Detection of Spin Orders with Cavity QED

Yunbo Zhang (Shanxi University)

Here I give an introduction to several detection methods of strongly correlated quantum phases in ultracold atomic gases. A general cavity-enhanced scheme is developed for detecting spin orders inside a two-component lattice gas of bosonic atoms.

第二届全国冷原子物理和量子信息青年学者学术讨论会 2008年9月15-19日 郑州

Collaborators and $$

- Liping Guo,… (SXU)
- Shu Chen (IOP, CAS)
- Su-Peng Kou (BNU)
- Su Yi (ITP, CAS)
- Li You (GaTech)
- NSFC
- 973
- …

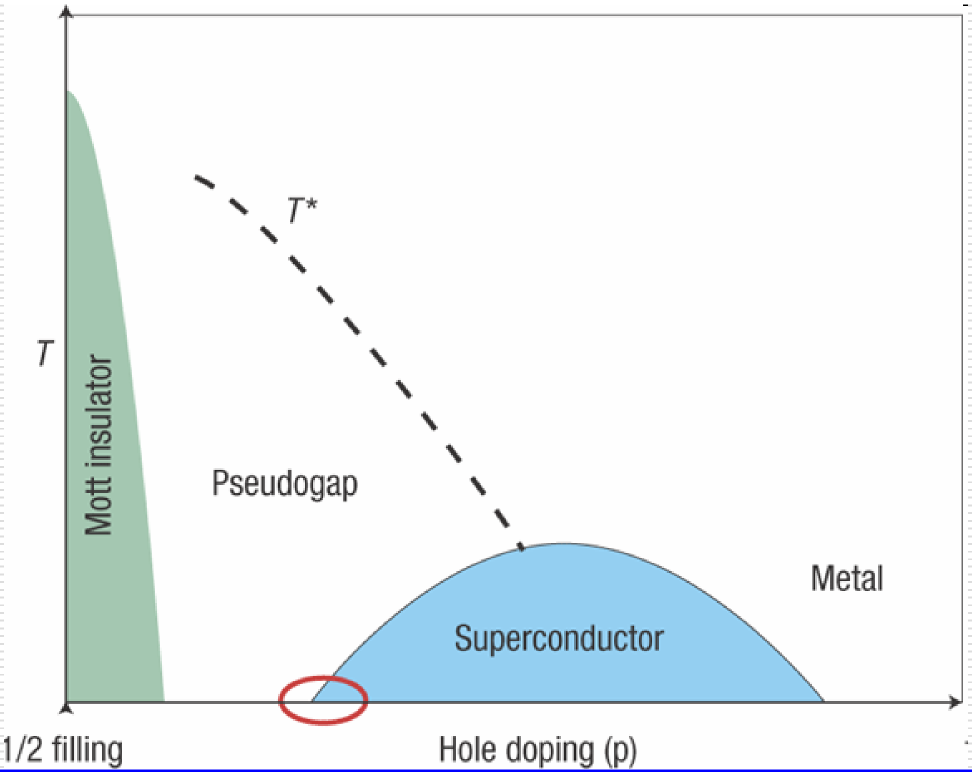
Talk Outline

- Several paradigm examples of strongly correlated states:
  - Mott insulator
  - Tonks gas
  - Bose glass

Talk Outline

- Novel Methods of Detection
  - Atomic Noise Interferometry - HBT
  - QND Detection 1D Quantum AF – Polarized Light
  - Cavity Optomechanics – Strong Interacting – Exotic Quantum Phases
  - Cavity Enhanced Detection

Mott Physics

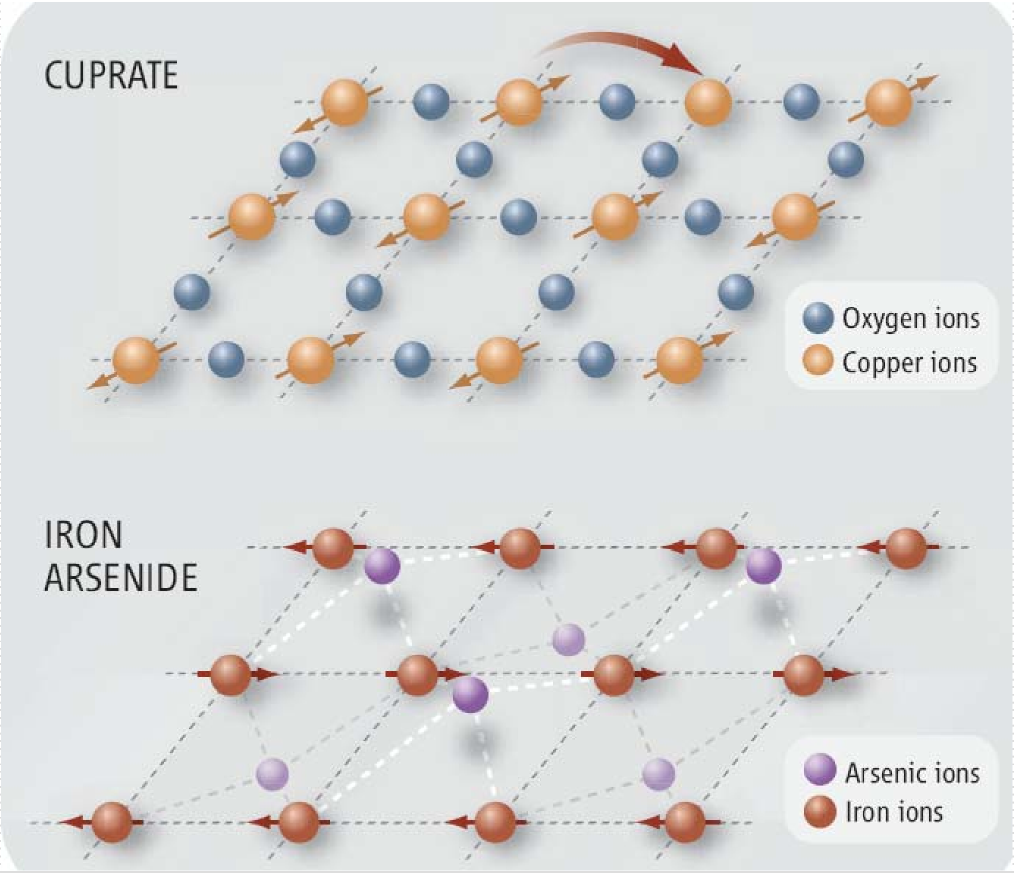


Phase diagram of a typical hole-doped copper oxide superconductor

Mott Physics

Undoped cuprate: Mott insulators

Undoped FeAs: Metals



Mott Physics

Bose-Hubbard model

$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + \sum_i \epsilon_i\hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$H_B = -J\sum_{\ell=-\infty}^{\infty}\left(a_\ell^\dagger a_{\ell+1} + a_{\ell+1}^\dagger a_\ell\right) + b\sum_{\ell=-\infty}^{\infty}\ell^2 a_\ell^\dagger a_\ell$$

$$V = U\sum_{\ell=-\infty}^{\infty} a_\ell^{\dagger 2} a_\ell^2$$

Nature **415** 39(2002)

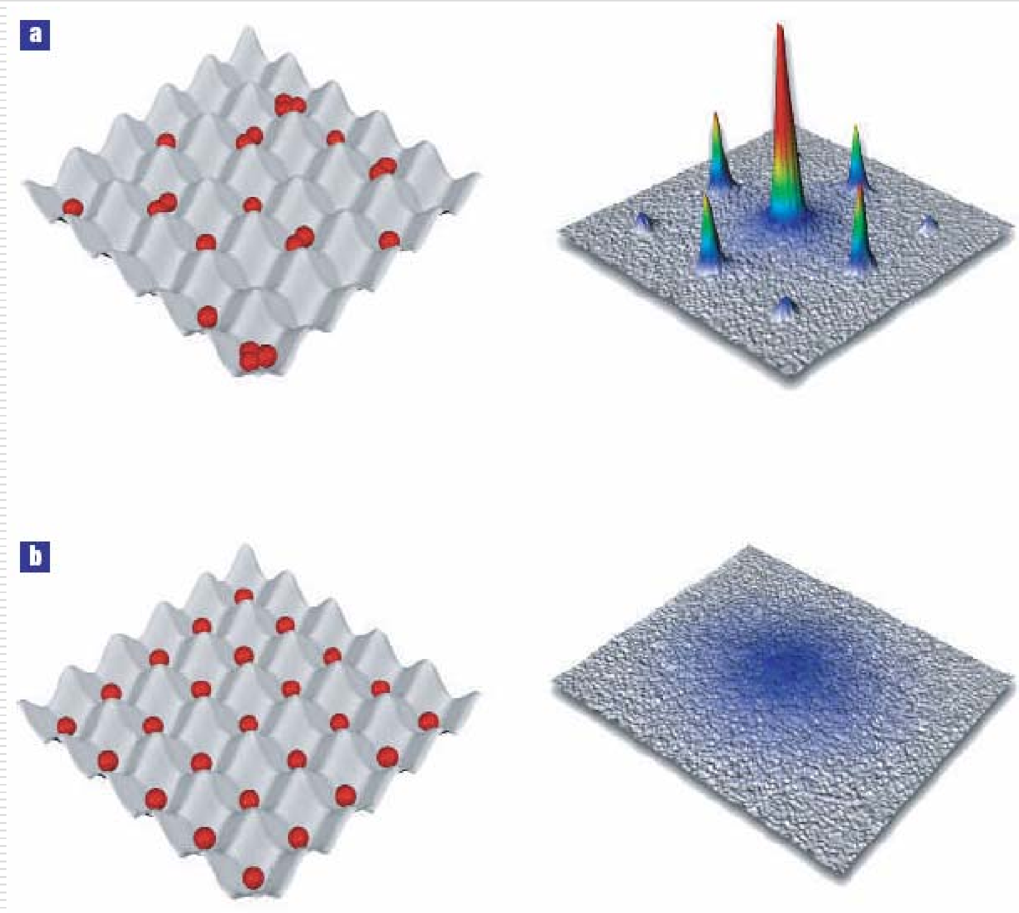
Mott Physics

Boson

Superfluid state BEC: Interference pattern

Mott insulating state: No interference

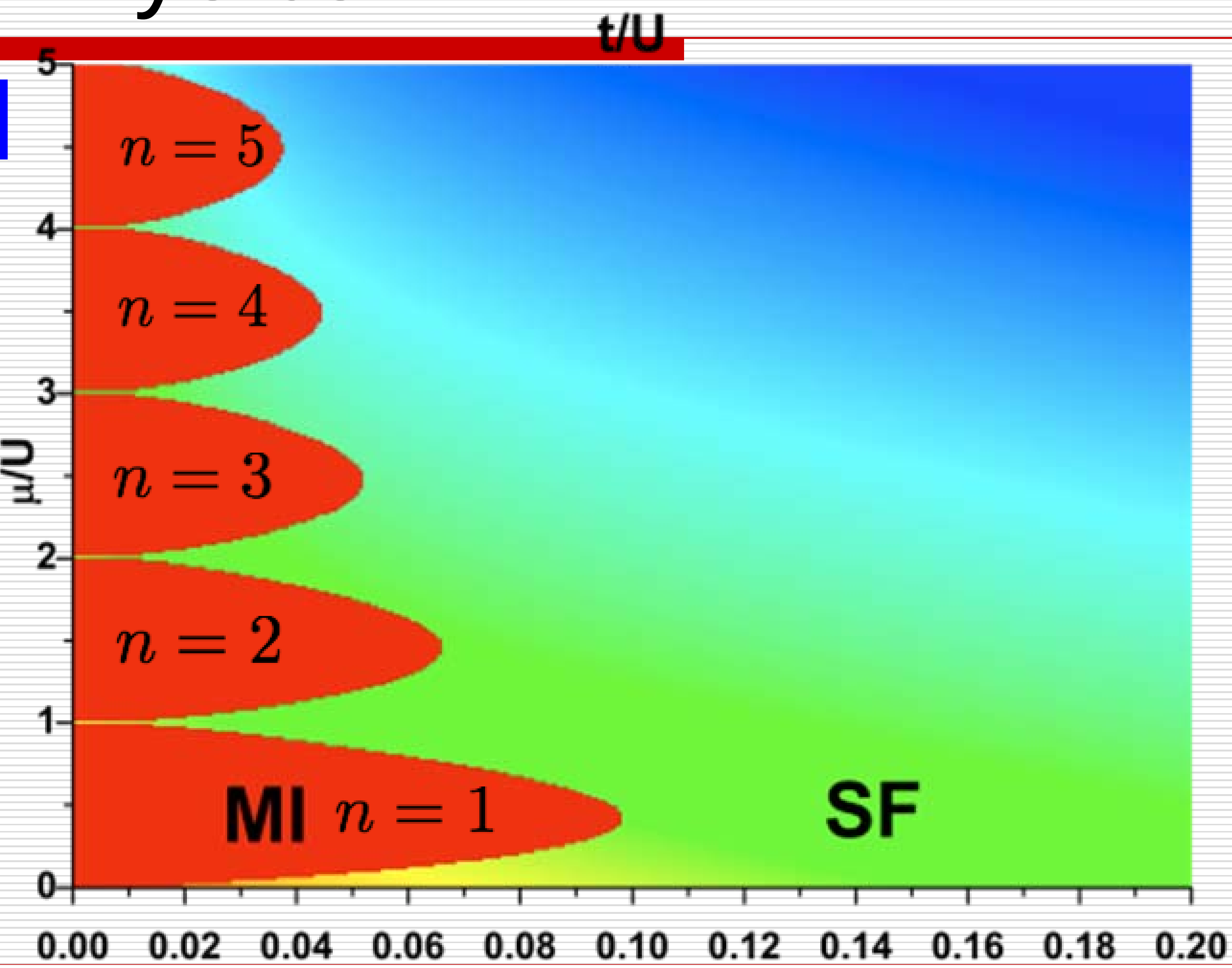
Nature **415** 39(2002)



Mott Physics

Boson

Global Phase Diagram



Mott Physics

ARTICLES

Three-body interactions with cold polar molecules

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$$V_{\rm eff}(\{\mathbf{r}_i\}) = \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k} W(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) + \cdots$$

Published online: 22 July 2007; doi:10.1038/nphys678

Fundamental interactions between particles, such as the Coulomb law, involve pairs of particles, and our understanding of the plethora of phenomena in condensed-matter physics rests on models involving effective two-body interactions. On the other hand, exotic quantum phases, such as topological phases or spin liquids, are often identified as ground states of hamiltonians with three- or more-body terms. Although the study of these phases and the properties of their excitations is currently one of the most exciting developments in theoretical condensed-matter physics, it is difficult to identify real physical systems exhibiting such properties. Here, we show that polar molecules in optical lattices driven by microwave fields naturally give rise to Hubbard models with strong nearest-neighbour three-body interactions, whereas the two-body terms can be tuned with external fields. This may open a new route for an experimental study of exotic quantum phases with quantum degenerate molecular gases.

