal{E}_I\left(\Phi^{(0)} + \vec{x}\right)$

$$= \mathcal{E}_I^{(0)}\left(\Phi^{(0)}\right) + \vec{x}^T \cdot M \cdot \vec{x} + \cdots$$

where, $\begin{pmatrix} \Phi_+^* \\ \Phi_0^* \\ \Phi_-^* \\ \Phi_+ \\ \Phi_0 \\ \Phi_- \end{pmatrix} = \begin{pmatrix} \Phi_+^{*(0)} \\ \Phi_0^{*(0)} \\ \Phi_-^{*(0)} \\ \Phi_+^{(0)} \\ \Phi_0^{(0)} \\ \Phi_-^{(0)} \end{pmatrix} + \begin{pmatrix} x_+^* \\ x_0^* \\ x_-^* \\ x_+ \\ x_0 \\ x_- \end{pmatrix}$

$$x_j \propto \exp(i\omega t - i\vec{k} \cdot \vec{r})$$

- Stable, if ω is real ($c_2 > 0$)
- Unstable, otherwise ($c_2 < 0$)

Linear response theory

Bogoliubov spectrum (I)

Characteristic eq. $$M-\hbar\omega\,\mathrm{I}_{6\times 6}=0,\quad M=\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix},$$

$$A=\varepsilon_k \mathrm{I}+\begin{pmatrix} (c_0+c_2)n_+ + c_2 n_0 & c_0\Phi_0^*\Phi_+ + c_2\Phi_0\Phi_-^* & (c_0-c_2)\Phi_-^*\Phi_+ \\ c_0\Phi_+^*\Phi_0 + c_2\Phi_0^*\Phi_- & c_0 n_0 + c_2(n_+ + n_-) & c_0\Phi_-^*\Phi_0 + c_2\Phi_0^*\Phi_+ \\ (c_0-c_2)\Phi_+^*\Phi_- & c_0\Phi_0^*\Phi_- + c_2\Phi_+^*\Phi_0 & (c_0+c_2)n_- + c_2 n_+ \end{pmatrix}$$

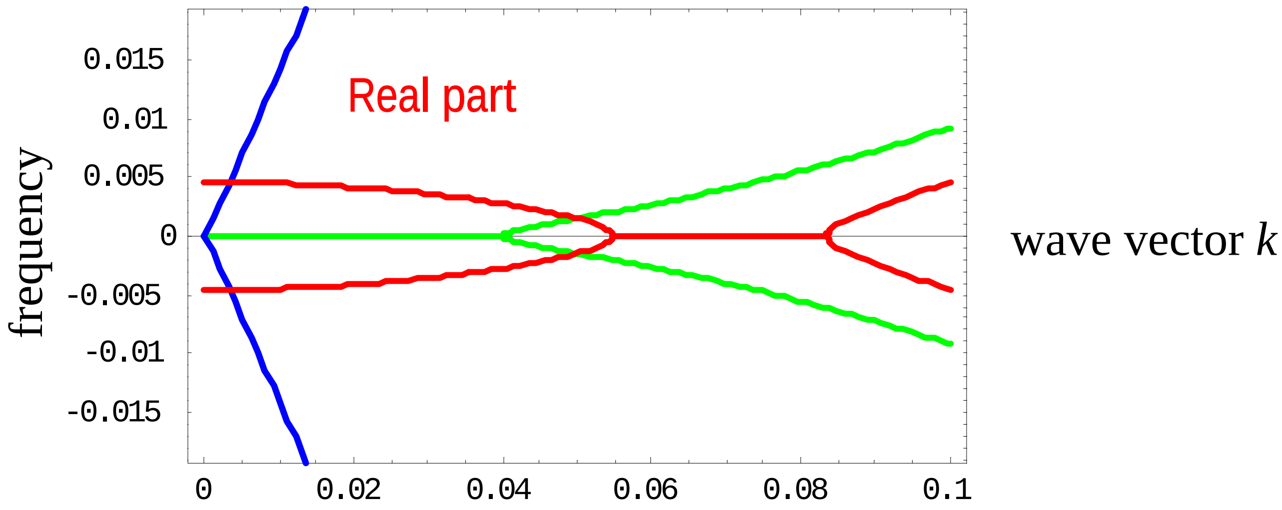
$$B=\begin{pmatrix} (c_0+c_2)\Phi_+^2 & (c_0+c_2)\Phi_+\Phi_0 & (c_0-c_2)\Phi_+\Phi_- + c_2\Phi_0^2 \\ (c_0+c_2)\Phi_+\Phi_0 & c_0\Phi_0^2 + 2c_2\Phi_+\Phi_- & (c_0+c_2)\Phi_0\Phi_- \\ (c_0-c_2)\Phi_+\Phi_- + c_2\Phi_0^2 & (c_0+c_2)\Phi_0\Phi_- & (c_0+c_2)\Phi_-^2 \end{pmatrix}$$