Mott Physics

Boson

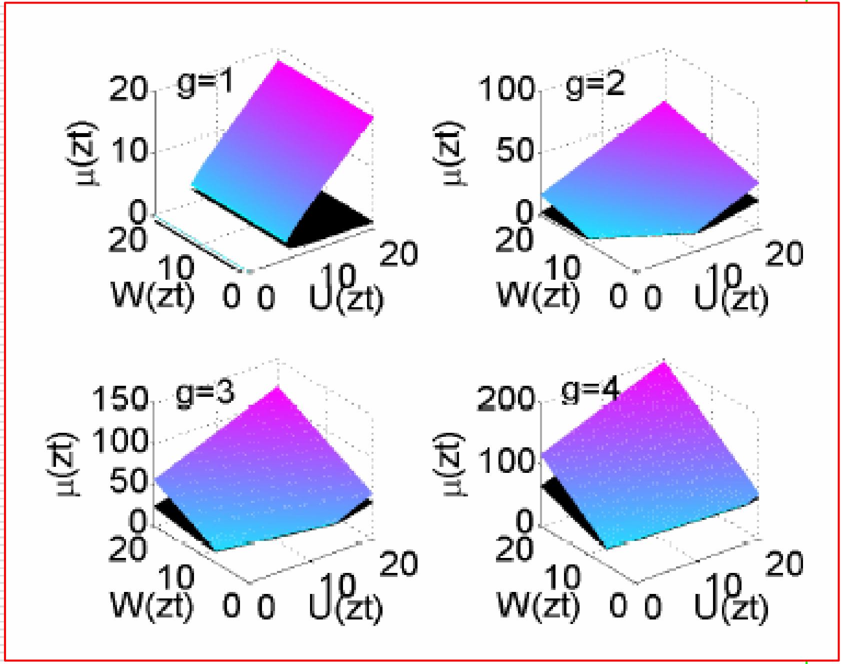
$$\hat{H} = -t\sum_{\langle i,j\rangle}\hat{c}_i^\dagger\hat{c}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + \frac{W}{6}\sum_i \hat{n}_i(\hat{n}_i - 1)(\hat{n}_i - 2) - \mu\sum_i \hat{n}_i,$$

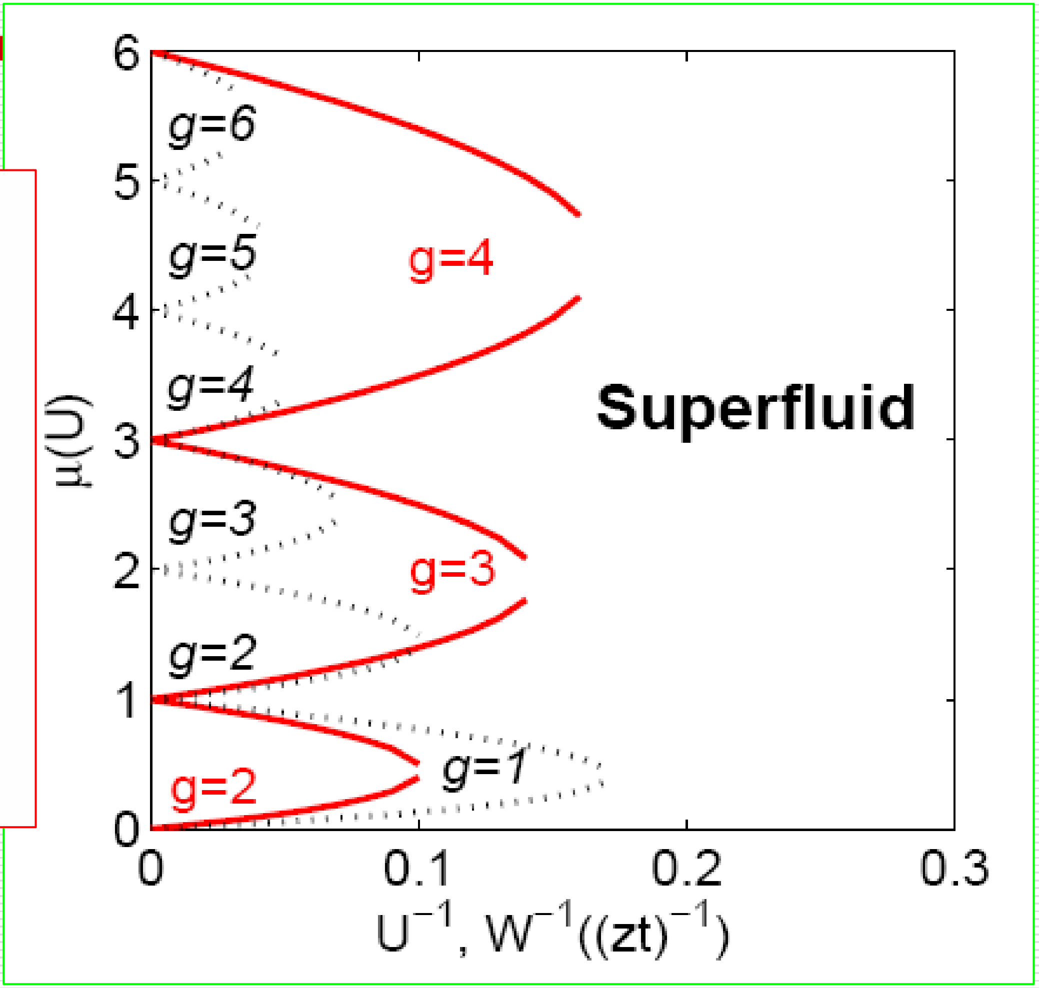
Global Phase Diagram with three-body interaction (MFT)

Chen, Huang, Kou, Zhang PRA to appear
Idea from Nature Physics **3**, 726(2007)

Mott Physics

Boson





Mott Physics

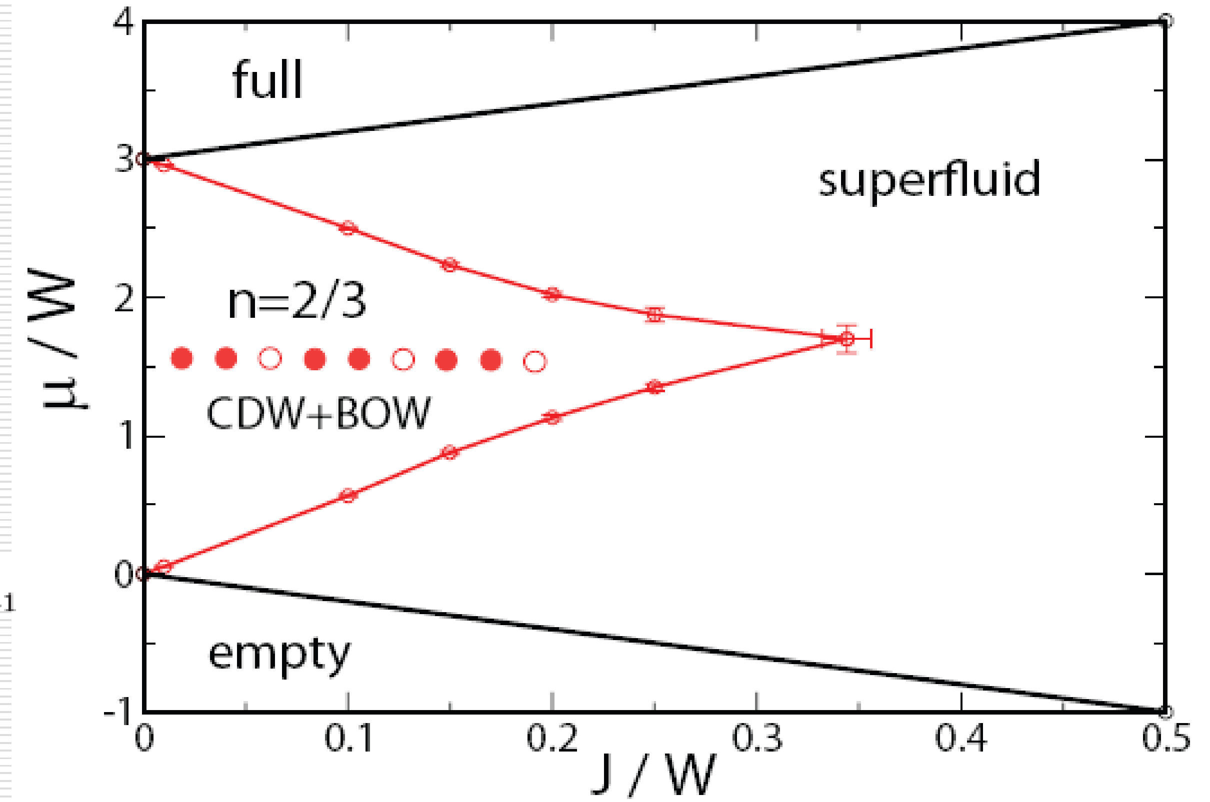
Boson

Related Work 0807.4563

$$S_{\mathrm{CDW}}(k) = \frac{1}{L}\sum_{j,l} \exp\left[ik(j-l)\right]\langle n_j n_l\rangle,$$

$$S_{\mathrm{BOW}}(k) = \frac{1}{L}\sum_{j,l} \exp\left[ik(j-l)\right]\langle K_j K_l\rangle,$$

bond operators $K_l = b_l^\dagger b_{l+1} + b_l b_{l+1}^\dagger$

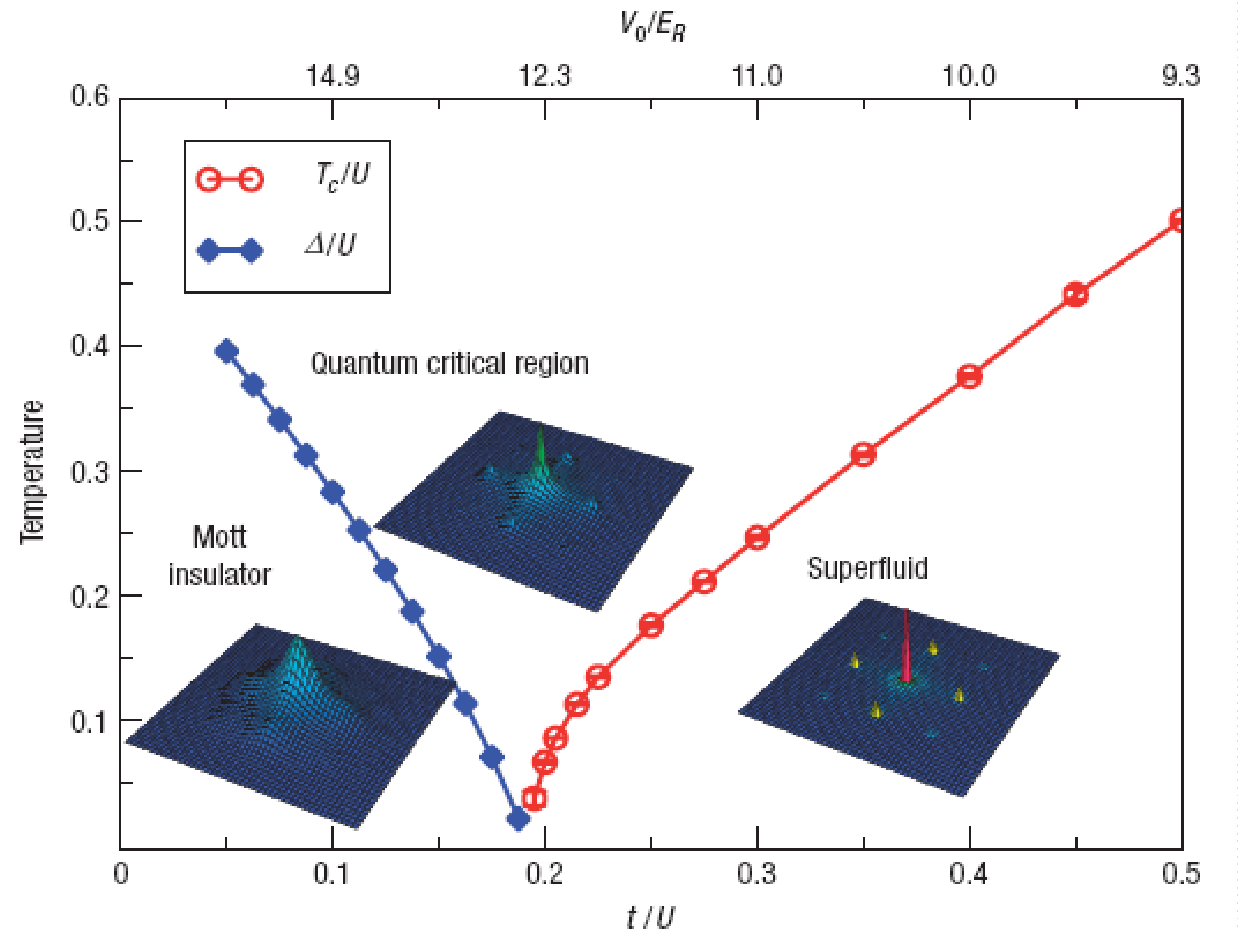


Quantum Monte-Carlo – Solid Phase at filling n=2/3

Mott Physics

Boson

Finite temperature

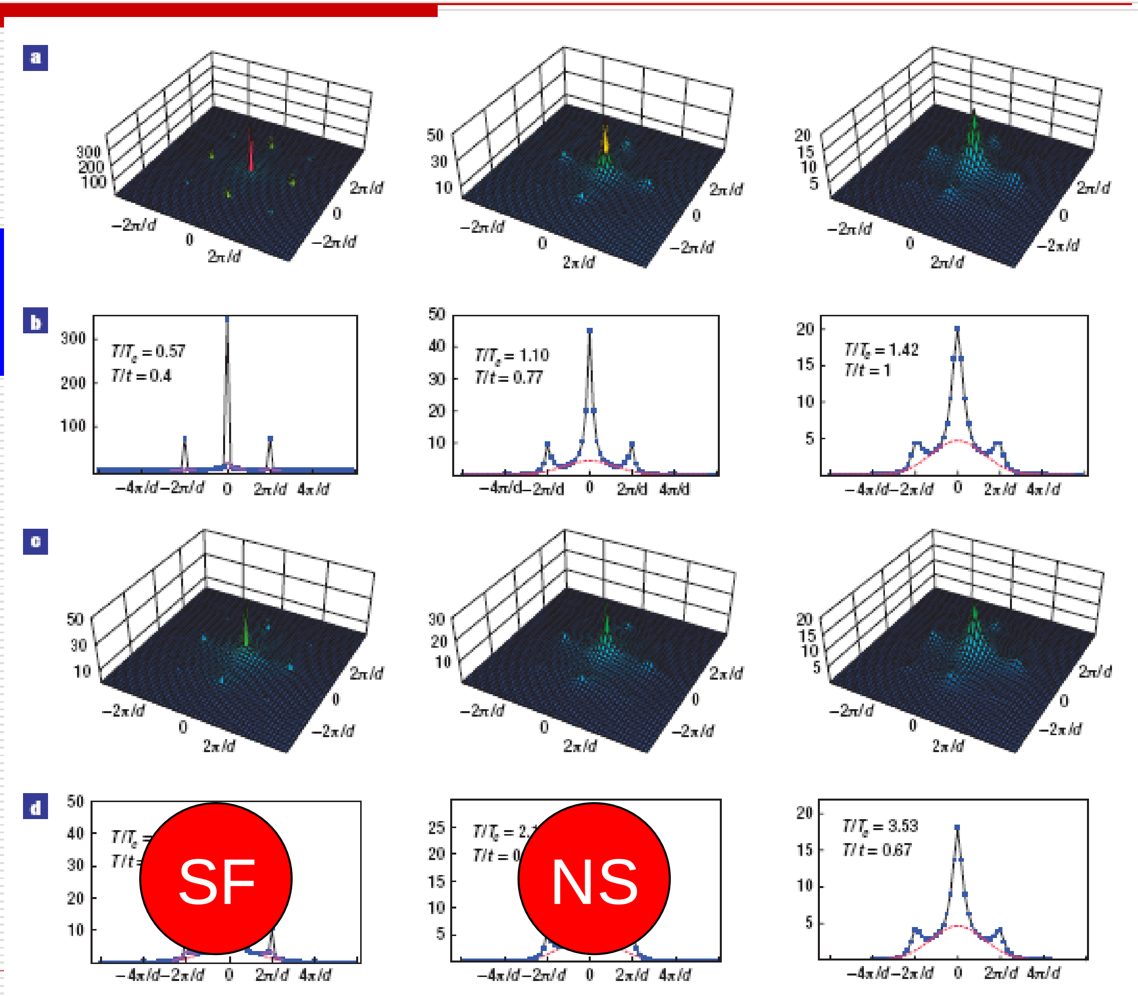


Nature Physics **4** 617(2008)

Mott Physics

Boson

More on interference pattern

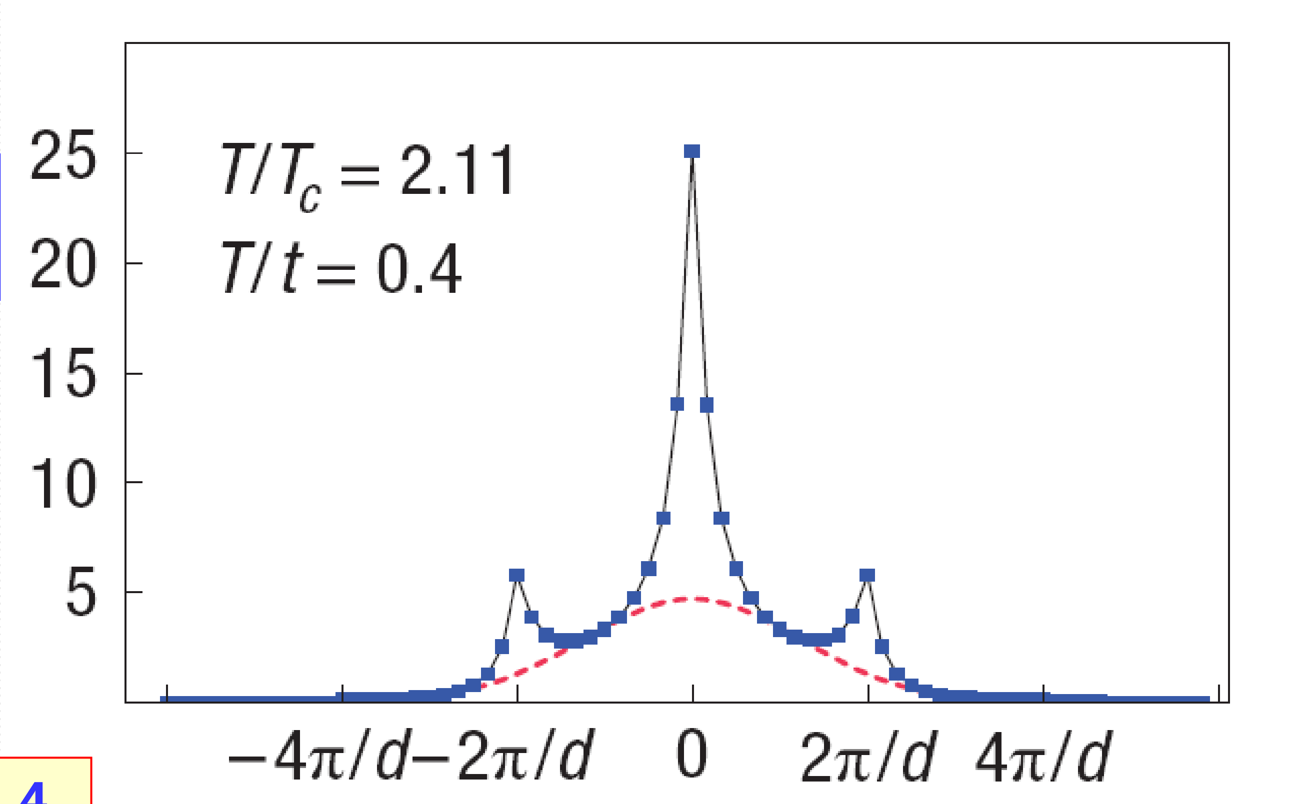


Nature Physics **4** 617(2008)

Mott Physics

Boson

Bimodal Normal State

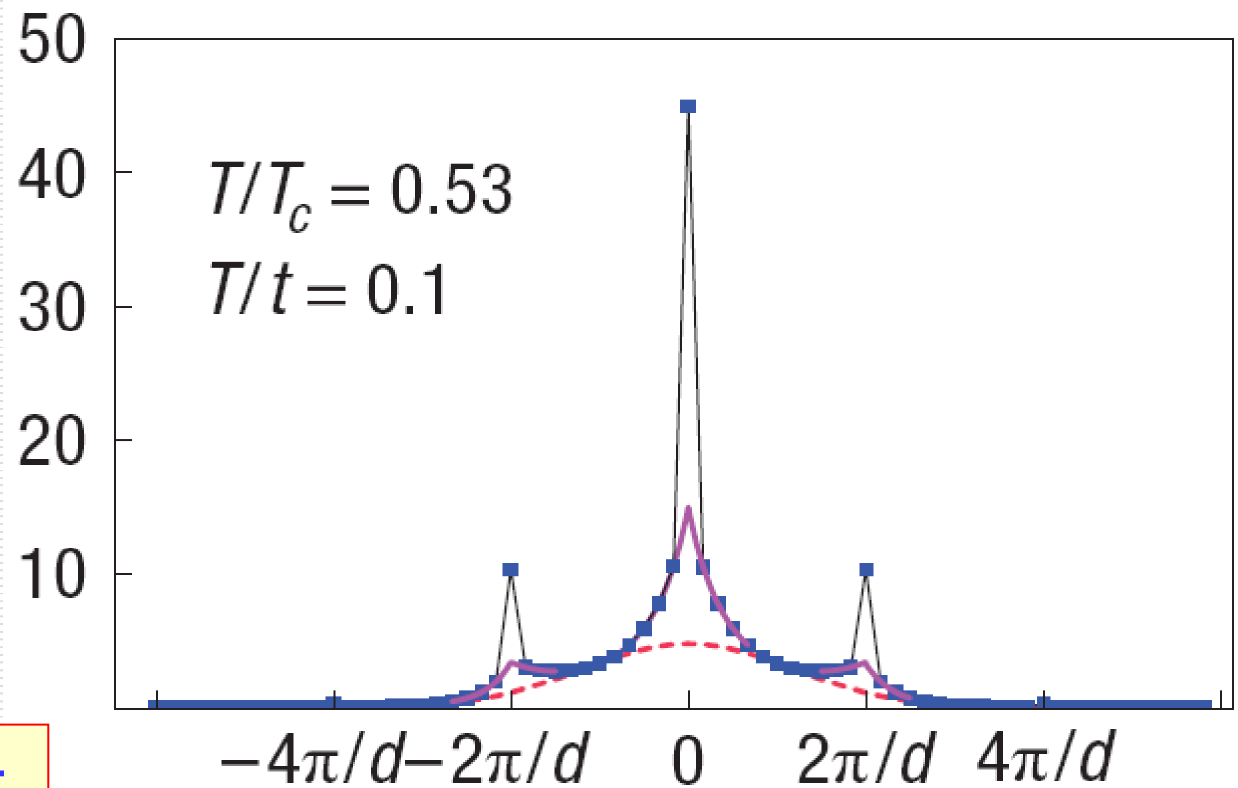


Nature Physics **4** 617(2008)

Mott Physics

Boson

Trimodal SF State



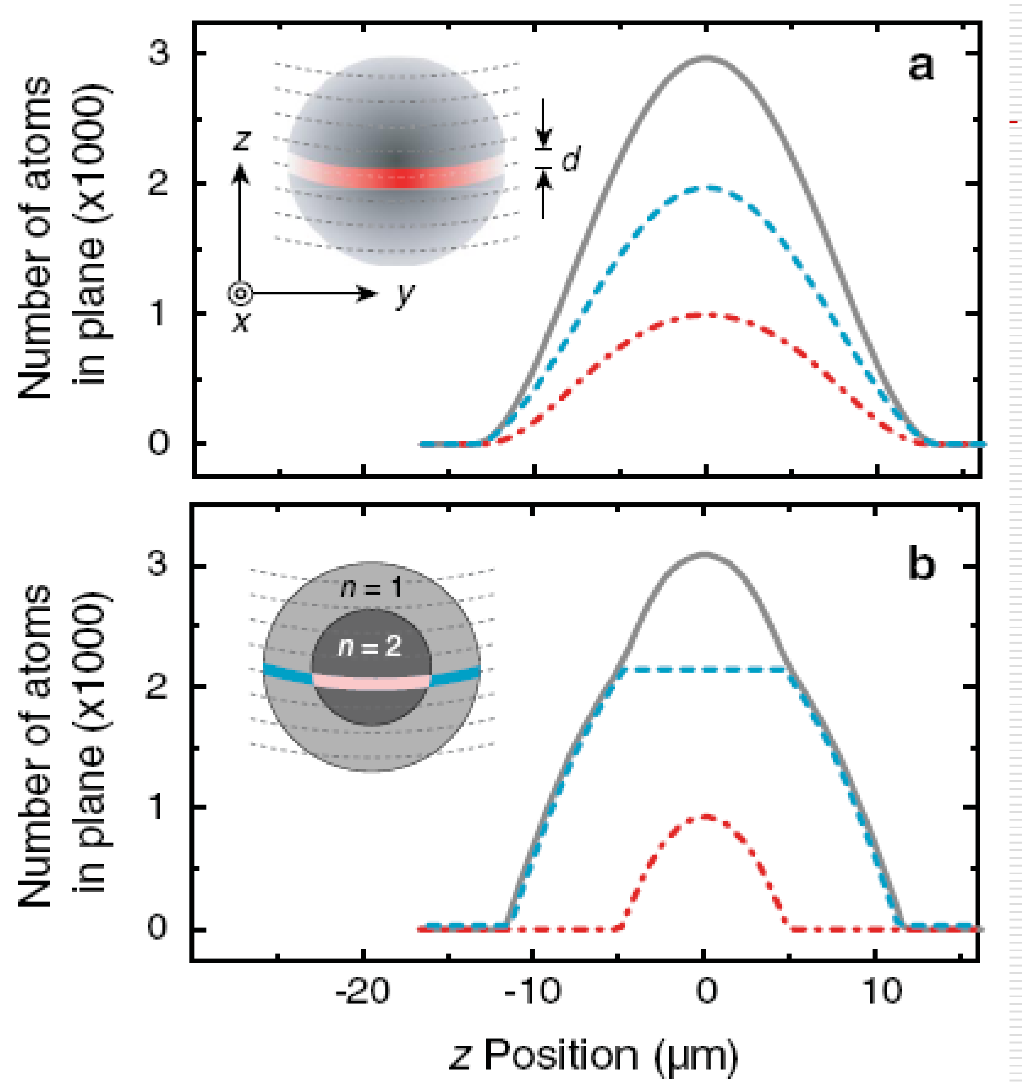
Nature Physics **4** 617(2008)

Mott Physics

Boson

Theory: V. A. Kashurnikov, et.al. PRA 2002
Experiments:
Bloch & Ketterle group 2006

Wedding cake structure in presence of trap



Mott Physics

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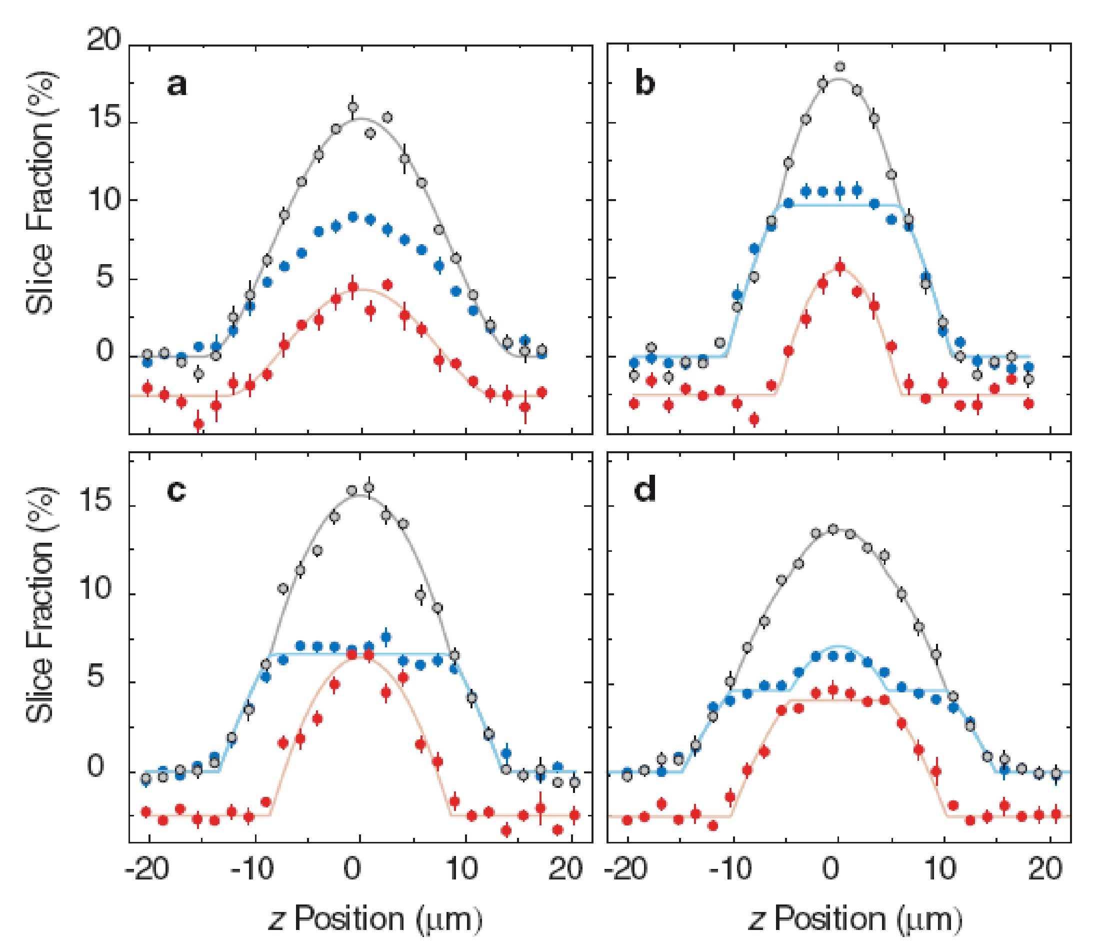
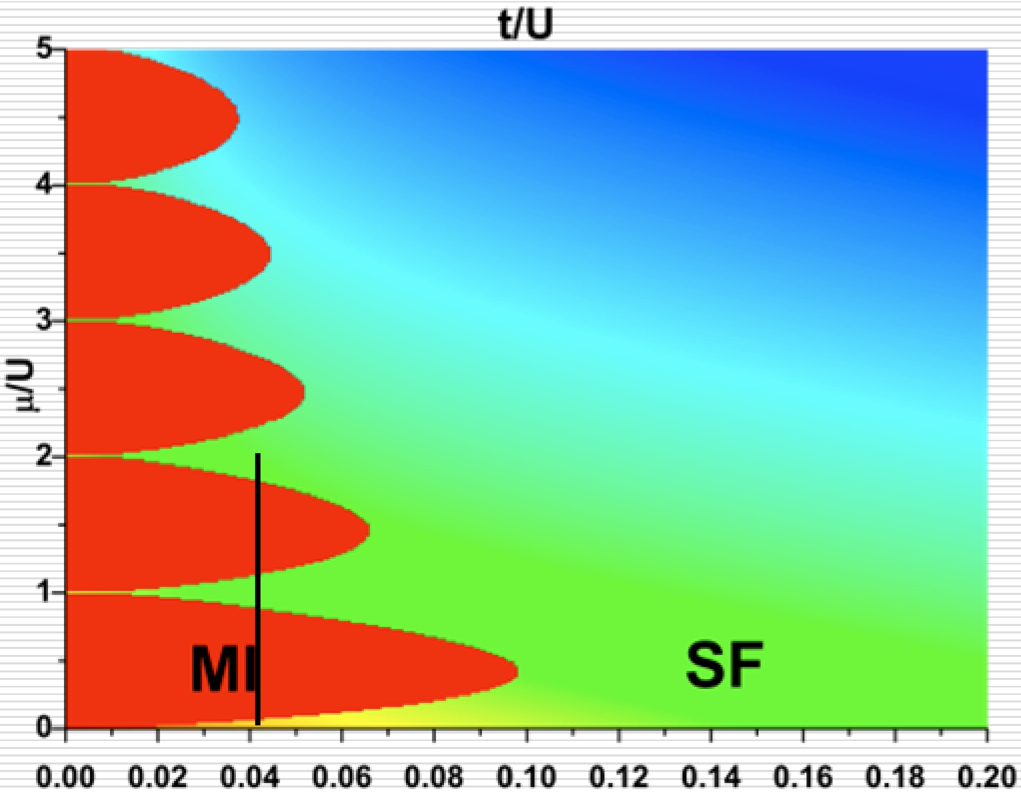
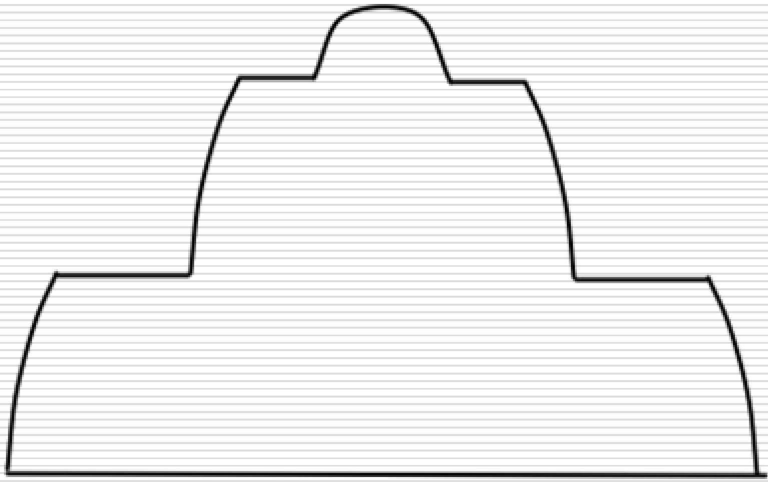