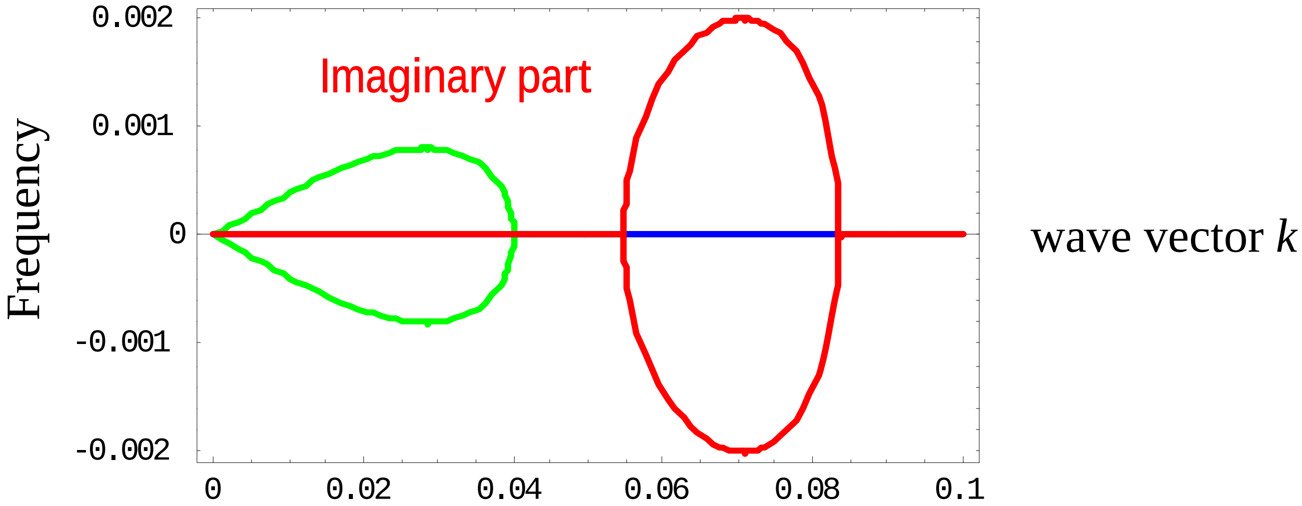
$$(\hbar\omega)_{1,2}^2=\varepsilon_k\left[(c_0 n + c_2 n + \varepsilon_k) + n\sqrt{(c_0-c_2)^2 + 4c_0 c_2 f^2}\right]$$

$$(\hbar\omega)_{3,4}^2=\varepsilon_k\left[(c_0 n + c_2 n + \varepsilon_k) - n\sqrt{(c_0-c_2)^2 + 4c_0 c_2 f^2}\right]$$