Mott Physics

Boson

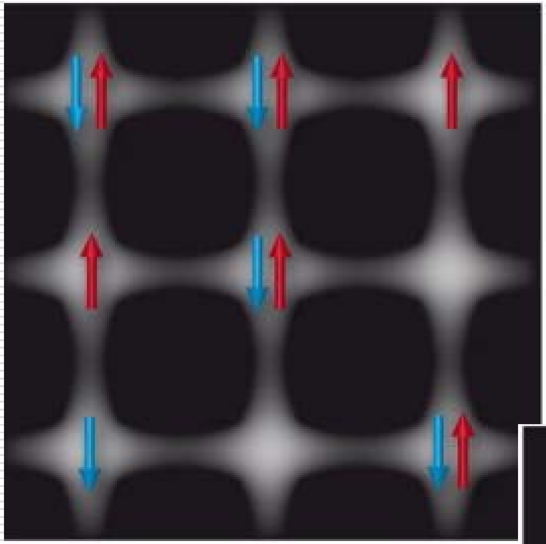
Wedding cake structure in presence of trap





Mott Physics

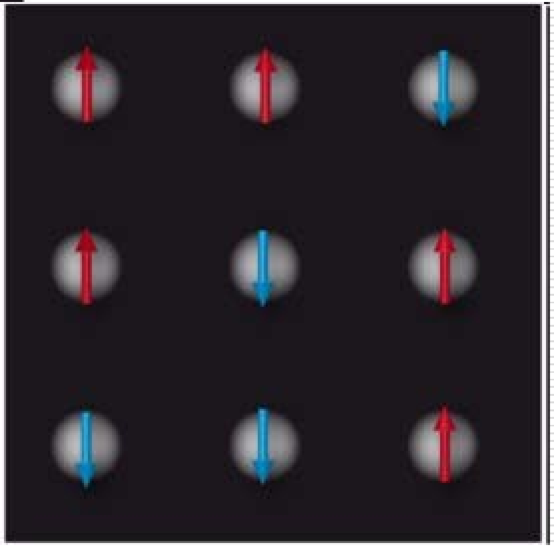
Fermion



metallic state:
Hopping, conducting

Mott insulating state:
No double occupancy, no hole

Nature **455** 204(2008)



Mott Physics

Fermion

$$\hat{H} = -J \sum_{\langle i,j\rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\downarrow}\hat{n}_{i,\uparrow} + V_t \sum_{i} (i_x^2 + i_y^2 + \gamma^2 i_z^2)\,(\hat{n}_{i,\downarrow} + \hat{n}_{i,\uparrow})\,.$$

3 energy scales: J, U, V_t

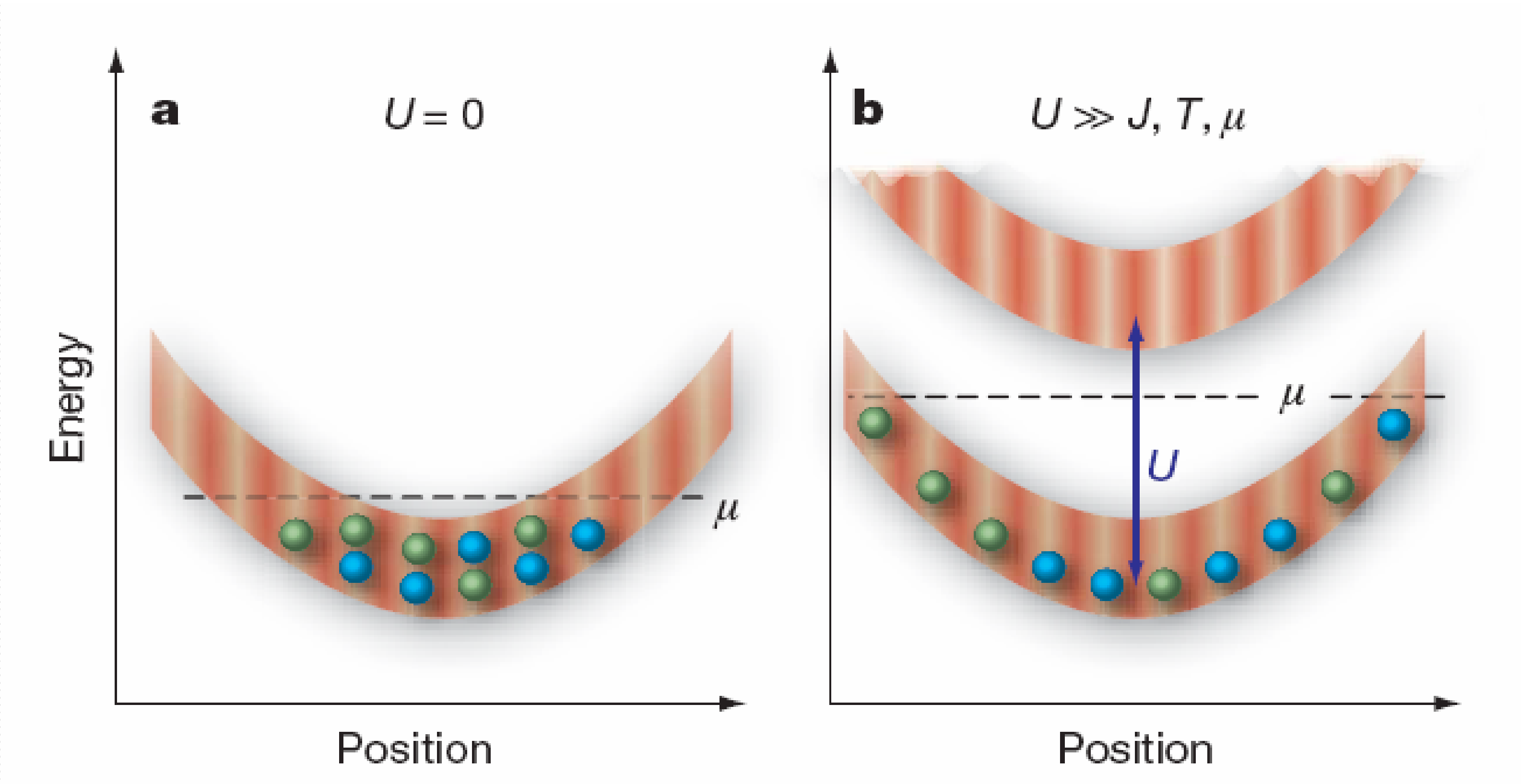
0809.1464

Confinement is more prominent in Fermion case

Mott Physics

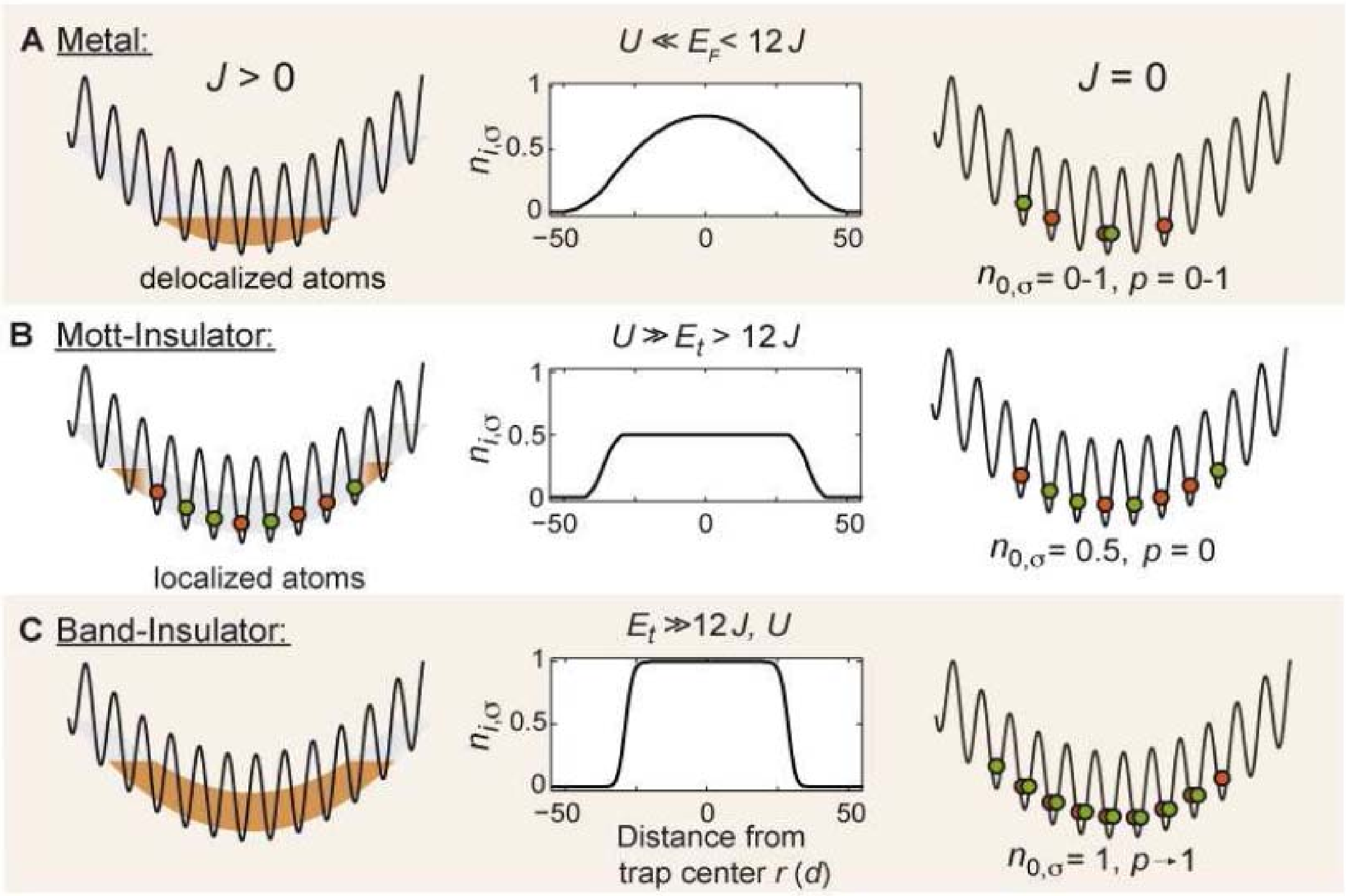
Fermion

Nature **455** 204(2008)



Mott Physics

Fermion



0809.1464

TG Gas

Lewi Tonks

Marvin D. Girardeau

TG Gas

Tonks 1936

Girardeau 1960

Paredes et.al. Nature 2004
Kinoshita et.al. Science 2004

Condensed-matter physics

Atomic beads on strings of light

Murray J. Holland

A new regime of strongly correlated quantum behaviour has been reached with the creation of a one-dimensional Tonks–Girardeau gas from ultracold atoms trapped within thin tubes of light.

TG Gas

In physics, a Tonks-Girardeau gas is a Bose-Einstein Condensate in which the repulsive interactions between bosonic particles confined to one dimension dominate the physics of the system

dimension dominate the physics of the system. In order to minimize their mutual repulsion, the bosons are prevented from occupying the same position in space. This mimics the Pauli exclusion principle for fermions, causing the bosonic particles to exhibit fermionic properties[1,2]. However, such bosons do not exhibit completely ideal fermionic (or bosonic) quantum behaviour; for example, this is reflected in their characteristic momentum distribution[3].

TG Gas

Bose-Fermi Mapping

$$\psi_{\mathrm{B}}(z_1, \ldots, z_N) = A(z_1, \ldots, z_N)\psi_{\mathrm{F}}(z_1, \ldots, z_N)$$

$$A(z_1, \ldots, z_N) = \prod_{1 \leqslant j < k \leqslant N} \mathrm{sgn}(z_k - z_j)$$

A Powerful Generalization

Cheon and Shigehara PRL 1999

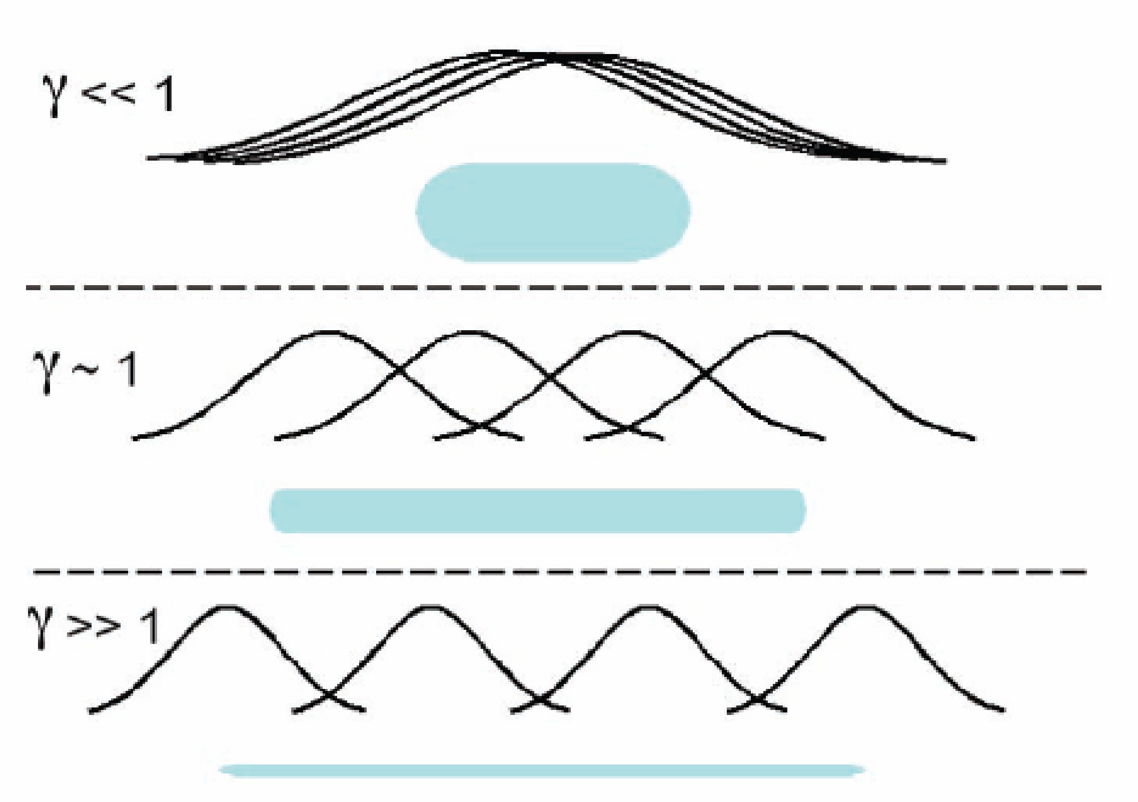
Bosons with strong but finite interactions map to spinless fermions with weak short-range interactions

TG Gas

$$\gamma = \varepsilon_{\rm int}/\varepsilon_{\rm kin}$$

$$\gamma = mg/\hbar^2 n$$

$$g = -\frac{\hbar^2}{\mu a_{\rm 1D}^{\rm B}}$$



The case of intermediate/finite interaction is more interesting …

1D Bose Gas - LL Model

$$H=\sum_i\left[\frac{p_i^2}{2m}+V_{\text{ext}}(x_i)\right]+\frac{\hbar^2}{m}c\sum_{i<j}\delta(x_i-x_j)$$

- 1D Bosons with repulsive δ interactions
- Ground- and excited-state of homogeneous system (V_{ext}=0) are exactly known from Bethe ansatz [Lieb, Liniger 1963]
- For interaction parameter $\gamma = mg/\hbar^2 n$>>1, problem is mapped exactly to TG gas

The case of intermediate/finite interaction is more interesting …

More 1D Models

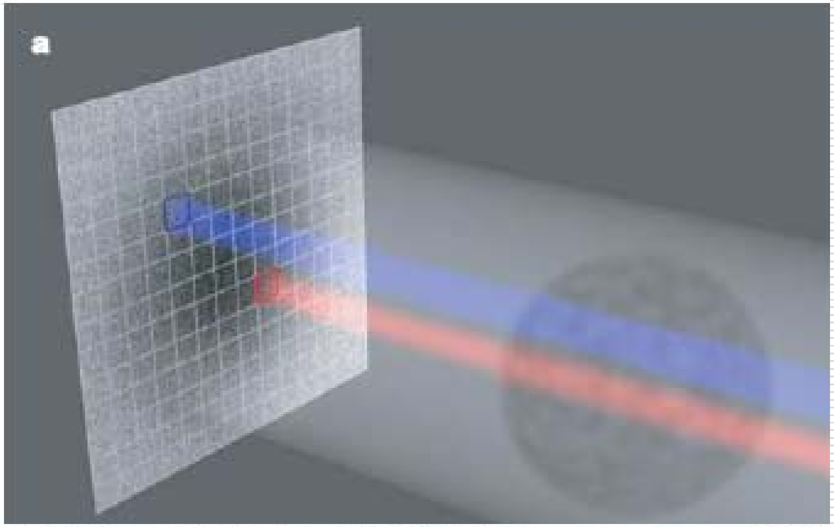
- 1D Bose Gas in Hard Wall
- p-wave interacting Fermion gas
- Potential with a δ-split
- Anyon gas
- 2 component, mixture
- …
- ⇨ Shu Chen's Talk

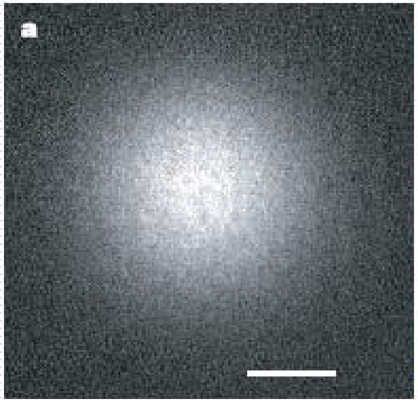
The case of intermediate/finite interaction is more interesting …

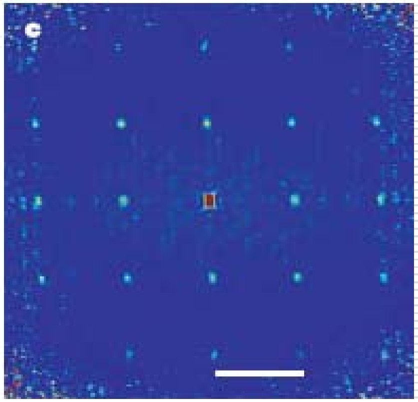
Mott Insulator and 1D Cold Atomic Gas are examples of strongly correlated states.

Novel Methods of Detection

- Standard Detection Method
  - Releasing the atoms from the trap
  - Destructive absorption spectroscopy
- Measurement
  - (Spin) density-density correlation
  - Higher order correlation functions







Novel Methods of Detection

- Novel Methods
  - atomic noise interferometry (Hanbury Brown-Twiss)

Altman *PRA* 70, 013603(2004) theoretical scheme

Fölling *Nature* 434, 481(2005) ^{87}Rb pair correlation (MI)

Rom *Nature* 444, 733(2006) ^{40}K anti-bunching (BI)

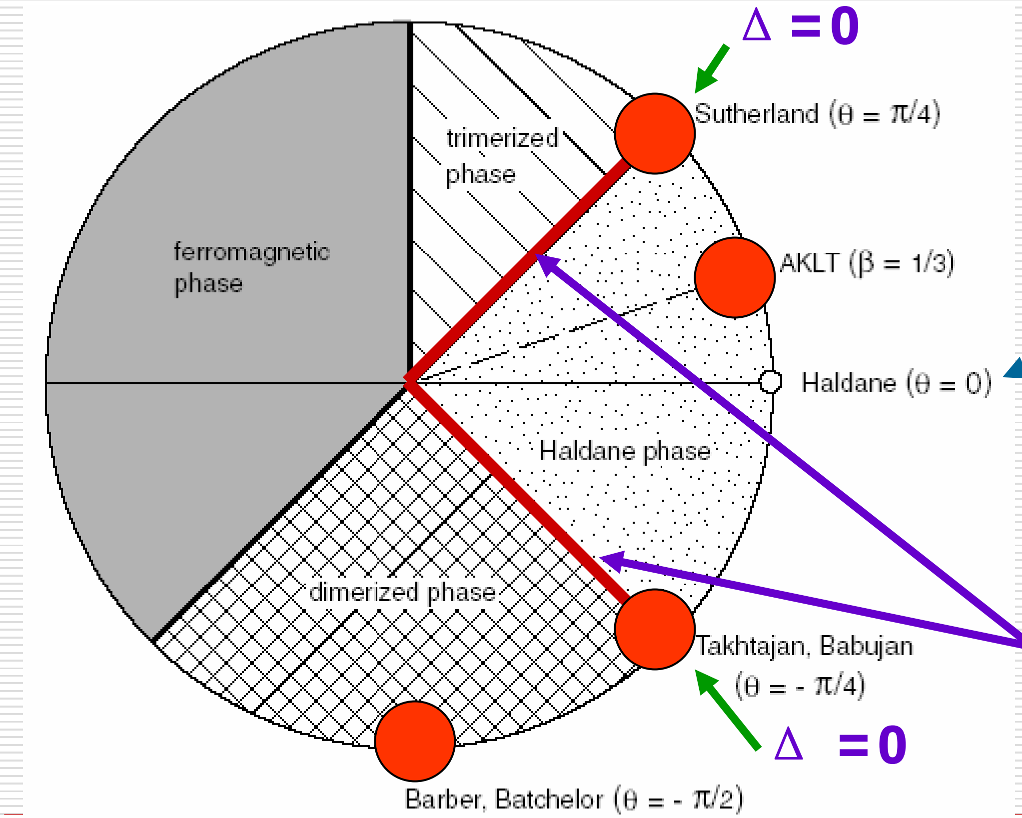
Schellekens *Science* 310, 648(2005) ^{4}He Bosonic HBT

Jeltes *Nature* 445, 402 (2007) ^{3}He Fermionic HBT

→ Hongwei Xiong’s Talk

QND detection 1D quantum AF

$$H = J\sum_j \left[(\cos\theta)\mathbf{S}_j\cdot\mathbf{S}_{j+1} + \sin\theta(\mathbf{S}_j\cdot\mathbf{S}_{j+1})^2\right]$$



Bilinear-Biquadratic **S=1** Heisenberg AFM chain

Haldane phase:
$-\pi/4 < \theta < \pi/4$

Δ =0.411J at θ = 0 for S=1, ξ=6

Δ =0.085J at θ = 0 for S=2, ξ =49

Critical points separating Haldane phase from others

QND detection 1D quantum AF

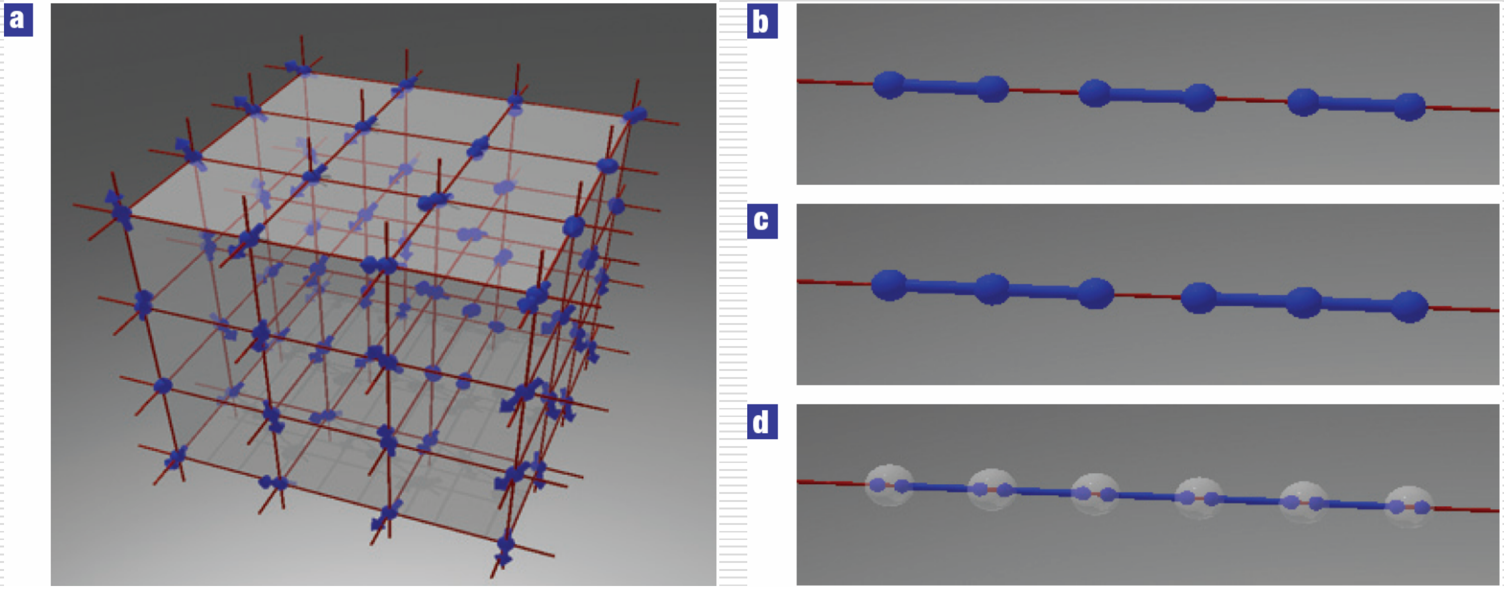


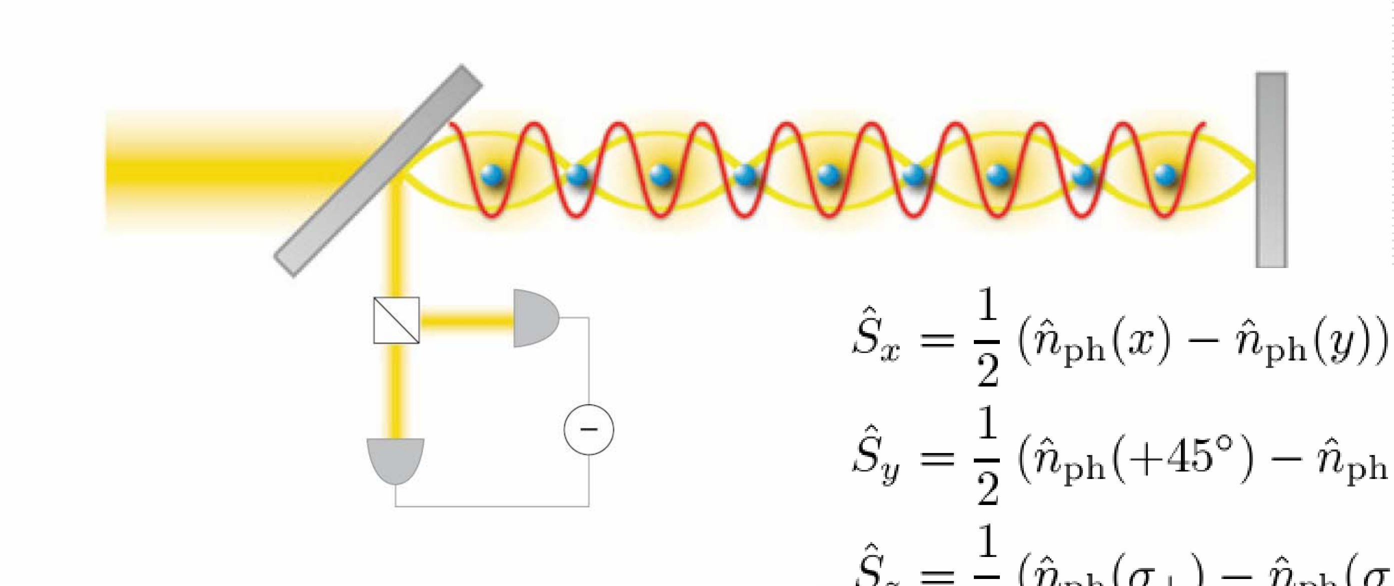
Figure 1 Antiferromagnetic states of spin-1 lattice systems. a, 3D cubic lattice with a paramagnetic state of unpolarized atoms. **b**, Dimerized state with pairs of neighbouring atoms forming singlets. **c**, Trimerized state with triples of neighbouring atoms forming singlets. **d**, AKLT state obtained from a concatenation of spin-1/2 singlets by projecting pairs of spins from different bonds into the subspace of total spin-1 (ref. 25).

nphy 776(2007)

QND detection 1D quantum AF

- Off-resonant interaction of spin-*F* atoms with a polarized light beam propagating in the *z* direction

$$\hat{H} = -\int_0^L \mathrm{d}z\,\rho A\left(a_0\hat{\phi} + a_1\hat{s}_z\hat{j}_z + a_2\left[\hat{\phi}\hat{j}_z^2 - \hat{s}_-\hat{j}_+^2 - \hat{s}_+\hat{j}_-^2\right]\right)$$



Stokes Vector

$$\hat{S}_x = \frac{1}{2}\left(\hat{n}_{\mathrm{ph}}(x) - \hat{n}_{\mathrm{ph}}(y)\right),$$

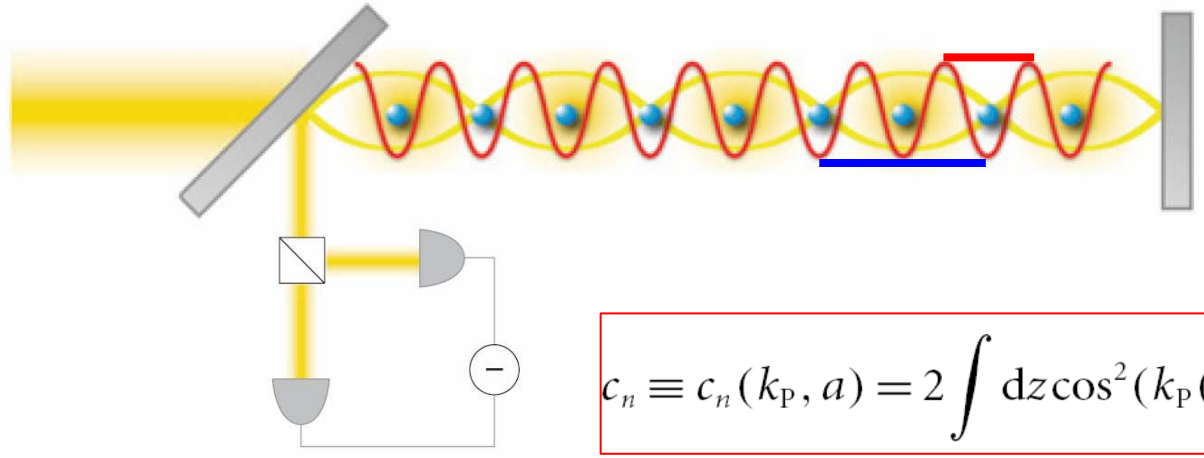
$$\hat{S}_y = \frac{1}{2}\left(\hat{n}_{\mathrm{ph}}(+45^\circ) - \hat{n}_{\mathrm{ph}}(-45^\circ)\right),$$

$$\hat{S}_z = \frac{1}{2}\left(\hat{n}_{\mathrm{ph}}(\sigma_+) - \hat{n}_{\mathrm{ph}}(\sigma_-)\right),$$

QND detection 1D quantum AF

- Off-resonant interaction of spin-*F* atoms with a polarized light beam propagating in the *z* direction

$$\hat{H} = -\kappa \hat{s}_3 \hat{J}_z^{\text{eff}} \qquad \hat{\mathbf{J}}^{\text{eff}} = \tilde{\sum}_n c_n \hat{\mathbf{j}}(z_n)$$



$$c_n \equiv c_n(k_P, a) = 2\int \mathrm{d}z \cos^2(k_P(z-a))|w(z-z_n)|^2$$

QND detection 1D quantum AF

- Off-resonant interaction of spin-F atoms with a polarized light beam propagating in the z direction

$$\begin{aligned}
\hat{S}_y^{\text{out}}(t) &= \hat{S}_y^{\text{in}}(t) + aS_x\hat{J}_z(t), \\
\hat{S}_z^{\text{out}}(t) &= \hat{S}_z^{\text{in}}(t), \\
\frac{\partial}{\partial t}\hat{J}_y(t) &= aJ_x\hat{S}_z^{\text{in}}(t), \\
\frac{\partial}{\partial t}\hat{J}_z(t) &= 0, \\
a &= -\frac{\gamma}{4A\Delta}\frac{\lambda^2}{2\pi}a_1.
\end{aligned}$$