$$(\hbar\omega)_{5,6}^2=(\varepsilon_k + c_2 n)^2 - c_2^2 n^2(1-f^2)$$

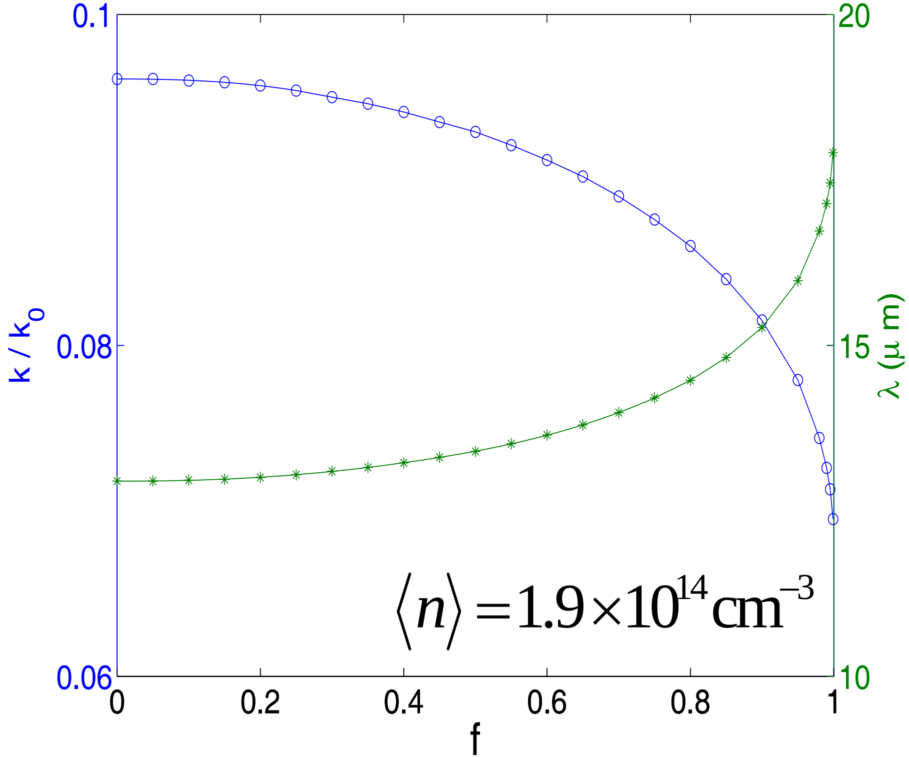
Bogoliubov spectrum (II)