QND

$$\hat{S}_i = \int s_i \mathrm{d}t$$

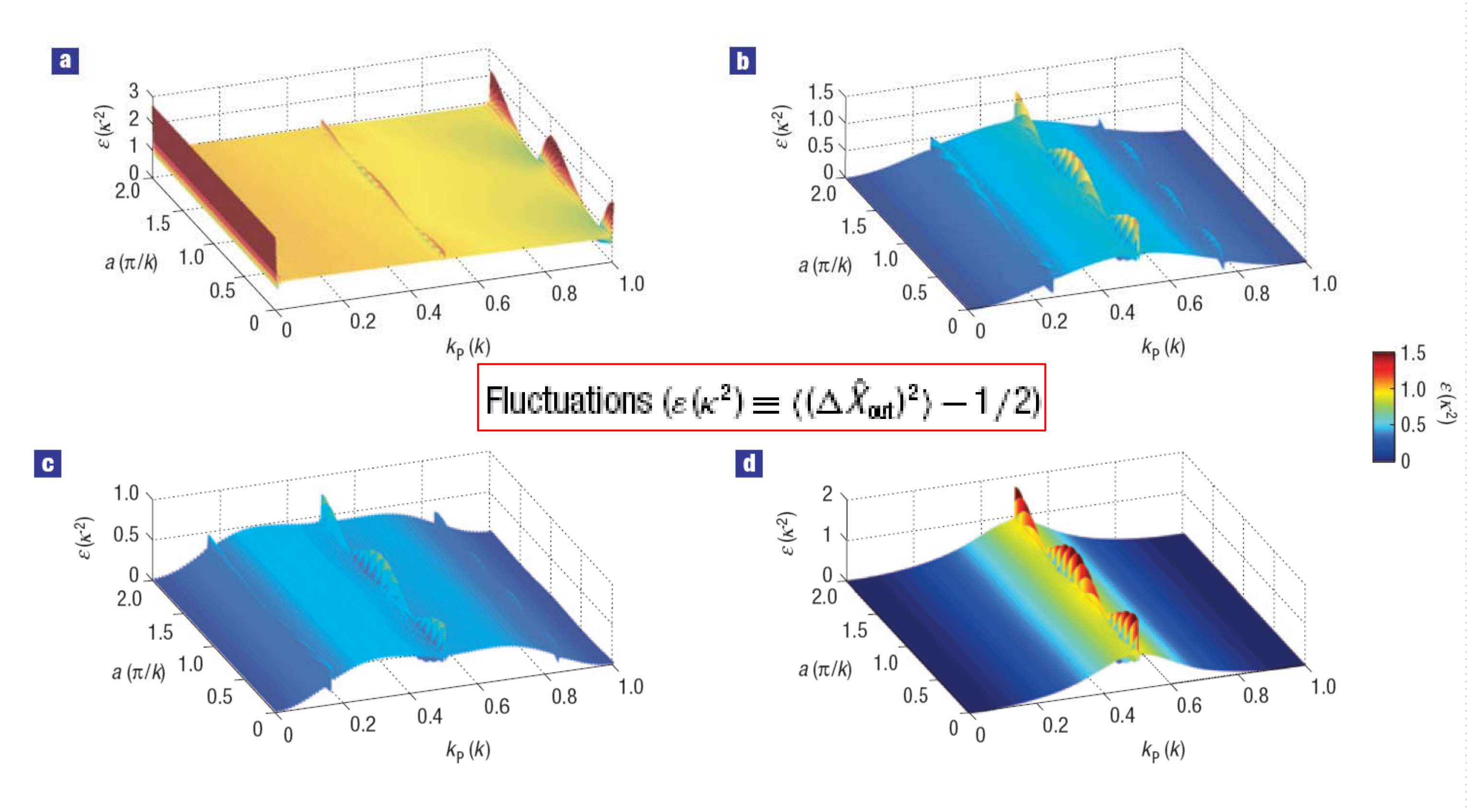
$$\langle \hat{S}_1 \rangle = N_{\text{ph}}/2 \gg 1$$

$$\hat{X} = \hat{S}_2/\sqrt{N_{\text{ph}}},\ \hat{P} = \hat{S}_3/\sqrt{N_{\text{ph}}}$$

$$\hat{X}_{\text{out}} = \hat{X}_{\text{in}} - \frac{\kappa}{\sqrt{FN_{\text{at}}}}\hat{J}_z^{\text{eff}}$$

J_z imprinted on X

QND detection 1D quantum AF

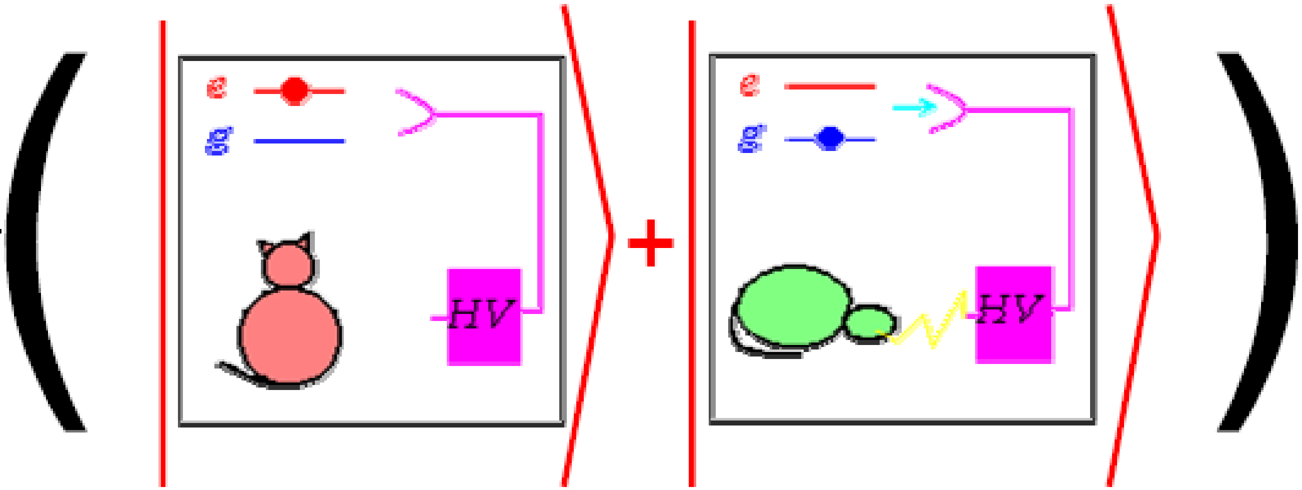


Detection of antiferromagnetic states of spin-1 lattice systems.

Cavity QED

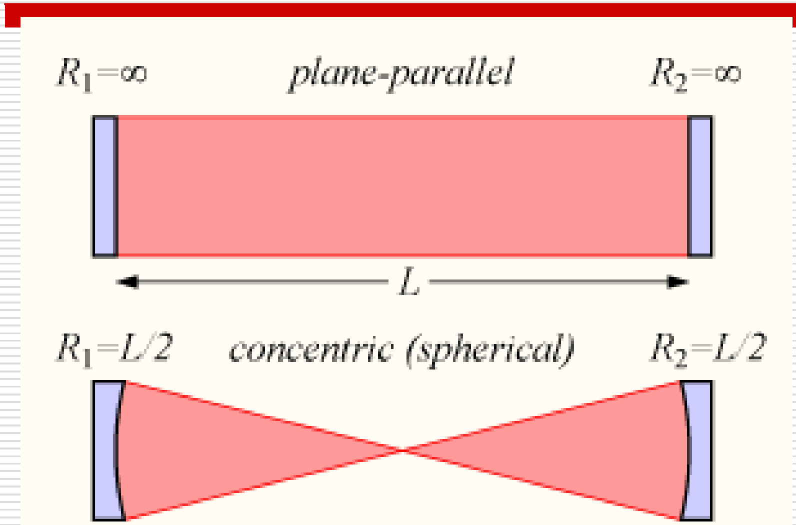
Cavity Quantum Electrodynamics
central paradigm for the study of
open quantum systems.

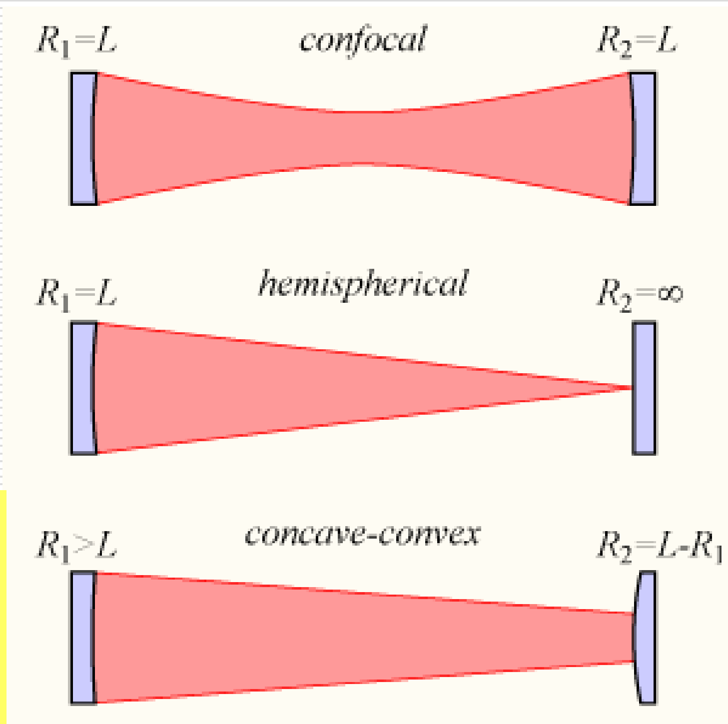
$$|\Psi_{at+\ldots+chat}\rangle = \frac{1}{\sqrt{2}}$$



Quantum measurement - decoherence

Cavity QED





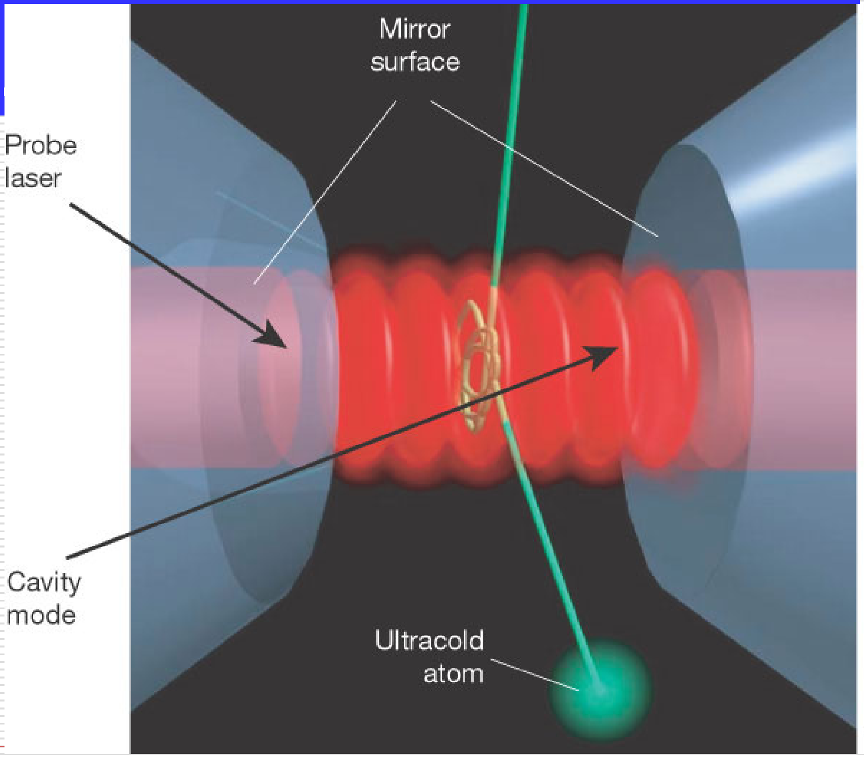
Cavity Mode
Detuning
Quality Factor
Finesse
Free Spectral Range (FSR)

Optical Cavity/Optical Resonator

Cavity QED

In addition to the coherent atom-field interactions, the system possesses two prominent decay channels: an excited atom may spontaneously emit light out of the cavity mode, and light may leak out through the cavity mirrors.

- Caltech: J. Kimble
- Munich: G. Rempe



The great advantage of employing a cavity is that these decoherence rates can be made small relative to the cavity mediated atom-field interaction.

$$\Gamma_{at}, \Gamma_{cav} << \Omega_0$$

Cavity QED

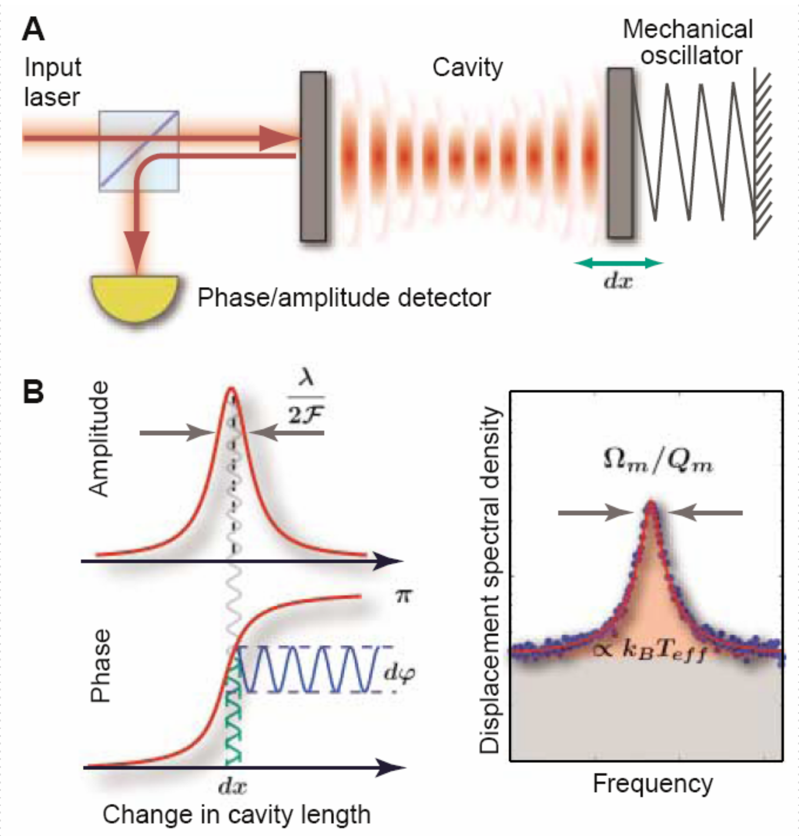
Most experimental and theoretical investigations in Cavity QED study the interactions of a single mode of the electro-magnetic field with a two-level atom. In this case, the dynamics may be modeled by the unconditional **master equation**

$$\dot{\rho} = -i[\mathcal{H}, \rho] + 2\kappa\mathcal{D}[a]\rho + \gamma_{||}\mathcal{D}[\sigma_-]\rho$$

where H is the Hamiltonian in a frame rotating at the frequency of the driving field

$$\mathcal{H} = \Delta_c a^\dagger a + \Delta_a \sigma_+ \sigma_- + i g_0 (a^\dagger \sigma_- - a\sigma_+) + i\mathcal{E}(a^\dagger - a)$$

Cavity Optomechanics

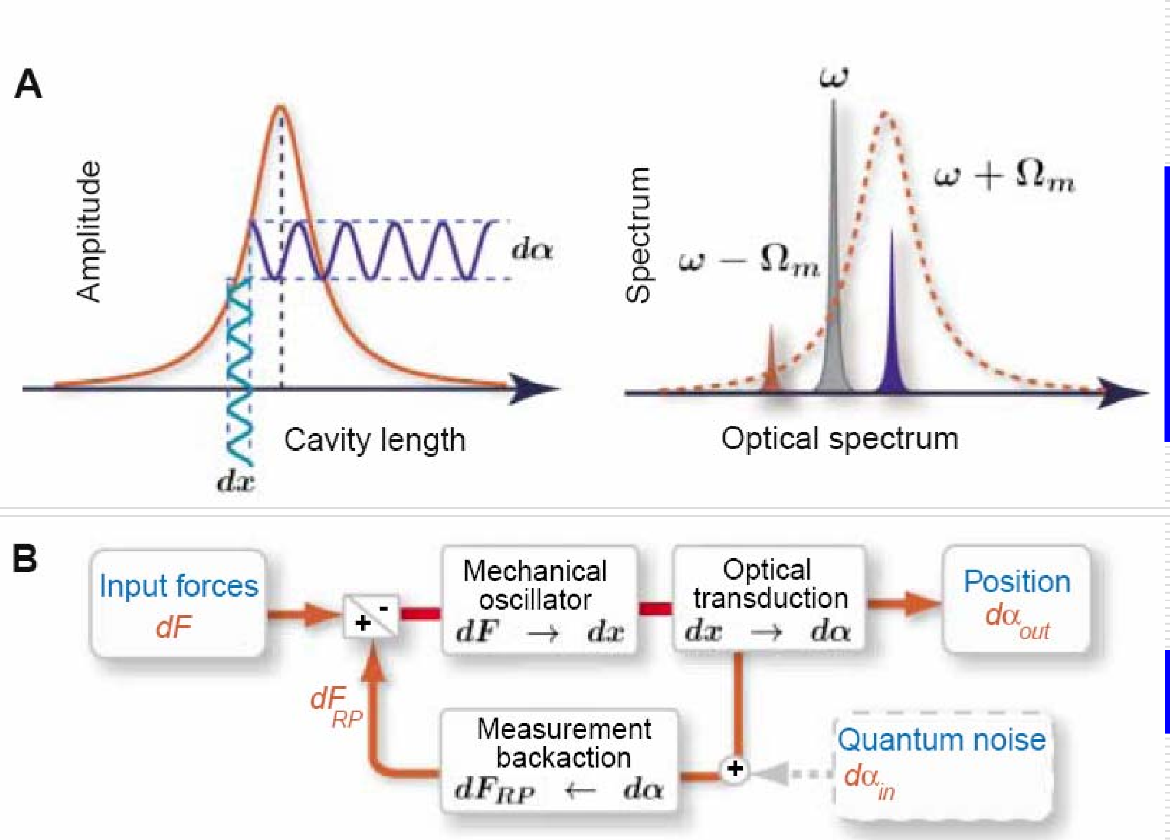


Optical resonator (Optical cavity)

On resonance dα = 0

Conversion of mechanical displacement *dx* to the phase change dφ

Cavity Optomechanics



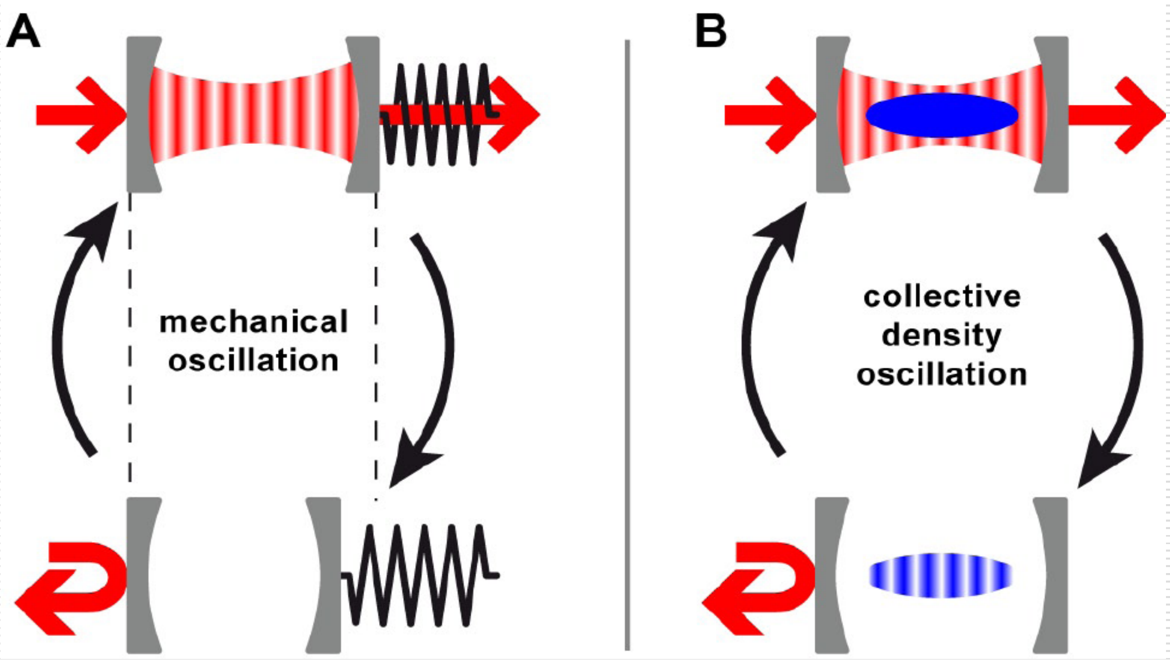
Laser is detuned with respect to the cavity resonance

Dynamic Back-Action

Coupling of optical dα and mechanical dx degrees of freedom

Cavity Optomechanics

Collective density excitation of BEC as mechanical oscillator



$$i\hbar\dot{\psi}(x)=\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}+\langle\hat{a}^\dagger\hat{a}\rangle\hbar U_0\cos^2(kx)+V_{ext}(x)+g_{1D}|\psi|^2\right]\psi(x)$$

$$i\dot{\hat{a}}=-\left[\Delta_c-U_0\langle\cos^2(kx)\rangle+i\kappa\right]\hat{a}+i\eta$$

Science Express 1163218
Science **321**, 1172 (2008)

Cavity Probing

LETTERS

Probing quantum phases of ultracold atoms in optical lattices by transmission spectra in cavity quantum electrodynamics

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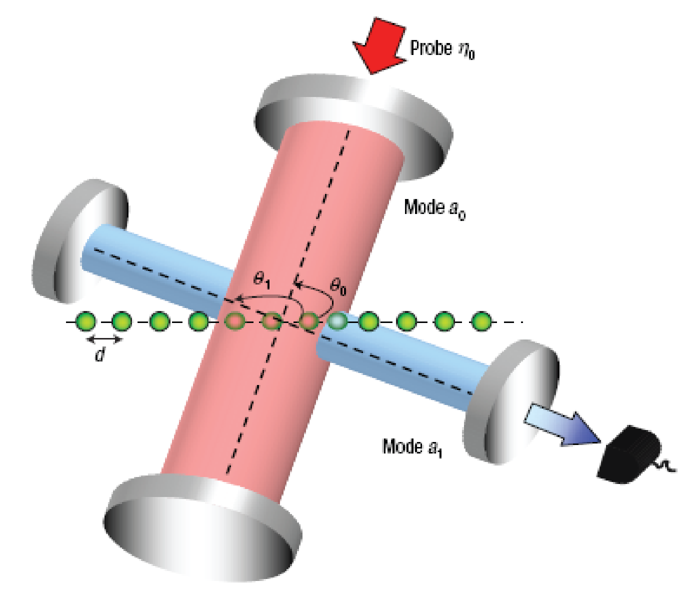
Atomic quantum statistics can be mapped on transmission spectra of high-*Q* cavities

A new detection scheme



Cavity Probing

$$\dot{a}_l = -i\left(\omega_l + \delta_l \hat{D}_{ll}\right) a_l - i\delta_m \hat{D}_{lm} a_m - \kappa a_l + \eta_l(t),$$

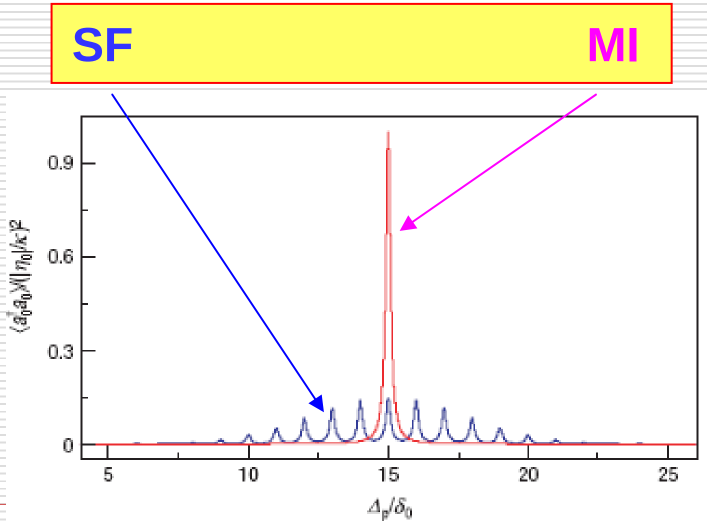
$$\text{with} \quad \hat{D}_{lm} \equiv \sum_{i=1}^{K} u_l^*(\mathbf{r}_i) u_m(\mathbf{r}_i) \hat{n}_i,$$

$$|\Psi\rangle_{\mathrm{MI}} = \prod_{i=1}^{M} |q_i\rangle_i \equiv |q_1, \ldots, q_M\rangle$$

$$|\Psi\rangle_{\mathrm{SF}} = \sum_{q_1,\ldots,q_M} \sqrt{N!/M^N} / \sqrt{q_1! \cdots q_M!} |q_1, \ldots, q_M\rangle$$

Only one cavity mode a_0 ($a_1 \equiv 0$)

$$a_0^\dagger a_0 = f(\hat{n}_1, \ldots, \hat{n}_M) = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$



Cavity Probing

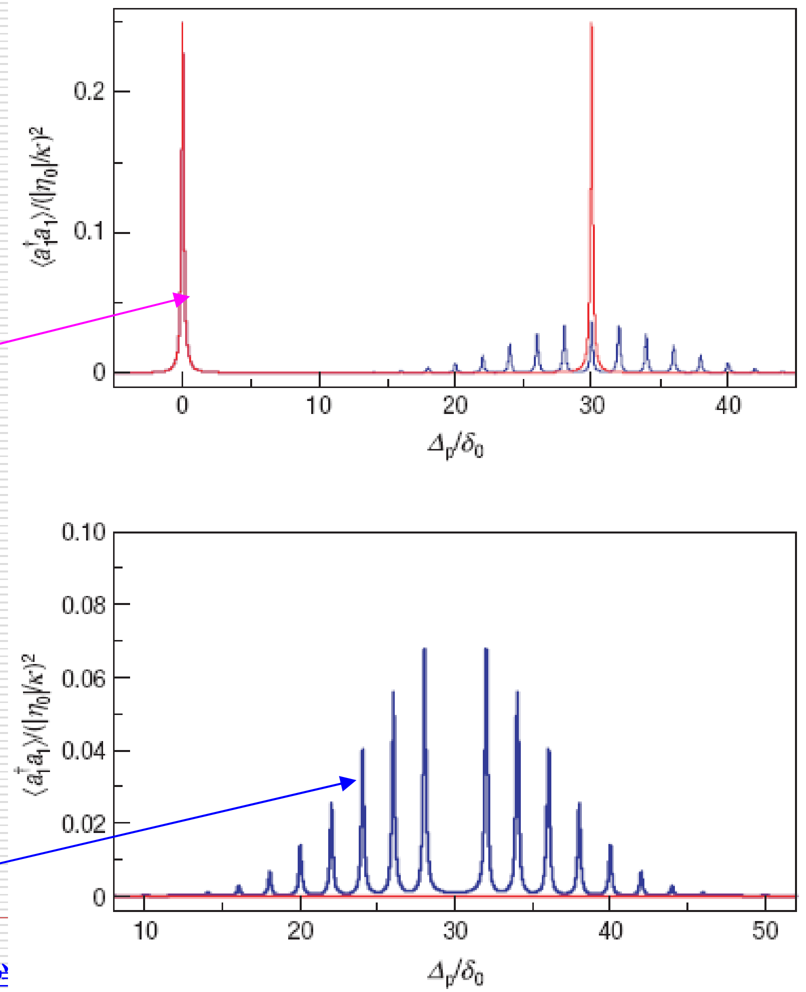
2 cavity modes a_0 , a_1

$$a_1^\dagger a_1 = \frac{\delta_1^2 \hat{D}_{10}^\dagger \hat{D}_{10} |\eta_0|^2}{[\hat{\Delta}_p'^2 - \delta_1^2 \hat{D}_{10}^\dagger \hat{D}_{10} - \kappa^2]^2 + 4\kappa^2 \hat{\Delta}_p'^2}$$

The transmission spectrum of a high-*Q* cavity directly maps the distribution function of ultracold atoms

MI

SF



Cavity-enhanced detection of magnetic orders in lattice spin models

2-species Bose-Hubbard model

VOLUME 90, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 14 MARCH 2003

Counterflow Superfluidity of Two-Species Ultracold Atoms in a Commensurate Optical Lattice

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In the Mott-insulator regime, two species of ultracold atoms in an optical lattice can exhibit the low-energy counterflow motion. We construct effective Hamiltonians for the three classes of the two-species (fermion-fermion, boson-boson, and boson-fermion–type) insulators and reveal the conditions when the resulting ground state supports super-counter-fluidity (SCF), with the alternative being phase segregation. We emphasize a crucial role of breaking the isotopic symmetry between the species for realizing the SCF phase.

Super-Counter-Fluidity (SCF)
Paired Superfluid Vacuum (PCF)

2-species Bose-Hubbard model

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Controlling Spin Exchange Interactions of Ultracold Atoms in Optical Lattices

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We describe a general technique that allows one to induce and control strong interaction between spin states of neighboring atoms in an optical lattice. We show that the properties of spin exchange interactions, such as magnitude, sign, and anisotropy, can be designed by adjusting the optical potentials. We illustrate how this technique can be used to efficiently "engineer" quantum spin systems with desired properties, for specific examples ranging from scalable quantum computation to probing a model with complex topological order that supports exotic anyonic excitations.