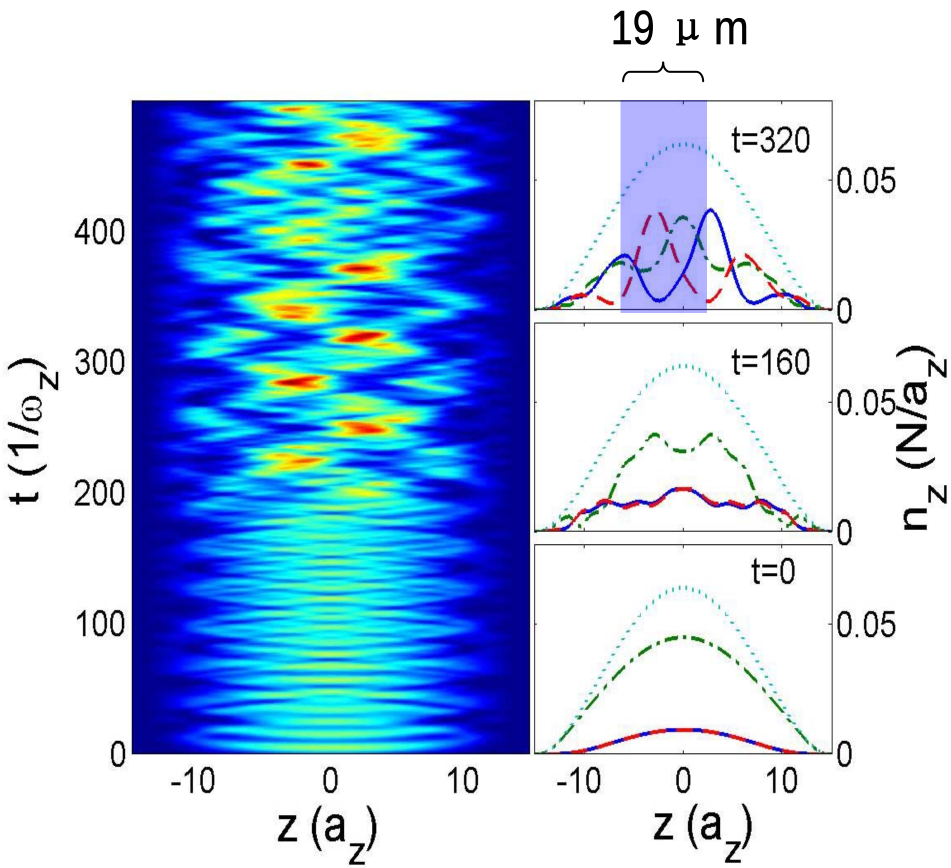


Results

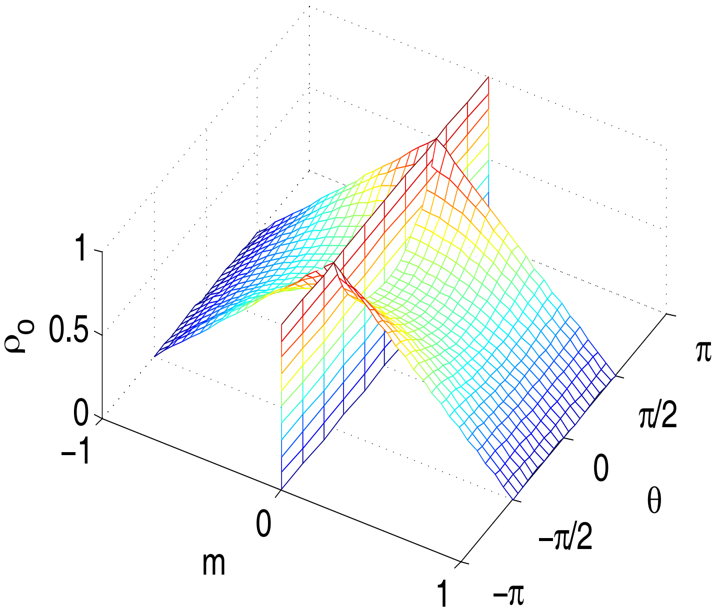
Analytical, uniform



$$\langle n \rangle = 1.9 \times 10^{14} \mathrm{cm}^{-3}$$



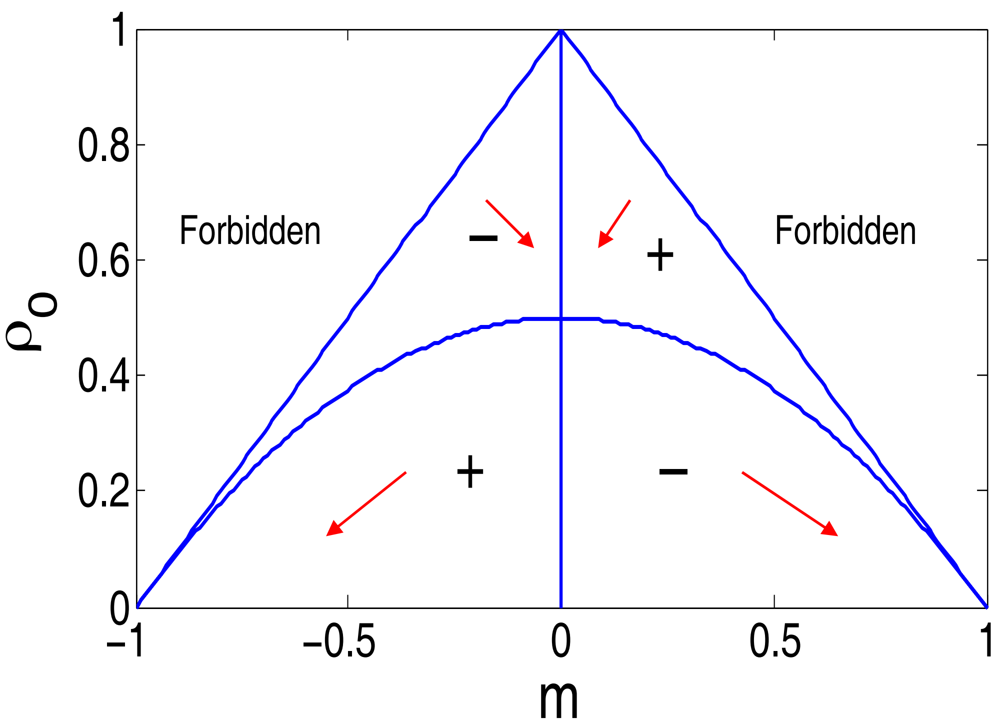
Domain formation



Surface of $d\varepsilon / dm = 0$

$d\varepsilon / dm > 0$, m decreases

$d\varepsilon / dm < 0$, m increases



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- ☺ Dr. D.-L. Zhou

$ NSF, NASA

Summary

- ✓ Coherent spin mixing dynamics in B fields
- ✓ Dynamical instability and spontaneous domain formation

Thanks.