XXZ

Kitaev Model

2-species Bose-Hubbard model

$$H = -\sum_{\langle ij\rangle\sigma}(t_{\mu\sigma}a^{\dagger}_{i\sigma}a_{j\sigma} + \text{H.c.}) + \frac{1}{2}\sum_{i,\sigma}U_{\sigma}n_{i\sigma}(n_{i\sigma}-1) + U_{\uparrow\downarrow}\sum_{i}n_{i\uparrow}n_{i\downarrow}, \quad (1)$$

Definition of spin order

$$\mathbf{S}_b = (1/2)\sum_{\sigma\sigma'}a^{\dagger}_{b\sigma}\vec{\sigma}_{\sigma\sigma'}a_{b\sigma'}$$

$$t_{\mu\sigma} \ll U_{\sigma}$$

$$H = \sum_{\langle i,j\rangle}[\lambda_{\mu z}\sigma^z_i\sigma^z_j \pm \lambda_{\mu\perp}(\sigma^x_i\sigma^x_j + \sigma^y_i\sigma^y_j)].$$

Phase Diagram

Phys. Rev. B. 72, 184507

- SF

$$\left|\Psi_{SF}\right\rangle = \left(1/\sqrt{M^{N_1}N_1!M^{N_2}N_2!}\right)\left(\sum_i b_{i1}^{+}\right)^{N_1}\left(\sum_i b_{i2}^{+}\right)^{N_2}\left|0\right\rangle$$

- MI

$$\left|\Psi\right\rangle = \prod_{i\in A, j\in B}\left|\psi_A\right\rangle_i\left|\psi_B\right\rangle_j$$

$$\left|\psi_{AB}\right\rangle = \cos\frac{\theta_{AB}}{2}\left|1,0\right\rangle_i + \exp\left(i\phi_{AB}\right)\sin\frac{\theta_{AB}}{2}\left|0,1\right\rangle_i$$

- 反铁磁（AF） $\theta_A\left(\theta_B\right)=0\left(\pi\right)$ or $\theta_B\left(\theta_A\right)=0\left(\pi\right)$
- 铁磁（FM） $\theta_A=\theta_B=0$
- XY相 $\theta_A=\theta_B\neq 0$

MI Phase Diagram

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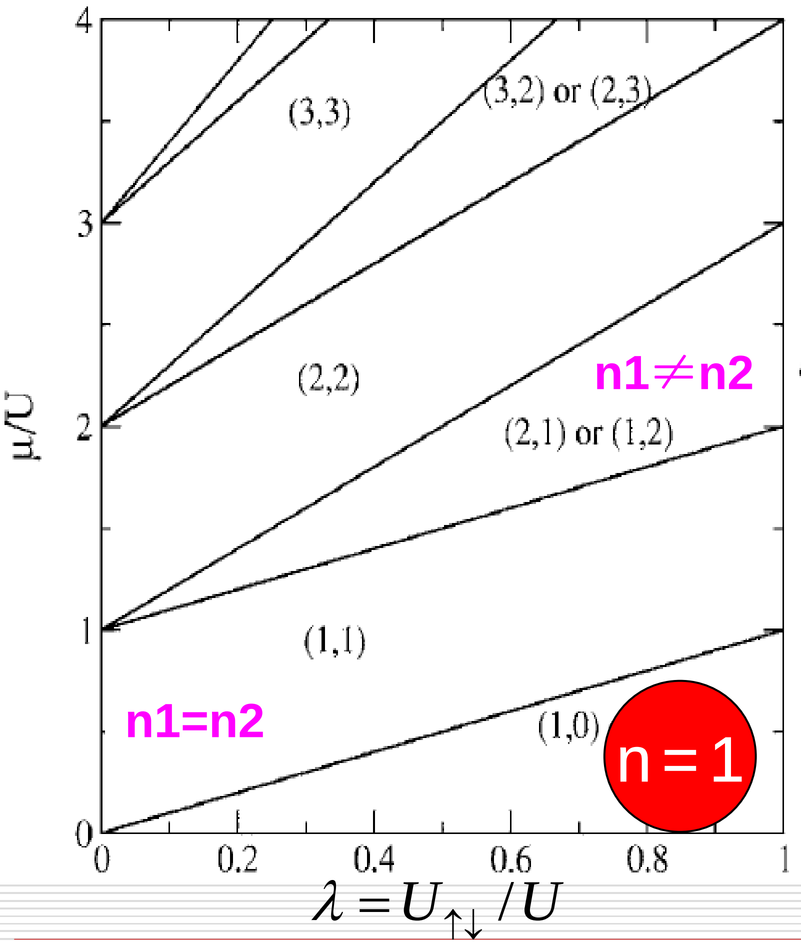
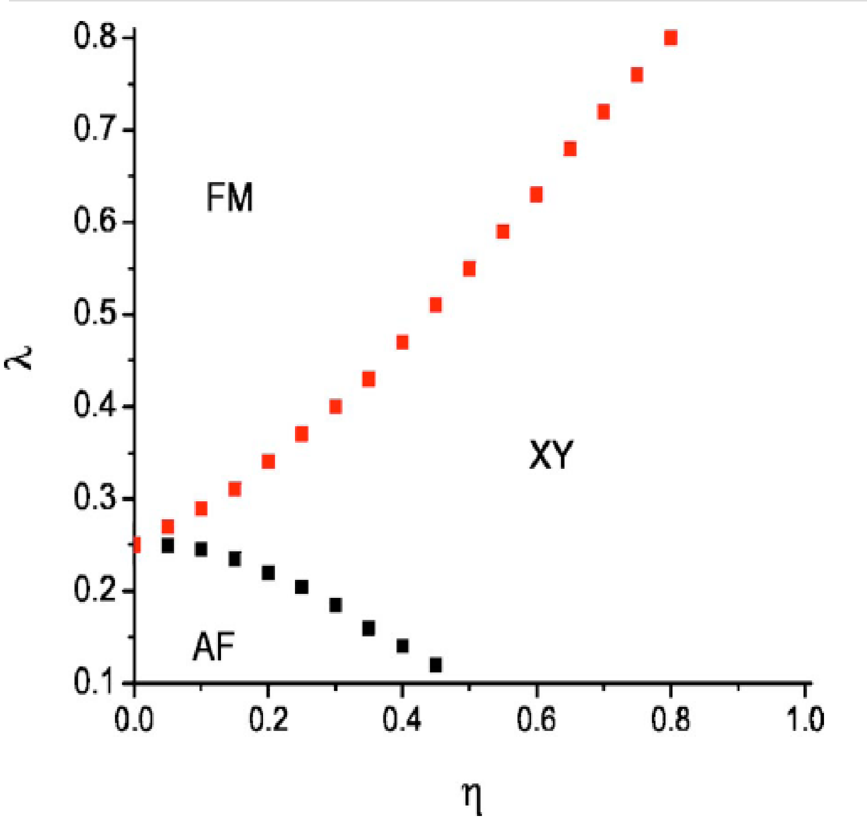
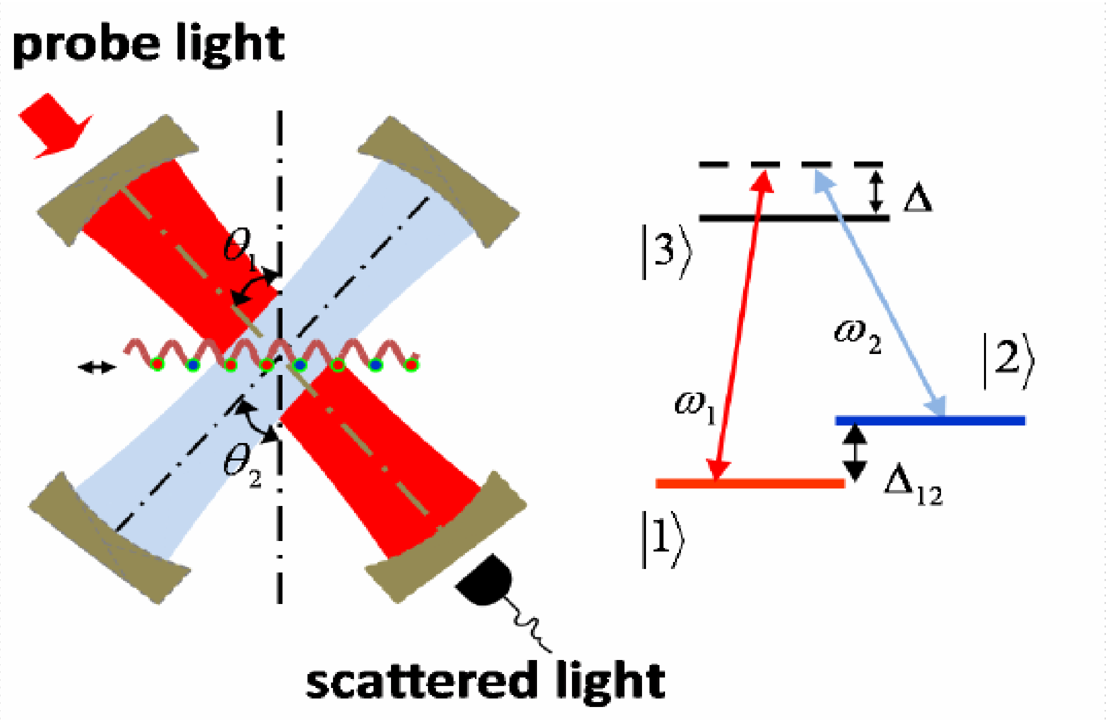




Phase diagram in MI with odd filling

Experimental Setup



Schematic illustration of the proposed experimental setup and the level diagram for a bosonic atom with two states resonantly coupled to the two cavities.

Model Hamiltonian

$$H = H_B + H_I$$

Bose-Hubbard Hamiltonian for two components

$$H_I = \sum_{l=1,2} \hbar \omega_l a_l^\dagger a_l - i\hbar\eta\left(a_1 e^{i\omega_{1p}t} - h.c.\right)$$
$$+\hbar\delta_1 \sum_{i=1}^{K} |u_1|^2 n_{i1} a_1^\dagger a_1 + \hbar\delta_2 \sum_{i=1}^{K} |u_2|^2 n_{i2} a_2^\dagger a_2$$
$$+\hbar\Omega \sum_{i=1}^{K} \left(A_i a_1^\dagger a_2 b_{i1}^\dagger b_{i2} + h.c.\right)$$

Coherent Pumping

Raman matched two-photon process

Semiclassical Approximation

No probe light η=0 and a_2 classical light - constant c-number

$$a_1^\dagger a_1 = |C|^2 \hat{D}^\dagger \hat{D}$$

$$C = -i\Omega a_2 / (i\Delta_{12} + \kappa)$$

$$\hat{D} = \sum_{i=1}^{K} A_i S_i^-$$

$$A_i(\theta_1, \theta_2) = u_1^*(\mathbf{r}_i) u_2(\mathbf{r}_i)$$

This operator is Spin related

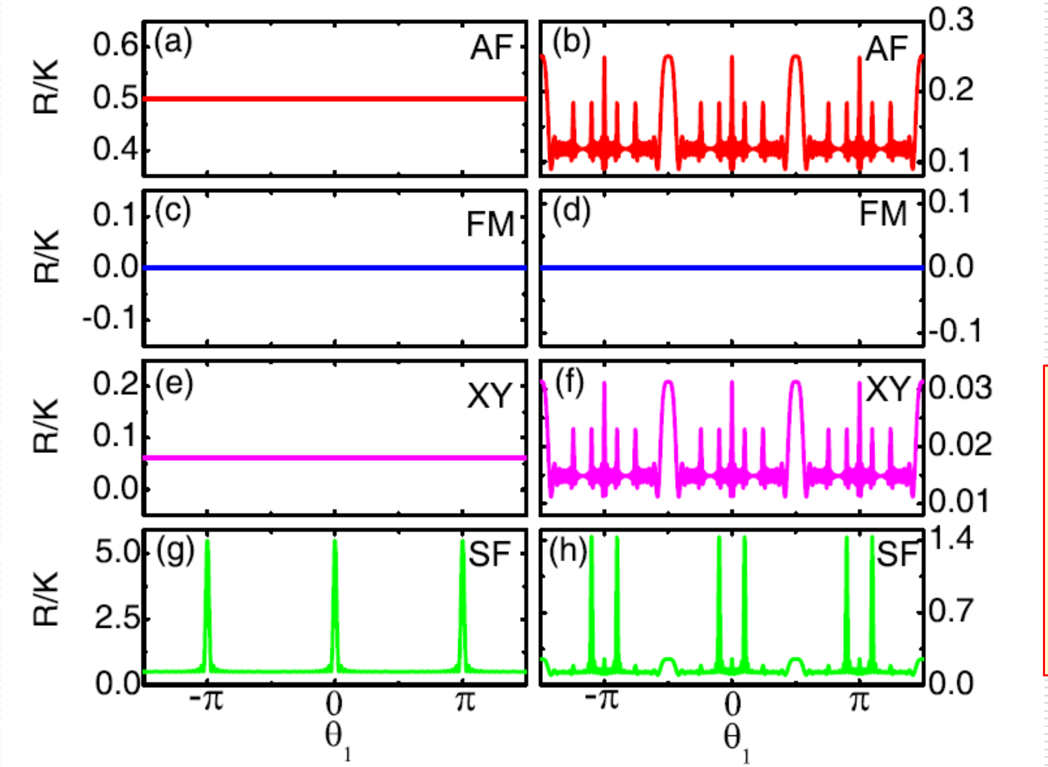
Semiclassical Approximation

	$\langle a_1^\dagger a_1 \rangle_{\theta_1=0}$	$\langle a_1^\dagger a_1 \rangle_{\theta_1=\pi/2}$
AF	$K\|C\|^2/2$	$K\|C\|^2/2$
FM	0	0
XY	$(K+3K^2)\|C\|^2/16$	$K\|C\|^2/16$
SF	$n_2(n_1K+1)K\|C\|^2$	$n_2K\|C\|^2$

TABLE I: Cavity 1 photon number for the four quantum phases of the two-component Bose-Hubbard model at the diffraction maxima (minima) with $\theta_1 = 0$ ($\theta_1 = \pi/2$) and $\theta_2 = 0$.

Scattered Photon Number

Semiclassical Approximation



$$R(\theta_1,\theta_2)=\langle D^\dagger D\rangle-\langle D^\dagger\rangle\langle D\rangle$$

Angular distribution of noise function R characterizes the quantum fluctuation of lattice spins

Traveling wave **Standing wave**

Coherent Pumping

$$\langle a_1^+ a_1 \rangle = \eta^2 \left(\kappa^2 + \zeta_2^2 \right) / B, \qquad \langle a_2^+ a_2 \rangle = \eta^2 \alpha^* \alpha / B$$

$$B = \kappa^4 + \kappa^2 \left(\zeta_1^2 + \zeta_2^2 + 2\alpha^* \alpha \right) + \left(\zeta_1 \zeta_2 - \alpha^* \alpha \right)^2$$

$$\alpha = \Omega \sum_i^K A_i \langle S_i^- \rangle, \qquad \Delta_{lp} = \omega_l - \omega_{1p}$$

$$\zeta_l = \Delta_{lp} + \delta_l \sum_i^K \langle n_{il} \rangle$$

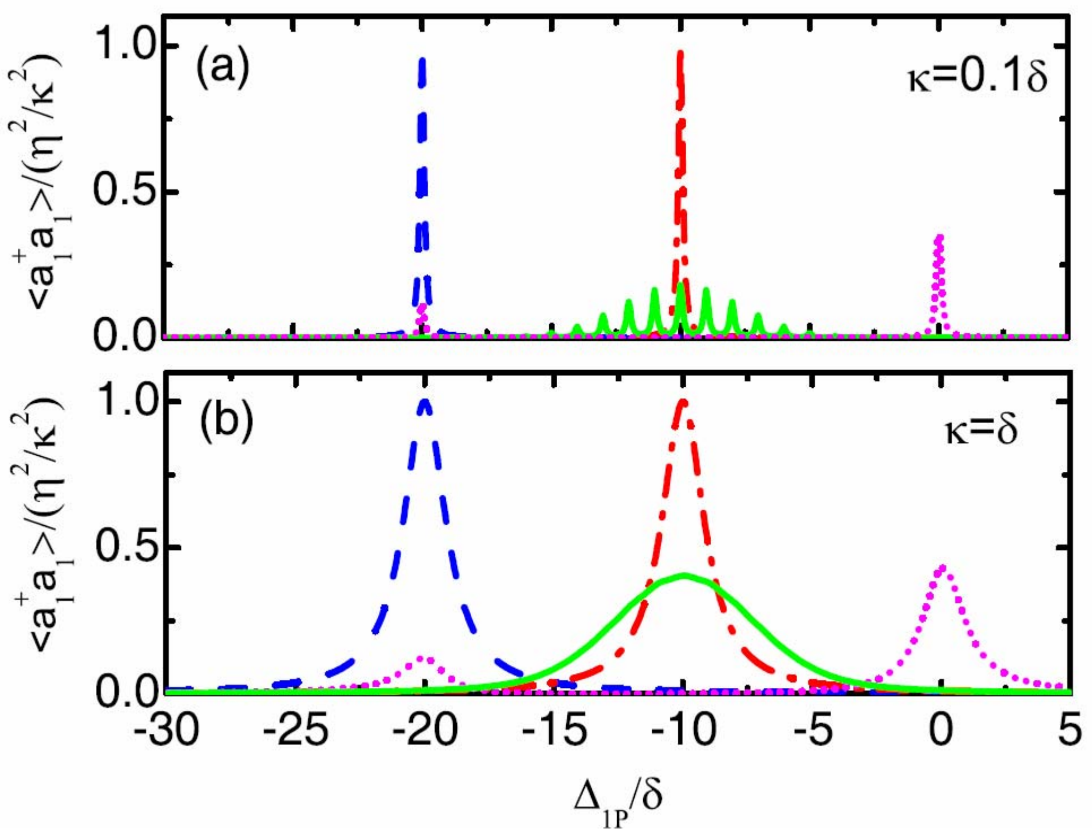
Coherent Pumping

AF

FM

XY

SF



Good cavity

Bad cavity

Cavity 1 photon numbers as a function of cavity-probe detuning

Summary

- Two examples of Strongly Correlated Quantum States in Ultracold Atoms
- Novel Detection Methods
- Cavity-enhanced Detection of Spin Orders in 2-component Lattice
- Application in Other Exotic Quantum Phases


THANK YOU